EE 470 – Extra Practice Problem Set #3

1. The CT signal $x(t) = \sin(400\pi t) + 2\cos(150\pi t)$ is sampled with an ideal impulse train. Sketch the CTFT of the sampled signal for the following values of the sampling rate:
   (a) $f_s = 100$ samples/s;
   (b) $f_s = 200$ samples/s;
   (c) $f_s = 400$ samples/s;
   (d) $f_s = 500$ samples/s.
   In each case, calculate the reconstructed signal using an ideal LPF with the transfer function given in Eq. (9.7) and a cut-off frequency of $\omega_s/2 = \pi f_s$.

2. A CT band-limited signal $x(t)$ is sampled at its Nyquist rate f_s and transmitted over a band-limited channel modeled with the transfer function

$$H_{ch}(\omega) = \begin{cases} 1 & 4\pi f_s \le |\omega| \le 8\pi f_s \\ 0 & \text{otherwise.} \end{cases}$$

   Let the signal received at the end of the channel be $x_{ch}(t)$. Determine the reconstruction system that recovers the CT signal $x(t)$ from $x_{ch}(t)$.

3. Consider a digital mp3 player that has 1024×10^6 bytes of memory. Assume that the audio clips stored in the player have an average duration of five minutes.
   (a) Assuming a sampling rate of 44 100 samples/s and 16 bits/sample/channel quantization, determine the average storage space required (without any form of compression) to store a stereo (i.e. two-channel) audio clip.
   (b) Assume that the audio clips are stored in the mp3 format, which reduces the audio file size to roughly one-eighth of its original size. Calculate the storage space required to store an mp3-compressed audio clip.
   (c) How many mp3-compressed audio files can be stored in the mp3 player?

4. Consider the input sequence $x[k]=2u[k]$ applied to a DT system modeled with the following input–output relationship:

$$y[k+1]-2y[k]=x[k],$$

and ancillary condition $y[-1]=2$.

(a) Determine the response $y[k]$ by iterating the difference equation for $0 \le k \le 5$.

(b) Determine the zero-state response $y_{zi}[k]$ for $0 \le k \le 5$.

(c) Calculate the zero-input response $y_{zs}[k]$ for $0 \le k \le 5$.

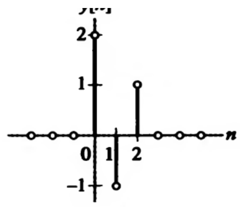
(d) Verify that $y[k]=y_{zi}[k]+y_{zs}[k]$.

5. **A discrete-time system is both linear and time invariant. Suppose the output due to an input $x[n]=\delta[n]$ is given in Fig. P1.77(a).**

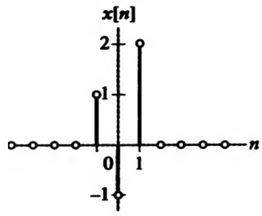
(a) Find the output due to an input $x[n]=\delta[n-1]$.

(b) Find the output due to an input $x[n]=2\delta[n]-\delta[n-2]$.

(c) Find the output due to the input depicted in Fig. P1.77(b).



(a)



(b)

6. For each of the following impulse responses, determine whether the corresponding system is causal.

$h[n]=(-1)^n u[-n]$

$h[n]=(1/2)^{|n|}$

$h[n]=\cos\left(\frac{\pi}{8}n\right)\{u[n]-u[n-10]\}$

$h[n]=2u[n]-2u[n-5]$

$h[n]=\sin\left(\frac{\pi}{2}n\right)$

$h[n]=\sum_{p=-1}^{\infty}\delta[n-2p]$

7. Determine the output of the systems described by the following difference equations with input and initial conditions as specified:

(a) $y[n]-\frac{1}{2}y[n-1]=2x[n]$,
$y[-1]=3, x[n]=\left(\frac{-1}{2}\right)^n u[n]$

(b) $y[n]-\frac{1}{9}y[n-2]=x[n-1]$,
$y[-1]=1, y[-2]=0, x[n]=u[n]$

(c) $y[n]+\frac{1}{4}y[n-1]-\frac{1}{8}y[n-2]=x[n]+x[n-1]$,
$y[-1]=4, y[-2]=-2, x[n]=(-1)^n u[n]$

(d) $y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n]$,
$y[-1]=1, y[-2]=-1, x[n]=2u[n]$

8. The systems that follow have input x[n] and output y[n]. For each system, determine whether it is (i) linear, (ii) time-invariant, (iii) causal:

$y[n] = 2x[n]u[n]$

$y[n] = \log_{10}(|x[n]|)$

$y[n] = \sum_{k=-\infty}^{n} x[k+2]$

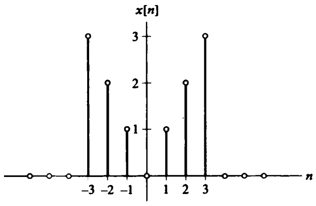
$y[n] = \cos(2\pi x[n+1]) + x[n]$

$y[n] = x[n]\sum_{k=-\infty}^{\infty} \delta[n-2k]$

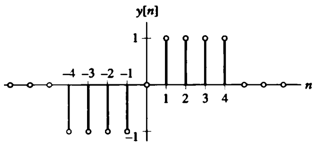
$y[n] = 2x[2^n]$

9. Given x[n] and y[n] as shown, sketch the following signals:

(a) $x[2n]$
(b) $x[3n-1]$
(c) $y[1-n]$
(d) $y[2-2n]$
(e) $x[n-2] + y[n+2]$
(f) $x[2n] + y[n-4]$
(g) $x[n+2]y[n-2]$
(h) $x[3-n]y[n]$
(i) $x[-n]y[-n]$
(j) $x[n]y[-2-n]$
(k) $x[n+2]y[6-n]$



(a)



(b)

10. Determine whether the following signals are periodic, and for those which are, find the fundamental period:

$x[n] = \cos\left(\frac{8}{15}\pi n\right)$

$x[n] = \cos\left(\frac{7}{15}\pi n\right)$

$x[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-3k] + \delta[n-k^2]\}$

$x[n] = \cos\left(\frac{1}{5}\pi n\right)\sin\left(\frac{1}{3}\pi n\right)$

$x[n] = (-1)^n$

$x[n] = (-1)^{n^2}$

$x[n] = \cos(2n)$

$x[n] = \cos(2\pi n)$