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OPTIMAL RADIOFREQUENCY PULSE DESIGN FOR MAGNETIC RESONANCE IMAGING USING GENETIC ALGORITHMS

Mohamed A. Alolfe, Abou-Bakar M. Youssef, and Yasser M. Kadah
System & Biomedical Engineering Department, Cairo University, Egypt

Abstract- The proper design of RF pulses in magnetic resonance imaging (MRI) has a direct impact on the quality of acquired images, and is needed for many applications. Several techniques have been proposed to obtain the RF pulse envelope given the desired slice profile. Unfortunately, these techniques do not take into account the limitations of practical implementation such as limited amplitude resolution. Moreover, implementing constraints for special RF pulses on most techniques are not possible. In this work, an approach is proposed to develop for designing optimal RF pulses under theoretically any constraints. The new technique poses the RF pulse design problem as a combinatorial optimization problem and uses efficient techniques from this area such as genetic algorithms (GA) to solve this problem. In particular, an objective function is proposed as the norm of the difference between the desired profile and the one obtained from solving the Bloch equations for the current RF pulse design values. The proposed approach is verified using analytical solution based RF simulations and compared to previous methods such as Shinnar-Le Roux (SLR) method, and analysis, selected, and tested the options and parameters that control the Genetic Algorithm (GA) can significantly affect its performance to get the best improved results. The results show a significant improvement over conventional design techniques, and suggest the practicality of using of the new technique for most important applications as slice selection for large flip angles, in the area of unconventional spatial encoding, and another clinical use.

I. INTRODUCTION

Magnetic resonance imaging (MRI) is an imaging technique using primarily in medical setting to produce high quality images of the inside of the human body. It offers true volumetric acquisition, ability to visualize and quantify flow, and spectroscopic imaging to image both anatomy and function. This technique relies on collecting the signal from an excited slice or volume within the human body. This excitation is achieved using special RF pulses that are designed to provide the required localization within the imaged volume. The proper design of RF pulses is important to avoid artifacts such as cross-talk in the acquired images.

The design of RF pulses is a rather difficult problem mathematically [1]. The basic goal of this design is to enable the desired slice profile to be achieved using the available RF pulse generation hardware. The practical implementation of RF pulse systems consists of a computer that stores the array of digital values representing the RF pulse envelope amplitudes. These values are converted using a digital-to-analog converter (DAC) into actual voltage levels. This DAC has a limited resolution in both amplitude and time. As a result, the generated envelope voltages appear like a piecewise constant curve with a fixed time step and limited stepwise amplitudes. These levels modulate the amplitude of the output of an RF generator before applying this output to the RF coils. The RF coils may be either linear (i.e., allowing only the real component to be applied) or quadrature (i.e., allowing both the real and imaginary components to be used). The actual frequency of the RF generator and the applied slice selection magnetic field gradient determine the position of excited slice. On the other hand, the amplitudes of the RF pulse envelope points determine the shape of the excitation profile as well as its flip angle.

Development of such pulses has been a topic of interest to researchers for over 20 years [2] and many methods have been developed. Hoult showed that at small tip angles (30 or less), the characteristic Bloch equations are nearly linear in nature [3]. As a result, assuming linear response in the spectrometer transmitter chain, the Fourier transform (FT) of the desired magnetization profile is a reasonable choice for the RF waveform. Pulses derived in this manner can produce acceptable results at larger tip angles, but as the tip angle

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approaches and exceeds 90, the Bloch equations become increasingly nonlinear, with significant distortion in both phase and magnitude [4].

Researchers have extensively investigated both direct and inverse approaches to RF pulse optimization. Hoult [3] and Mansfield *et al.* [6] were pioneers in the direct solution of the Bloch equations for shaped pulse design, whose work was verified numerically by Locher [7]. Techniques for analytical inversion of the Bloch equations were also proposed by Caprihan [8] and Silver *et al.* [9]. Caprihan concluded that numerical methods are the key to effective selective pulse synthesis. Conolly *et al.* [10] and Murdoch *et al.* [11] explored optimal control adaptations to RF pulse refinement. This work was reinforced by Ngo and Morris [12], who showed that the Bloch equations respond linearly to large excitations if they are treated as a superposition of perturbations of small tip angle. Among the many design methodologies that have been proposed in the last decade, the Shinnar-Le Roux (SLR) method is probably the most widely used [13]. This method works by transforming the problem into one of designing a finite impulse response (FIR) filter. The solution of this problem is obtained as an FIR filter coefficients and subsequently transformed back into the desired RF pulse envelope.

Given the extensive literature on FIR filter design, the strength of the SLR technique is that it taps into some of the most powerful FIR filter design techniques to solve the original problem of RF pulse design [13]. Nevertheless, it still has some limitations that did not enable its performance to be optimal in practice. In particular, this technique does not take into consideration the limited amplitude resolution in designing the RF filter. As it is well known in FIR filter design literature, the limited precision (i.e., quantization or limited word length effects) in implementing the digital filter may substantially deteriorate its performance [14]. Even if the FIR filter design technique is modified to take care of this problem and provides optimal filter coefficients for a given precision, there is no guarantee that the backward transformation to RF pulse coefficients would preserve this property for RF pulse coefficients. In other words, the characteristics of the SLR transformation do not allow such constraints to be imposed. In fact, it is generally difficult to impose any type of constraints on the solution (like for example adiabatic constraints). As a result, the obtained design may in fact be suboptimal in many cases that are common in practical use. An example where difficulty to obtain accurate slice profiles is reported is the use of unconventional spatial encoding techniques such as wavelet encoding and pseudo-Fourier imaging [15]. The implementation of such techniques had to compromise between the need to use low flip angles to obtain accurate slice profiles for correct encoding and the need for high flip angles for better signal-to-noise ratio. Therefore, a new RF pulse design technique that can incorporate practical constraints thus offering a true optimal performance under the practical implementation constraints would be rather helpful to solve these problems.

In this work, the problem of RF pulse design is formulated as a combinatorial optimization problem with an arbitrary number of constraints and uses efficient techniques from this area such as genetic algorithms (GA) to solve this problem. This formulation takes into account the limited precision of RF pulse generation and provides the optimal results at any given precision. Unlike SLR technique, the objective function will be proposed as the norm of the difference between the desired profile and the one obtained from solving the Bloch equations for the current RF pulse design values, which offers a feedback loop to improve the results. The proposed approach will be verified using analytical solution based RF simulations and compared to previous methods such as Shinnar-Le Roux (SLR) method, and analysis, selected, and tested the options and parameters of genetic algorithms to get the best improved results. The detailed implementation is provided details for each technique and present their results compared to those of the SLR technique.

II. THEORY

Given the definition of the RF pulse, it is possible to compute the expected slice profile using the solution to the Bloch equations. This solution relies on using the analytical form for the slice profile from a single rectangular pulse of arbitrary magnitude [1]. In traditional practical cases, the gradient is kept constant and the relaxation times (T_1&T_2) are neglected. The Bloch equations relate magnetization values with these assumptions is given by:

$$\frac{d}{dt}\begin{bmatrix} Mx \\ My \\ Mz \end{bmatrix} = \gamma \cdot \begin{bmatrix} 0 & G.z & -By \\ -G.z & 0 & Bx \\ By & -Bx & 0 \end{bmatrix}\begin{bmatrix} Mx \\ My \\ Mz \end{bmatrix} = A\vec{M} \tag{1}$$

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Here, Bx and By are the two quadrature components of the RF pulse, γ is the gyromagnetic ratio, and $G.z$ is the slice selection gradient. The gradient is assumed to lie along the $+z$ axis. For a rectangular pulse, the solution can be simply computed as,

$$\vec{M}(z) = \vec{M}_0(z).Exp(-A.t),\tag{2}$$

Where t is the duration of the rectangular RF pulse and the matrix exponent can be computed analytically for this problem as in [1].

Keeping in mind the practical implementation of RF pulses in the form of piecewise constant envelope pulses (i.e., a sequence of rectangular pulses of arbitrary amplitudes), the output magnetization from one piece serves as the initial condition for the next. Hence, given any design for the RF pulse, the slice profile can be computed this method. Given that the amplitudes of the RF pulses must be represented within a certain number of bits, the problem now becomes the one of finding the optimal combination of amplitudes that would give a slice profile closest to the desired. This problem description shows that this problem is indeed a combinatorial optimization problem. Using the rich literature of this area, the solution can be obtained efficiently and accurately. In this work, the one of the most prominent techniques is explored in this area, namely, genetic algorithm (GA).

III. METHODS

The one of the most prominent techniques was used in the area of a combinatorial optimization problem, namely, genetic algorithm (GA). The Genetic algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the "fitness" (i.e., minimizes the cost function) [16]. GA accommodate all the facets of soft computing, namely uncertainty, imprecision, non-linearity, and robustness.

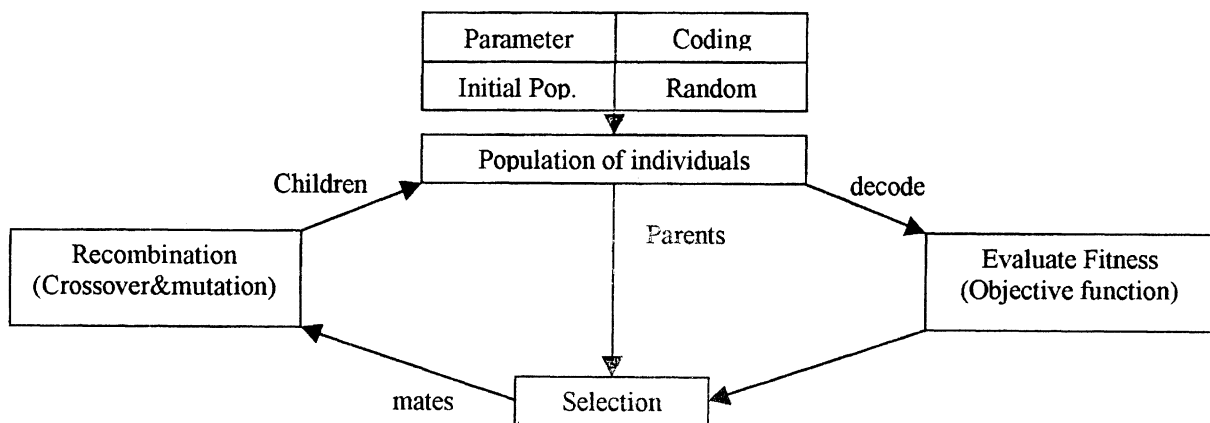The Figure (1) summarizes how the genetic algorithm works:

Figure 1: Computational Flows in Genetic Algorithms

A solution is encoded as a string of genes to form a chromosome representing an individual. In many applications the gene values are [0, 1] and the chromosomes are simply bit strings. An objective function, f, is supplied which can decode the chromosome and assign a fitness value to the individual the chromosome represents.

Given a population of chromosomes the genetic operator's crossover and mutation can be applied in order to propagate variation within the population. Crossover takes two parent chromosomes, cuts them at some random gene/bit position and recombines the opposing sections to create two children. Mutation is a background operator, which selects a gene at random on a given individual and mutates the value for that gene (for bit strings the bit is complemented).

The use of GA is robust in that they are not affected by spurious local optima in the solution space. Nevertheless, the parameters that control the GA can significantly affect its performance, and there is no

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guidance in theory as to how properly select them but in this work, the options and parameters of genetic algorithms are selected, and tested to get the best improved results and compared to previous works in this field. The most important parameters are the population size, the fitness scaling, the selection function, the crossover function, and the mutation function. Details of the proposed algorithm are as follows:

1) *Population of the chromosomes*: Population represents the size of the solutions that we are working with and specifies how many individuals there are in each generation. With a large population size, the genetic algorithm searches the solution space more thoroughly, thereby reducing the chance that the algorithm will return a local minimum that is not a global minimum. However, a large population size also causes the algorithm to run more slowly.

2) *Population Initialization*: The populations of chromosomes are initialized randomly by the GA. A chromosome that represents the SLR solution is added to the initial population to produce the initial population. This means that we start with the best solution in the literature and try to get a more optimal solution that produces a better slice profiles.

3) *Chromosome structure*: Binary chromosome is used. The RF pulse is encoded into the chromosome as follows. The real RF pulse values are converted into discrete ones according to the resolution of the D/A of the MRI machine (12 or 16 bit for example). Every bit represents a gene. The most significant bits of all values are placed adjacent to each other, then the second most bits and so on until placing the least significant bits together at the end. Hence, the chromosome size equals the number of the RF pulse envelope values times the bit resolution (usually 12 or 16).

4) *Fitness criterion*: The chromosome is decoded to obtain the corresponding RF pulse envelope amplitudes and its slice profile is computed by solving the Bloch equations. Then, an error measure is calculated for the difference between the response of this RF pulse and the desired response. This measure is usually taken as either the 1-norm or 2- norm of the difference vector. The results of this paper were obtained using the 1-norm. The evaluation function is used:

$$F = |D_x - M_x| + |D_y - M_y| + |D_z - M_z| \tag{3}$$

5) *Fitness scaling:* Fitness scaling converts the raw fitness scores that are returned by the fitness function to values in a range that is suitable for the selection function. The range of the scaled values affects the performance of the genetic algorithm. If the scaled values vary too widely, the individuals with the highest scaled values reproduce too rapidly, taking over the population gene pool too quickly, and preventing the genetic algorithm from searching other areas of the solution space. On the other hand, if the scaled values vary only a little, all individuals have approximately the same chance of reproduction and the search will progress very slowly. Options for fitness scaling in the GA is specified :

- *Rank:* Rank, scales the raw scores based on the rank of each individual instead of its score. The rank of an individual is its position in the sorted scores. The rank of the most fit individual is 1, the next most fit is 2, and so on. Rank fitness scaling removes the effect of the spread of the raw scores.

- *Proportional:* Proportional scaling makes the scaled value of an individual proportional to its raw fitness score.

- *Top:* Top, scales the top individuals equally. Selecting Top displays an additional field, Quantity, which specifies the number of individuals that are assigned positive scaled values. Quantity can be an integer between 1 and the population size or a fraction between 0 and 1 specifying a fraction of the population size.

- *Shift linear*: Shift linear, scales the raw scores so that the expectation of the fittest individual is equal to a constant multiplied by the average score. You specify the constant in the Max survival rate field, which is displayed when you select Shift linear.

6) *Selection:* Selection options specify how the genetic algorithm chooses parents for the next generation. You can specify options for selection in the GA:

- *Stochastic uniform*: stochastic uniform, lays out a line in which each parent corresponds to a section of the line of length proportional to its scaled value. The algorithm moves along the line in steps of equal size. At each step, the algorithm allocates a parent from the section it lands on. The first step is a uniform random number less than the step size.

- *Remainder:* Remainder, assigns parents deterministically from the integer part of each individual's scaled value and then uses roulette selection on the remaining fractional part.

- *Roulette:* Roulette, chooses parents by simulating a roulette wheel, in which the area of the section of the wheel corresponding to an individual is proportional to the individual's expectation. The algorithm uses a random number to select one of the sections with a probability equal to its area.

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- *Tournament:* Tournament selection chooses each parent by choosing Tournament size players at random and then choosing the best individual out of that set to be a parent. Tournament size must be at least 2.

7) *Reproduction:* Reproduction options specify how the genetic algorithm creates children for the next generation. You can specify options for reproduction in the GA:

- *Elite count:* specifies the number of individuals that are guaranteed to survive to the next generation. Set Elite count to be a positive integer less than or equal to the population size. The default value is 2.

- *Crossover fraction:* specifies the fraction of the next generation, other than elite children, that are produced by crossover. Set Crossover fraction to be a fraction between 0 and 1. The default value is 0.8.

8) *Crossover:* Crossover options specify how the genetic algorithm combines two individuals, or parents, to form a crossover child for the next generation. You can specify options for crossover in the GA:

- *Scattered:* creates a random binary vector and selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form the child.

- *Single point:* chooses a random integer n between 1 and Number of variables.

- *Two point:* Selects two random integers' m and n between 1 and Number of variables.

- *Intermediate:* Creates children by taking a weighted average of the parents. You can specify the weights by a single parameter, Ratio, which can be a scalar or a row vector of length Number of variables. The function creates the child from parent1 and parent2 using the following formula. *Child = parent1 + rand * Ratio * (parent2 - parent1).*

9) *Mutation:* Mutation options specify how the genetic algorithm makes small random changes in the individuals in the population to create mutation children. Mutation provides genetic diversity and enables the genetic algorithm to search a broader space. You can specify options for mutation in the GA:

- *Gaussian:* Gaussian, adds a random number taken from a Gaussian distribution with mean 0 to each entry of the parent vector. The variance of this distribution is determined by the parameters Scale and Shrink, and by the Initial range setting in the Population options.

- *Uniform:* Uniform mutation is a two-step process. First, the algorithm selects a fraction of the vector entries of an individual for mutation, where each entry has a probability Rate of being mutated. The default value of Rate is 0.01. In the second step, the algorithm replaces each selected entry by a random number selected uniformly from the range for that entry.

VI. RESULTS AND DISCUSSION

The proposed Genetic Algorithm method was applied to design RF pulses with rectangular spatial profile at $\pi/2$ and π flip angle. The outcome of the Genetic method is compared to the outcome of the SLR technique. Firstly to get the best results from the genetic algorithm, it is usually needed to experiment with different options of GA. Selecting the best options for a problem involves trial and error. 64 points for RF pulse design is used and applied Genetic Algorithm to design RF pulses with rectangular spatial profile at π flip angle. The following sections describe some ways you can change options to improve results.

1) Population Size and Initial Range

One of the most important factors that determine the performance of the genetic algorithm performs is the diversity of the population. If the average distance between individuals is large, the diversity is high; if the average distance is small, the diversity is low. Getting the right amount of diversity is a matter of trial and error. If the diversity is too high or too low, the genetic algorithm might not perform well. This section explains how to control diversity by setting the Initial range of the population, and also explains how to set the population size. The effects of the initial range and population size on the performance of the genetic algorithm are illustrated in Table (1).

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Table (1): Best values obtained for changing Population size and initial range

No.	Pop.Size	Norm-1 Errors(Best values)		
		Initial range [-0.0001,0.0001]	Initial range [-0.01,0.01]	Initial range [-0.1,0.1]
1.	16	4.9923	5.0223	8.557
2.	32	4.9256	4.8973	5.1912
3.	64	4.9951	4.9348	4.8993
4.	128	4.986	5.026	5.0184
5.	256	4.9337	4.935	6.6971

2) The fitness scaling

The range of the scaled values affects the performance of the genetic algorithm. If the scaled values vary too widely, the individuals with the highest scaled values reproduce too rapidly, taking over the population gene pool too quickly, and preventing the genetic algorithm from searching other areas of the solution space. On the other hand, if the scaled values vary only a little, all individuals have approximately the same chance of reproduction and the search will progress very slowly. The effects of the fitness scaling on the performance of the genetic algorithm are illustrated in Table (2), and Figure (2).

Table (2): Best values obtained for changing the Fitness function

No.	Scaling Function	Norm-1 Errors (Best Values)
1.	Rank	4.8973
2.	Proportional	4.9541
3.	Top	4.9981
4.	Shift Linear	4.927

3) The function of selection

The effects of changing the selection function on the performance of the genetic algorithm are illustrated in Table (3), and Figure (3).

Table (3): Best values obtained for changing the Selection function

No.	Selection Function	Norm-1 Errors (Best Values)
1.	Stochastic uniform	4.8973
2.	Remainder	5.0248
3.	Roulette	5.0006
4.	Tournament	4.9209

4) The function of crossover

The effects of changing the crossover function on the performance of the genetic algorithm are illustrated in Table (4), and Figure (4).

Table (4): Best values obtained for changing the Crossover function

No.	Crossover Function	Norm-1 Errors (Best Values)
1.	Scattered	4.8973
2.	Single point	5.0653
3.	Two point	4.9948
4.	Intermediate	5.2007

5) The function of mutation

The effects of changing the mutation function on the performance of the genetic algorithm are illustrated in Table (5), and Figure (5).

Table (5): Best values obtained for changing the mutation function

No.	Mutation Function	Norm-1 Errors (Best Values)
1.	Gaussian	4.8973
2.	Uniform	5.05

Secondly, after the test options of GA to get the best results, we are applied Genetic Algorithm with the best options to design two RF pulses with rectangular spatial profiles at $\pi/2$ and π flip angle. The outcome of the Genetic Algorithm is compared to the outcome of the SLR technique, and compared the outcome of this work (used the best options for GA selected from above) to outcome of previous work (used the following options for

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GA: proportional scaling function, Roulette selection function, single point crossover function, probability of crossover is taken as 90%, uniform mutation function, and probability of mutation is taken as 1%). The results are shown in Table (6) and Figures (6), and (7). As can be observed, the GA technique show a significant improvement over the design computed using the SLR technique. Also, the outcome of this work appears better and faster to reach the solution than that from previous work in the number of experiments we performed.

Table (6): Norm-1 Error resulting for $\pi/2$ and π flip angle

No.	Flip angle	Norm-1 Errors for 128 points			Norm-1 Errors for 256 points		
		SLR work	previous work	Our work	SLR work	previous work	Our work
1.	$\pi/2$	2.787	1.304	1.2698	2.5955	1.0979	0.88545
2.	π	5.6079	3.9839	3.8768	3.5083	3.0929	2.7724

One of the most important applications for the proposed technique is in the area of unconventional spatial encoding where complex RF pulse are to be generate at high accuracy. The proposed technique is expected to enable better reconstruction accuracy, less image artifacts, and higher signal-to-noise ratio. The study of this area requires the investigation of several of these techniques and the assessment of the results and is left to future work.

V. CONCLUSIONS

In this paper, a new optimized RF pulse design in MRI using the GA is presented. The new technique relies on posing the problem as a combinatorial optimization problem and uses GA to compute the solution under any type of constraints, and select the best options for parameters of GA to get the more improvement results. The results demonstrate the success of the new approach and suggest its potential for practical use in clinical magnetic resonance imaging.

REFERENCES

[1] E.T. Jaynes, "Matrix treatment of nuclear induction," *Phys. Rev.*, vol. 98, no. 4, pp. 1099-1105, May 1955.

[2] A. N. Garroway, P. K. Grannell, and P. Mansfield, "Image formation in NMR by a selective irradiative process," *J. Phys. C*, vol. 7, pp. L457–L462, 1974.

[3] D. I. Hoult, "The solution of the Bloch equations in the presence of a varying B field-an approach to selective pulse analysis," *J. Magn. Reson.*, vol. 35, pp. 69-86, 1979.

[4] Eliot T. Lebsack, Steven M. Wright, "Iterative RF Pulse Refinement for Magnetic Resonance Imaging," *IEEE Transactions on Biomedical Engineering*, vol. 49, no.1, pp. 41-48,January 2002.

[5] J. B. Weaver, Y. Xu, D. M. Healy, and J. R. Driscoll, "Wavelet-encoded MR imaging," *Magn. Reson. Med.*, vol. 24, pp. 275–287, 1992.

[6] P. Mansfield, A. A. Maudsley, P. G. Morris, and I. L. Pykett, "Selective pulses in NMR imaging: A reply to criticism," *J. Magn. Reson.*, vol. 33, pp. 261–274, 1979.

[7] P. R. Locher, "Computer simulation of selective excitation in n.m.r. imaging," *Phil. Trans. R. Soc. Lond.*, vol. B.289, pp. 537–542, 1980.

[8] A. Caprihan, "Effect of amplitude modulation on selective excitation in NMR imaging," *IEEE Trans. Med. Imag.*, vol. MI-4, pp. 169–175, 1983.

[9] M. S. Silver, R. I. Joseph, and D. I. Hoult, "Highly selective $\pi/2$ and π pulse generation," *J. Magn. Reson.*, vol. 59, pp. 347–351, 1984.

[10] S. Conolly, D. Nishimura, and A. Macovski, "Optimal control solutions to the magnetic resonance selective excitation problem," *IEEE Trans. Med. Imag.*, vol. MI-2, pp. 106–115, 1986.

[11] J. B. Murdoch, A. H. Lent, and M. K. Kritzer, "Computer-optimized narrowband pulses for multislice imaging," *J. Magn. Reson.*, vol. 74, pp. 226–263, 1987.

[12] J. T. Ngo and P. G. Morris, "General solution to the NMR excitation problem for no interacting spins," *Magn. Reson. Med.*, vol. 5, pp. 217–237, 1987.

[13] J. Pauly, P. Le Roux, D. Nishimura, and A. Macovski, "Parameter relations for the Shinnar-Le Roux selective excitation pulse design algorithm," *IEEE Trans. Med. Imag.*, vol. 10, no. 1, pp. 53-65, March 1991.

[14] C.S. Burrus, *et al*, *Computer-based Exercises for Signal Processing*, Prentice Hall, New Jersey, 1994.

[15] Y.M. Kadah and X. Hu, "Pseudo-Fourier imaging: A technique for spatial encoding in MRI," *IEEE Trans. Med. Imag.*, vol. 16, no. 6, pp.893-902, Dec. 1997.

[16] Randy L. Haupt, Sue Ellen Haupt, Practical Genetic Algorithm, John Wiley & Sons, 2004

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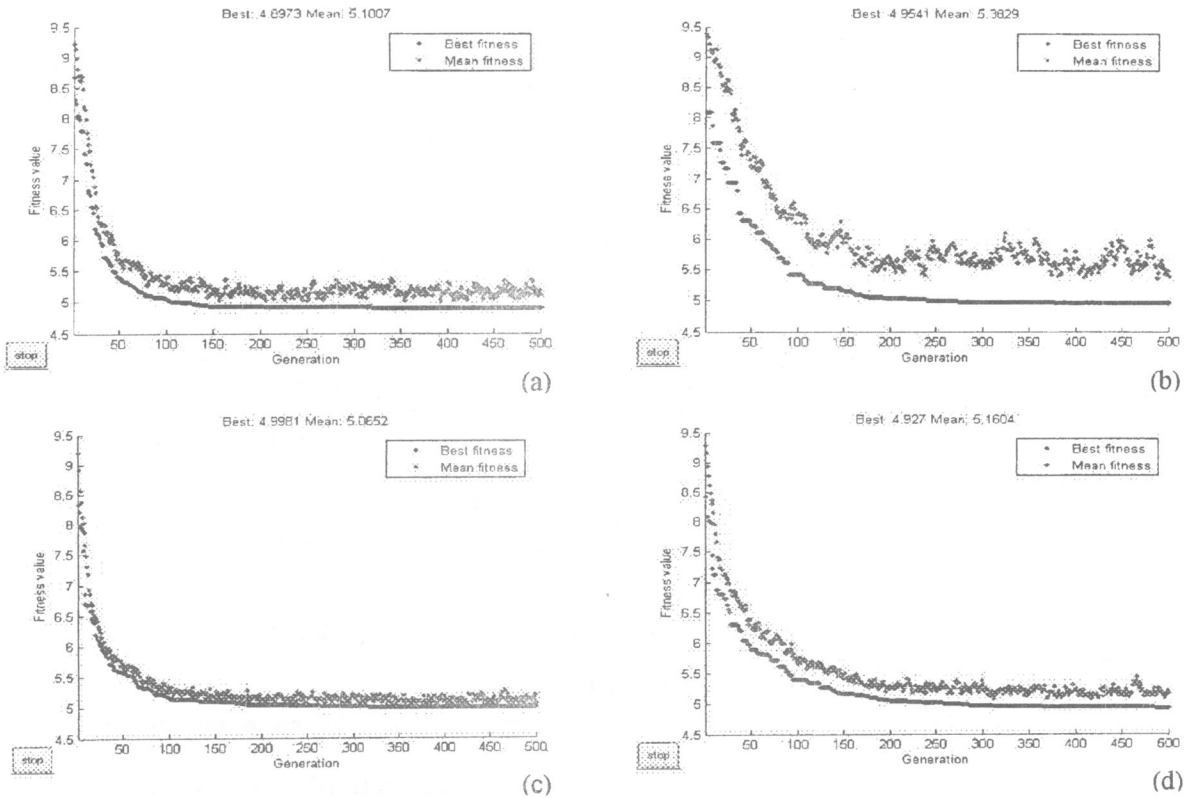Faculty of Electronic Engineering, Menoufiya University, Egypt.

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Figure (2): (a) Norm-1 Error with rank function, (b) Norm-1 Error with proportional function, (c) Norm-1 Error with top function, and (d) Norm-1 Error with shift linear function.
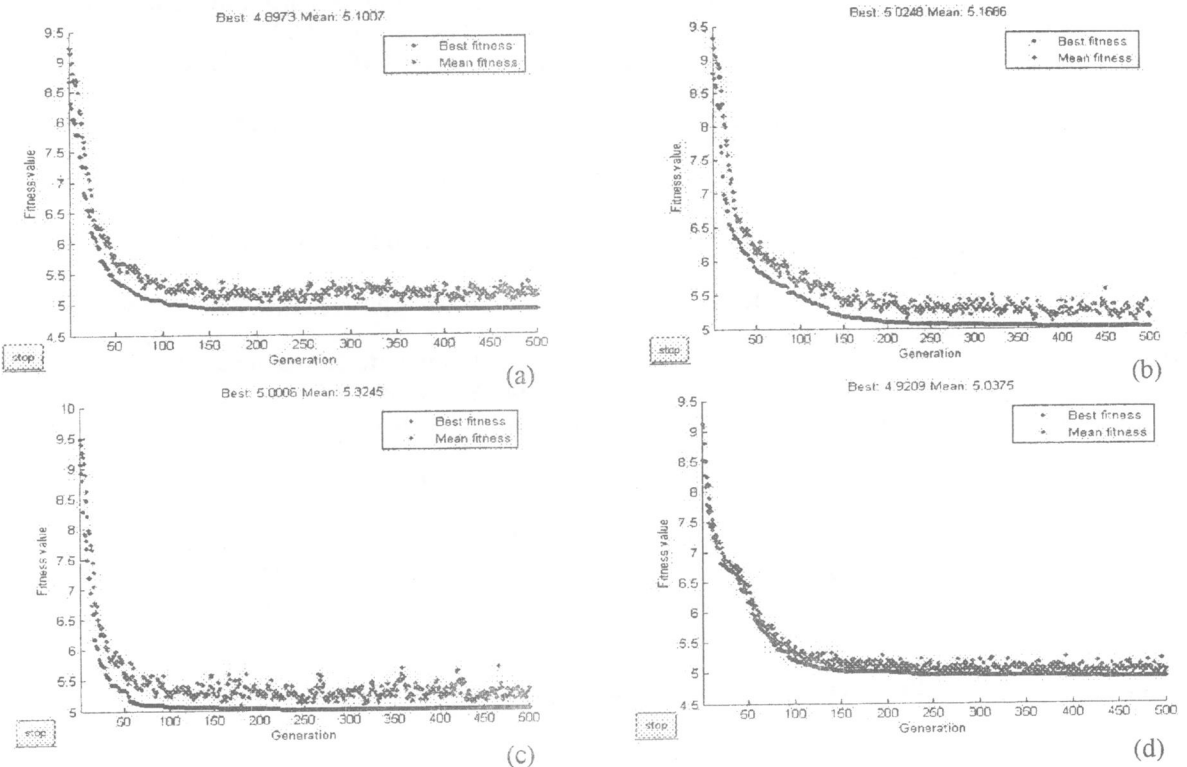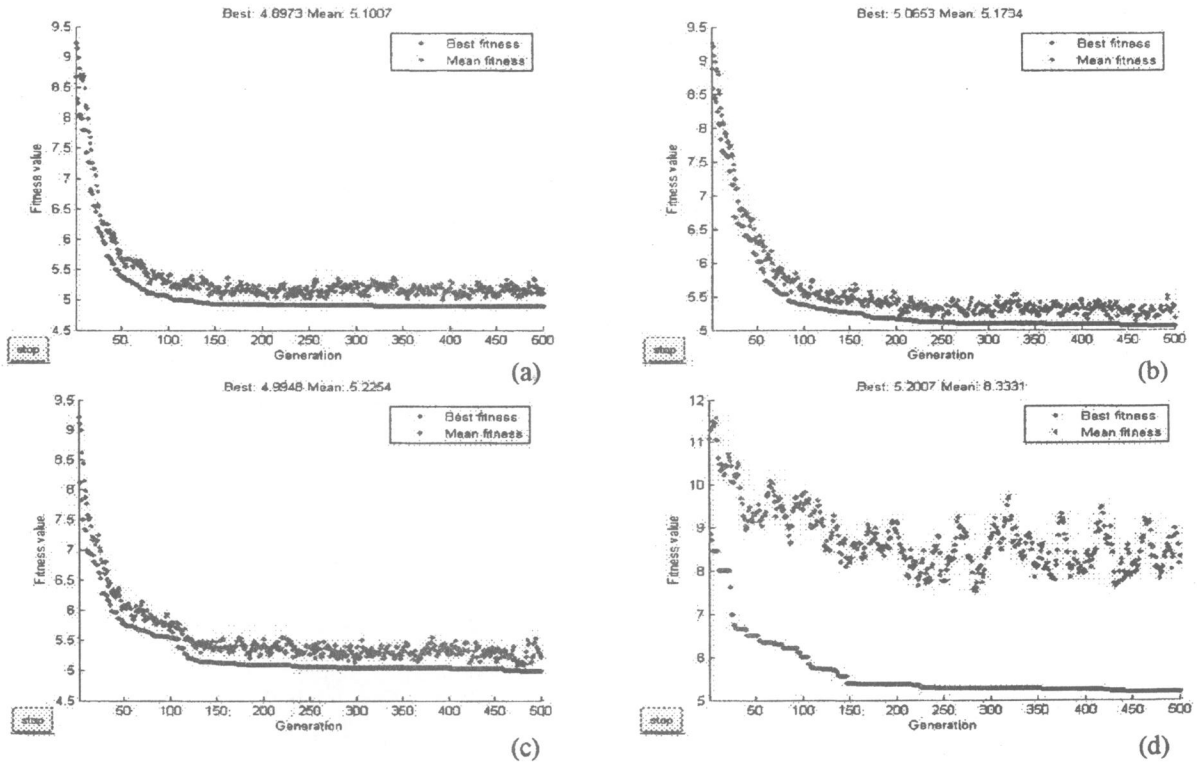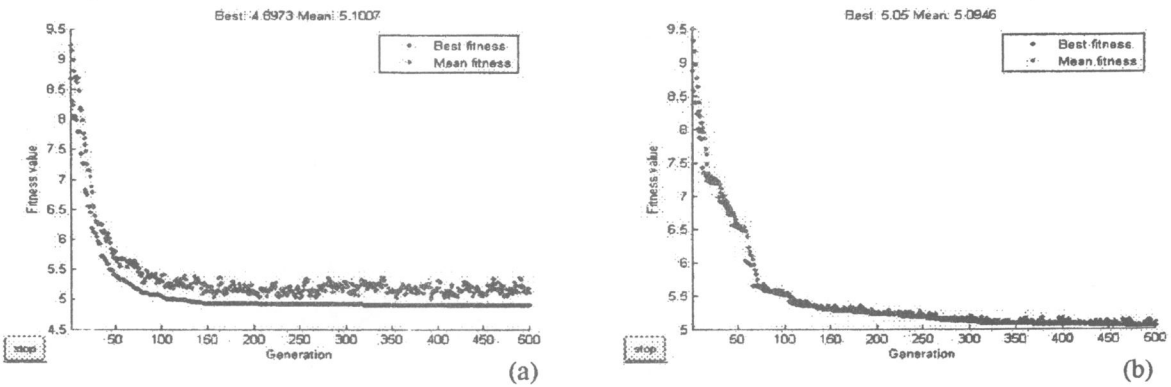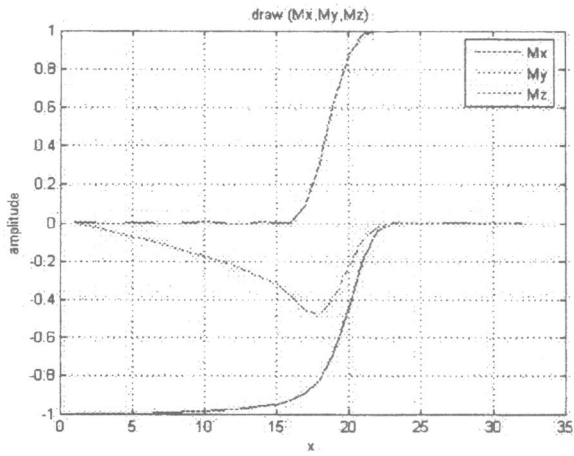

Figure (3): (a) Norm-1 Error with stochastic uniform function, (b) Norm-1 Error with remainder function, (c) Norm-1 Error with roulette function, and (d) Norm-1 Error with tournament function.

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Figure (4): (a) Norm-1 Error with scattered function, (b) Norm-1 Error with single point function, (c) Norm-1 Error with two point function, and (d) Norm-1 Error with Intermediate function.
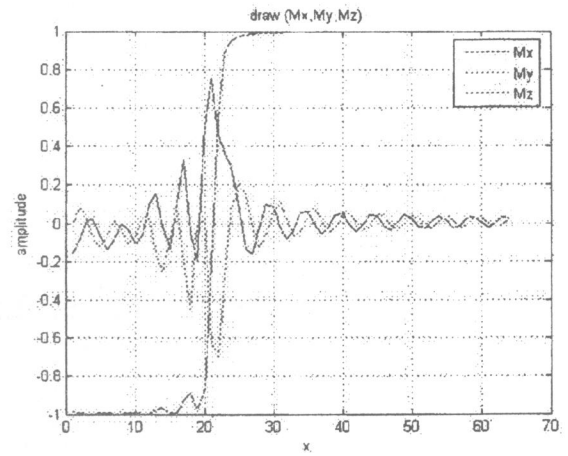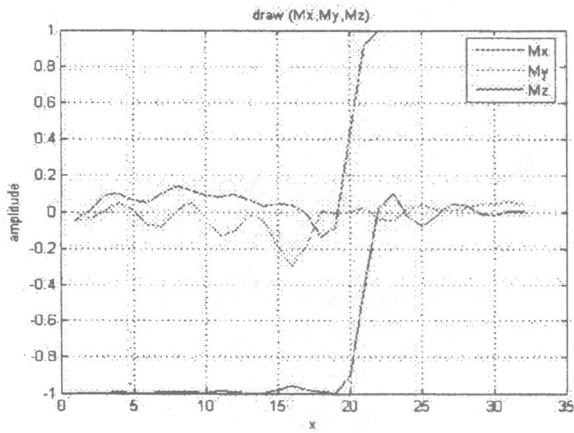
Figure (5): (a) Norm-1 Error with Gaussian function, (b) Norm-1 Error with uniform function.

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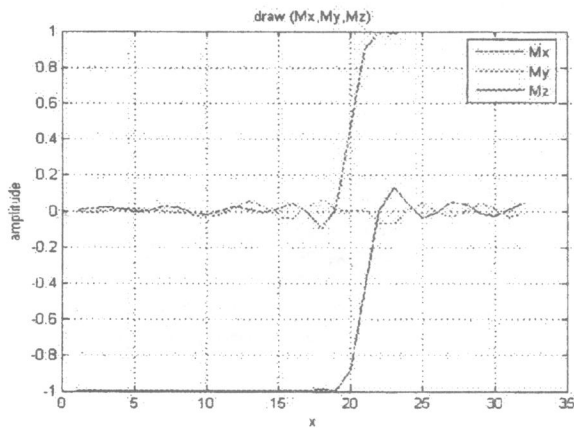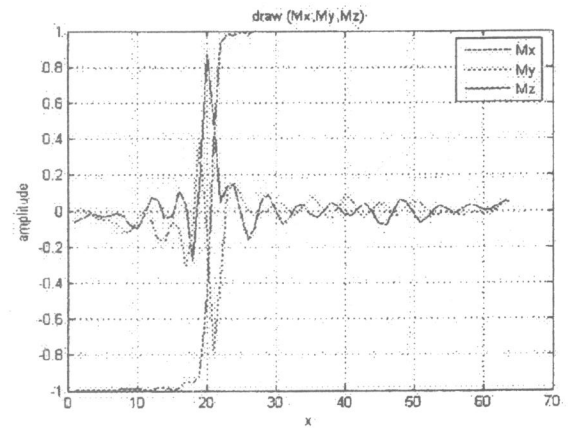
Figure (6): (a, b) slice profile from SLR before GA for $\pi/2$, π flip angles, and 256 points RF pulse, (c, d) slice profile from previous work after applied GA, and (e, f) slice profile from our work after applied GA
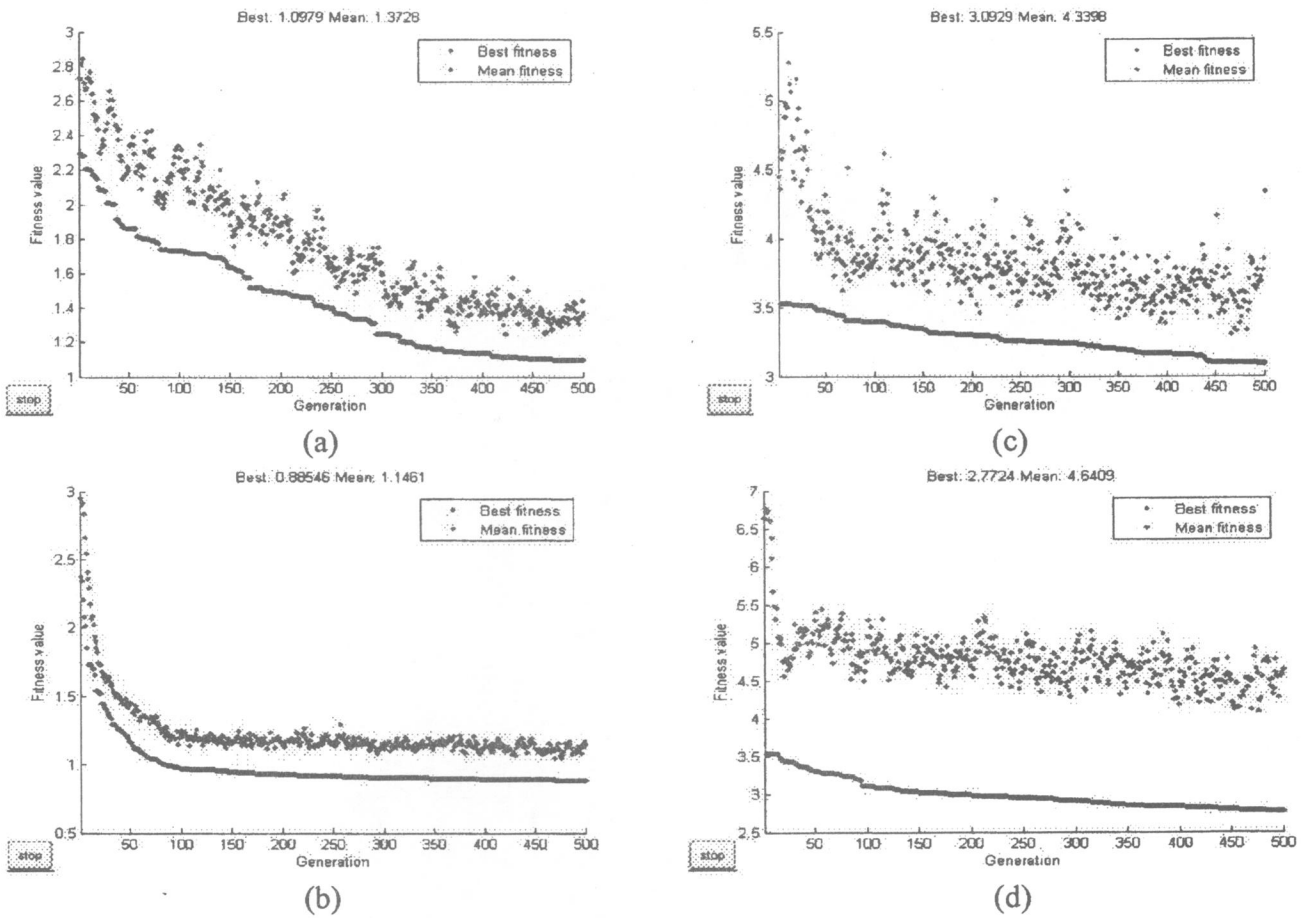
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Figure (7): (a) Norm-1 Error for π/2 flip angle and 256 points from (previous work), (b) Norm-1 Error for π/2 flip angle and 256 points from our work, (c) Norm-1 Error for π flip angle and 256 points from (previous work), and (d) Norm-1 Error for π flip angle and 256 points from our work.