INTERNATIONAL SYSTEM OF MEASUREMENT

EE 306

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International System of Units (SI)

- SI is based on the measures of six basic physical quantities:

Quantity	Symbol	Unit
Mass	m	kilogram (kg)
Length	l	meter (m)
Time	t	second (s)
Electric current	I	ampere (A)
Absolute temperature	θ	kelvin (K)
Luminous intensity	I_v	candela (cd)

- All other units are derived units from the above base units and are related to base units by their definition

SI Derived Units

Quantity	Symbol	Formula	SI Unit
Area	A	$A = l \times l$	m^2
Volume	V	$V = l \times l \times l$	m^3
Velocity	u	$u = l \,/\, t$	m/s
Acceleration	a	$a = du/dt$	m/s^2
Angular velocity	ω	$\omega = angle/\, t$	rad/s
Force (Weight)	F	$F = m \times a$	newton (N)
Work / Energy	W	$W = F \times l$ $= \frac{1}{2} m \times u^2$	joule (J)
Power	P	$P = W \,/\, t = F \times u$	watt (W)
Torque	T (or M)	$T = F \times l$	N . m

SI Prefixes

- Source: http://physics.nist.gov/cuu/Units/prefixes.html

Factor	Name	Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da

Factor	Name	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

Relevant Definitions

- Mass unit of megagram is also known as the tonne (t)
- Celsius scale of temperature: shifted version of kelvin (same change)

$$0\ °C = 273.15\ K$$

- Weight: Force on body due to its mass in presence of gravity
  - Weight (N) = Mass x g (gravitational acceleration = 9.81 m/s^2)
- Pressure = Force / Area (normal)   (N/m^2)
- Shear = Force / Area (tangent)   (N/m^2)
- Potential energy: energy acquired/lost when body moves in force field
  - Assuming body moving distance h against gravity, $W = m \cdot g \cdot h$
- Kinetic Energy

$$\text{Kinetic energy} = \tfrac{1}{2}mu^2 \text{ joules}$$

$$W = \tfrac{1}{2}mu^2$$

Relevant Definitions

- Power is the rate of doing work (SI unit of J/s or W)

$$P = \frac{W}{t} = \frac{F \cdot l}{t} = F \cdot \frac{l}{t}$$

$$P = Fu$$

- For rotating electrical machine:

$$P = M\omega = \frac{2\pi N_r M}{60}$$

Rotational speed Symbol: N_r Unit: revolution per minute (r/min)

Rotational speed Symbol: n_r Unit: revolution per second (r/s)

- Commercial energy unit: kWh
  - Work done by a source of power one kilowatt for period of one hour

$$1 \text{ kW h} = 1000 \times 3600 \text{ watt seconds or joules} = 3\,600\,000 \text{ J} = 3.6 \text{ MJ}$$

Efficiency (η)

- When a device converts or transforms energy, some of the input energy is consumed to make the device

$$\text{Efficiency} = \frac{\text{energy output in a given time}}{\text{energy input in the same time}} = \frac{W_o}{W_{in}}$$

$$= \frac{\text{power output}}{\text{power input}} = \frac{P_o}{P_{in}}$$

- Units of efficiency: None (dimensionless)

Example (H1.3)

A body having a mass of 30 kg is supported 50 m above the earth's surface. What is its potential energy relative to the ground?

If the body is allowed to fall freely, calculate its kinetic energy just before it touches the ground. Assume gravitational acceleration to be 9.81 m/s^2.

$$\text{Weight of body} = 30\,[\text{kg}] \times 9.81\,[\text{N/kg}] = 294.3\ \text{N}$$

$$\therefore \qquad \text{Potential energy} = 294.3\,[\text{N}] \times 50\,[\text{m}] = \mathbf{14\ 700\ J}$$

If u is the velocity of the body after it has fallen a distance l with an acceleration g

$$u = \sqrt{(2gl)} = \sqrt{(2 \times 9.81 \times 50)} = 31.32\ \text{m/s}$$

and

$$\text{Kinetic energy} = \tfrac{1}{2} \times 30\,[\text{kg}] \times (31.32)^2\,[\text{m/s}]^2 = \mathbf{14\ 700\ J}$$

Example (H1.6)

A generating station has a daily output of 280 MW h and uses 500 t (tonnes) of coal in the process. The coal releases 7 MJ/kg when burnt. Calculate the overall efficiency of the station.

Input energy per day is

$$W_{in} = 7 \times 10^6 \times 500 \times 1000$$

$$= 35.0 \times 10^{11}\ \text{J}$$

Output energy per day is

$$W_o = 280\ \text{MW h}$$

$$= 280 \times 10^6 \times 3.6 \times 10^3 = 10.1 \times 10^{11}\ \text{J}$$

$$\eta = \frac{W_o}{W_{in}} = \frac{10.1 \times 10^{11}}{35.0 \times 10^{11}} = 0.288$$

Example (H1.7)

A lift of 250 kg mass is raised with a velocity of 5 m/s. If the driving motor has an efficiency of 85 per cent, calculate the input power to the motor.

Weight of lift is

$$F = mg = 250 \times 9.81 = 2452 \text{ N}$$

Output power of motor is

$$P_o = Fu = 2452 \times 5 = 12\,260 \text{ W}$$

Input power to motor is

$$P_{in} = \frac{P_o}{\eta} = \frac{12\,260}{0.85} = 14\,450 \text{ W} = \mathbf{14.5\ kW}$$

Example (H1.8)

An electric heater contains 40 litres of water initially at a mean temperature of 15 °C; 2.5 kW h is supplied to the water by the heater. Assuming no heat losses, what is the final mean temperature of the water?

A useful constant to note is that it takes 4185 J to raise the temperature of 1 litre of water through 1 °C.

$$W_{in} = 2.5 \times 3.6 \times 10^6 = 9 \times 10^6 \text{ J}$$

Energy to raise temperature of 40 litres of water through 1 °C is

$$40 \times 4185 \text{ J}$$

Therefore change in temperature is

$$\Delta\theta = \frac{9 \times 10^6}{40 \times 4185} = 53.8 \text{ °C}$$

$$\theta_2 = \theta_1 + \Delta\theta = 15 + 53.8 = 68.8 \text{ °C}$$

Summary of Important Formulas

F [newtons] = m [kilograms] × a [metres per second squared] [1.1]

i.e. $F = ma$

Torque $T = Fr$ (newton metres) [1.3]

Work $W = Fl$ (joules) [1.4]

Work = Energy

Kinetic energy $W = \frac{1}{2}mu^2$ [1.5]

Power $P = Fu$ (watts) [1.7]

$= T\omega = M\omega = 2\pi nT$ [1.8]

Efficiency $\eta = P_o / P_{in}$ [1.9]

Summary of Terms and Concepts

Force, when applied to a body, causes the body to accelerate.

Weight is the gravitational force exerted by the earth on a body.

Torque, when applied to a body, causes the body to rotationally accelerate.

Energy is the capacity to do work. When selling energy, it is measured in kilowatt hours rather than joules.

Power is the rate of working.

Efficiency is the ratio of output power to input power. The difference between output and input is usually due to wastage.

Suggested Readings and Exercises

- Hughes textbook – Chapter 1
- Exercise 1 (Hughes)
  - Problems 6-10