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Orthogonal Matching Pursuit & Compressive Sampling Matching Pursuit for Doppler Ultrasound Signal Reconstruction

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***Abstract*—In this work we want to make use of a novel framework of compressed sensing (CS) sampling theory to reconstruct the Doppler ultrasound signal. CS aim to reconstruct signals and images from significantly fewer measurements. Doppler ultrasound is one of the most non-invasive diagnostic techniques. The present data acquisition methods use much data to acquire the image, this cause in increasing the process time and heating. To overcome this limitation we propose a framework of CS. The result shows that the reconstruction performed perfectly with high quality in very short time, by using two CS reconstruction algorithms, orthogonal matching pursuit and compressive sampling matching pursuit algorithms. There is no significant difference in the quality of the resulting images reconstructed by using both reconstruction algorithms.**

***Index Terms*—Compressed sensing; orthogonal matching pursuit; compressive sampling matching pursuit; Doppler ultrasound; reconstruction; sampling theory.**

I. Introduction

Doppler ultrasound is one of the most important non-invasive techniques for measuring and monitoring blood flow within the body. Doppler generates either continuous wave (CW) or pulsed wave (PW) ultrasound. During the acquisition of Doppler data a train of pulses transmitted repeatedly to be acquired from selected region of interest. In most cases of Doppler signal acquisition done in more than one mode, this leads to reduction the frame rates for other modes, this reduction limit the ability of follow event in real-time. Also, rapid transmitting of ultrasound pulses to the same location increase the average power per unit area.

Compressed sensing (CS) is a new sampling framework; state that images and signals can be reconstructed by using fewer numbers of measurements (what was previously believed to be highly incomplete measurements). CS was first proposed when Donoho published his paper [1] in 2006. CS is the process for acquiring and reconstructing a signal that is supposed to be sparse or compressible. CS is useful in applications where one cannot afford to collect or transmit a lot of measurements in applications such as medical imaging and data compression. There are rapidly growing application of CS in the field of medical imaging and image processing.

CS methods provide robust framework for reducing the number of measurements require to summarize the sparse signals [2, 3]. For this reason, CS methods are useful in areas where analog-to-digital costs are high. The signals acquired directly in CS without going through the intermediate stage of acquiring N samples.

The main problem in CS is to recover sparse signal θ from a set of measurements δ, assume that δ is a linear measurement given by $\delta = \mu\theta$; where θ is an *N-by-1*, δ is an *M-by-1* vector containing the compressive measurements and μ is an *M-by-N* measurement matrix. The most difficult part of signal reconstruction is to identify the location of the largest component in the target signal. The signal reconstruction algorithm must take the M measurement in the vector δ, where $N>>M$. Several methods for recovering sparse θ from a limited number of measurements have been proposed, such as convex optimization, combinatorial methods and greedy algorithms. In this work we want to use two types of Greedy algorithm to reconstruct the Doppler ultrasound signal, the algorithms are orthogonal matching pursuit (OMP) and compressive sampling matching pursuit (CoSaMP).

In this work we want to make use of novel sampling framework, compressive sensing for reconstructing Doppler ultrasound spectrum to reduce the number of acquisitions and eliminate the sampling randomly. Applying CS will reduce the acquisition samples, which lead to overcome the limitation of present Doppler acquisition. This algorithm successfully demonstrated by using real Doppler ultrasound data. The result shows that the CS sampling framework recovered Doppler spectrogram perfectly.

II. Reconstruction Algorithms

In this part we want to go through CS recovery algorithms in brief and we will concentrate on the algorithms used in this work. There are many approaches that have been proposed for reconstructing signals and images with compressive sensing theory [4, 5]. Most of the algorithms solve constrained optimization problem. Commonly used algorithms are based on convex optimization, greedy algorithm and combinatorial algorithms.

We will focus our attention on two algorithms of the greedy algorithm, OMP and CoSaMP. Greedy algorithms rely on iterative approximation of the signal coefficients and support, either by iteratively identifying the support of the signal until a convergence criterion is met, or alternatively by obtaining an improved estimate of the sparse signal at each iteration that attempts to account for the mismatch to the measured data.

A. *Orthogonal Matching Pursuit (OMP)*

Orthogonal matching pursuit (OMP) was proposed in [4], this algorithm combines the simplicity and the fastness for high-dimensional sparse signal recovery. Hence, it is easy to implement in practice. Tropp and Gilbert [5] proved that OMP can be used to recover a sparse signal with high probability. The algorithms begins by finding the column of a mostly related to the measurements and then repeated this step by correlating the columns with the signal residual, which is obtained by subtracting the contribution of a partial estimate of the signal from the original measurement vector.

Suppose that θ is an arbitrary *k*-spares in $\mathbb{R}^M$, and let $\{a_1,\ldots,a_N\}$ be a family of *N* measurement vectors. From an *N* x *M* matrix Φ whose rows are the measurement vectors, and observe that the *N* measurement of the signal can be collected in *N*-dimensional data vector.

$$\delta = \mu\theta \qquad (1)$$

We refer to μ as the measurement matrix and denote its columns by $a_1,\ldots,a_M$.

It is natural to think of signal recovery as a problem dual to sparse approximation. Since θ has only *k* nonzero components, the data vector (1) is a nonlinear computation of *k* columns from μ. In this language of approximation, we say θ has *k*-term representation over the dictionary μ.

Therefore, sparse approximation algorithms can be used for recovering sparse signal. To identify the ideal signal θ, we need to determine which columns of μ participate in measurement vector θ. The idea behind the algorithm is to pick a column in a greedy fashion. At each iteration, we chose the column of μ that is most strongly correlated with the remaining part of *x*. Then we subtract off it is a contribution to θ and iterate on the residual. After *k* iteration, the algorithms suppose to identify the correct set of columns.

B. *Compressive sampling matching pursuit (CoSaMP)*

CoSMP is one of the greedy pursuit recovery algorithm proposed in [6]. CoSaMP is an iterative recovery algorithm, recovers the signals using measurement matrices that satisfy the restricted isometry property (RIP). The algorithm fundamentally is based on OMP [5], also it constitute several ideas from the others algorithm to so as to a chive stronger guarantee. This algorithm is same as OMP, but does a limit search at each step, it adds more than one coordinate at a time, then it discards the least useful coordinates.

Suppose that μ is an *M* x *N* measurement matrix satisfies the RIP with constant $\lambda_{2k} \leq C$. The sample vector represented by $\delta = \mu\theta + \tau$, where τ is the error. For a given precision parameter Ω, the algorithm CoSaMP produce a *k*-sparse approximation ω that satisfies

$$\left\|\theta-\omega\right\|_2 \leq C.\max\{\eta, \frac{1}{\sqrt{k}}\left\|\theta-\theta_{k/2}\right\|_1 + \left\|\tau\right\|_2\} \qquad (2)$$

Where $\theta_{k/2}$ is a pest *k/2*-sparse approximation to θ.

CoSaMP use the largest coordinates, and approximate the signal at each iteration. After each new residual is formed, reflecting the missing portion of the signal, the measurements are updated. This is repeated until all the recoverable portion of the signal is found.

III. METHODS

Before applying the algorithms for reconstruction we first need to create the measurement matrix μ, later we create the sparse coefficients. We intend to reconstruct a vector θ, the Doppler ultrasound signal in our case, with fewer numbers of non-zero components, that is, with OMP and CoSaMP recovery algorithms.

In this part we want to show how to apply an orthogonal matching Pursuit algorithm and compressive sampling matching pursuit algorithm from sparse approximation to the Doppler ultrasound signal recovery problem. Doppler ultrasound signal was sampled randomly and constructed by using CS via OMP and CoSaMP algorithms to regenerate a reconstructed Doppler signal, which is used to generate a Doppler ultrasound spectrogram using much fewer numbers of measurements *M*.

The Doppler ultrasound data constructed using OMP, which begins by finding the column of μ most related to the measurements. Then the algorithm repeats this step by correlating the columns to the signal residual, which is obtained by subtracting the contribution of a partial estimate of the signal from the original measurement vector. The measurement model represented as: δ = μ θ.

Where μ is a measurement matrix in *N by M***,** δ is an *M*-dimensional and θ is a sparse signal with *k* nonzero.

The signal θ reconstructs by solving the relation (1), the resulting signal used to generate Doppler spectrogram.

For reconstructing Doppler ultrasound signals with compressive sampling matching pursuit algorithm. Doppler signal with a length of *N* was sampled randomly and constructed by CoSaMP using fewer numbers of measurements *M*. To reconstruct the signal as we mention before we select a measurement matrix μ randomly and then reconstruct the signal by solving the measurement vector δ. The measurement model represented as: $\delta = \mu\ \theta$.

Applying CoSaMP to reconstruct the Doppler data by solving the measurement vector (1), lead to a good approximation of Doppler signal θ. By using the largest coordinates, an approximation of the signal is found at each iteration. After each new residual is formed, reflecting the missing portion of the signal, the measurements are updated. This is repeated until all the recoverable portion of the signal is found.

Two different quantitative measures were used to evaluate the performance and accuracy of the reconstruction, Root mean Square Error (RMSE) and Peak Signal-to-Noise Ratio (PSNR).

The recovery algorithms run on a TOSHIBA laptop with duo-core @ 2.3 GHz and 2 GB of main memory.

IV. Results and Discussion

The experiment validated using Doppler ultrasound imaging spectrum data downloaded from Torpp group website. The data are FR data of length 2032.

Reconstruction of the Doppler ultrasound data performed with two different CS reconstruction algorithms, OMP algorithm and CoSaMP algorithm.

OMP algorithm used to identify the nonzero elements of the signal iteratively and reconstruct signal using the pseudo-inverse. The data with length of *N* sampled randomly, different number of measurements *M* were used for reconstruction (128, 406, 800, 1219 and 1625 points). Both reconstructions and Doppler spectrogram were performed with software program written in Matlab (Mathworks, MA).

Figure 1, shows the recovered Doppler ultrasound spectrum using different number of measurements generated from the recovered signal via OMP algorithm. The result shows that the spectrum was reconstructed perfectly even by using fewer numbers of measurements.

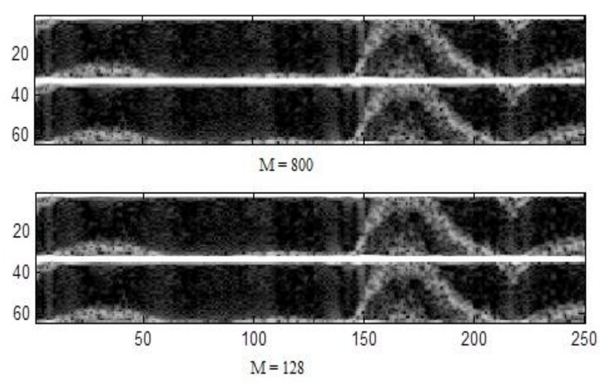

Figure 1.reconstructed Doppler ultrasound spectrum via OMP using two different numbers of points (M = 128, 800)

The error from the recovered images was calculated to compare to the original image, the result illustrated in figure 2. The result shows that the images reconstructed with higher number of measurements gives error lower than that reconstructed with fewer numbers of measurements.

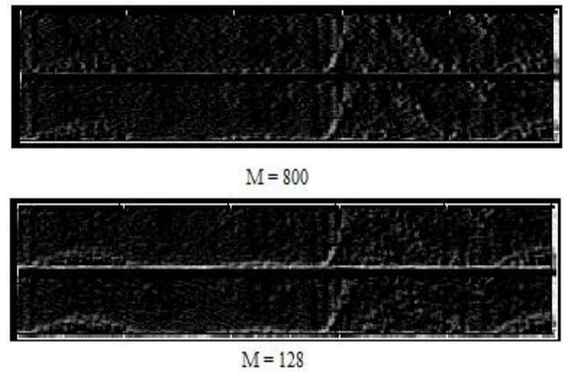

Figure 2. The error from the recovered spectrum via OPM for different numbers of points

When Doppler data processed by using CoSaMP, which is used to identify the nonzero elements of the signal in an iteratively and reconstruct signal using the pseudo-inverse. The data with the same length and number of points were used for reconstruction. The same program was used for reconstruction and generation of Doppler spectrogram.

The recovered signal illustrated in Figure 3. Doppler ultrasound signal recovered using different number of measurements, which is used then to generate the Doppler spectrogram. The recovered signal was performed via CoSaMP algorithm. The result shows that the spectrogram was reconstructed with good performance.

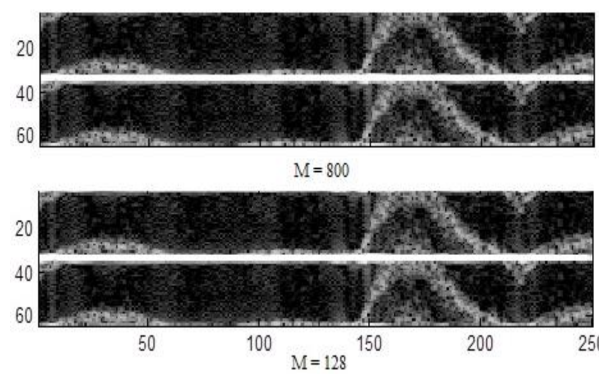

Figure 3.reconstructed Doppler ultrasound spectrum via CoSaMP using two different numbers of points (M = 128, 800)

The error from the result images were calculated by subtracting the recovered image from the original image, the results shown in figure 4. The result shows that the error in the image decreased by increasing the number of measurements. This result will be judged by quantitative performance evaluation.

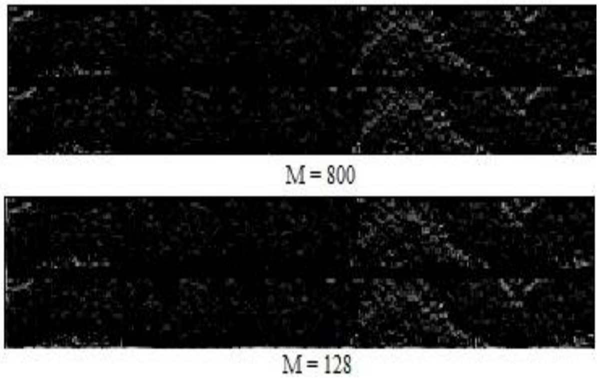

Figure 4. The error from the recovered spectrum via CoSaPM for different numbers of points

The process time for both algorithms was calculated for different random samples of measurements, the result of elapsed time shown in table 1. The result shows that when fewer numbers of points were used the reconstruction will be performed faster (take very low time) and when the higher numbers of points were used the reconstruction takes more time (the reconstruction time increased). In other words we can say the reconstruction time decreased by decreasing the number of reconstruction points.

The time taken during reconstruction with OMP compared to the time taken with CoSaMP, the result shows that OMP gives better reconstruction time.

The analysis of the results was performed by calculating the root mean square error and peak signal-to-noise ratio.

The RMSE for both algorithms was shown in table 1. The result shows that fewer numbers of points give very high root mean square error. The error decreased by increasing the number of measurements. Comparing the RMSE from both algorithms, the result shows that CoSaMP gives lower RMSE.

Peak signal-to-noise ratio (PSNR) from each recovered image was calculated for both algorithms the result shown in table 1. The result shows that fewer numbers of measurements, gives lower PSNR. While PSNR values increased by increasing the number of reconstruction points. This indicates that the quality of the image increased by increasing the number of measurements.

Table 1. Number of measurements versus elapsed time, RMSE and PSNR

Number of Measurement %		5	20	40	60	80
Elapse Time second	**OMP**	0.6	0.86	1.24	1.64	2.08
	CoSaMP	0.99	1.74	2.62	3.08	4.29
RMSE	**OMP**	16.37	15.84	15.49	15.37	14..17
	CoSaMP	12.17	12.14	12.08	12.08	11.73
PSNR	**OMP**	23.85	24.13	24.33	24.40	25.10
	CoSaMP	26.42	26.44	26.48	26.48	26.75

The PSNR form both algorithms compared, the result show that CoSaMP algorithm reconstructs the image with better quality.

V. Conclusion

Reconstruction of signal from a few numbers of measurements is a unique in signal processing. Reduction of measurements can be done by using the novel sampling theory, CS theory. In this work we tested two CS reconstruction algorithms to reconstruct Doppler ultrasound RF signal, orthogonal matching pursuit and compressive sampling matching pursuit. We have shown that Doppler signal can be reconstructed with high performance by using either orthogonal matching pursuit reconstruction algorithm or compressive sampling matching pursuit algorithm. The result shows that the signal can be reconstructed even by using a very few number of points. The reconstruction by using OMP performed faster than CoSaMP, but gives error higher than CoSaMP. Generally, both algorithms reconstruct the Doppler ultrasound signal within a very short time and good quality even when fewer numbers of measurements used.

Acknowledgment

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