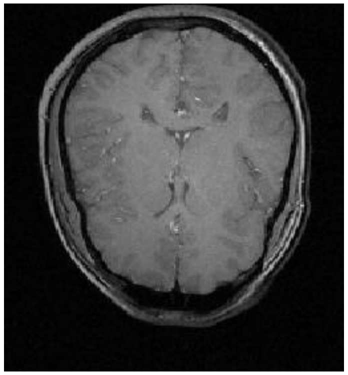
Computed Tomography

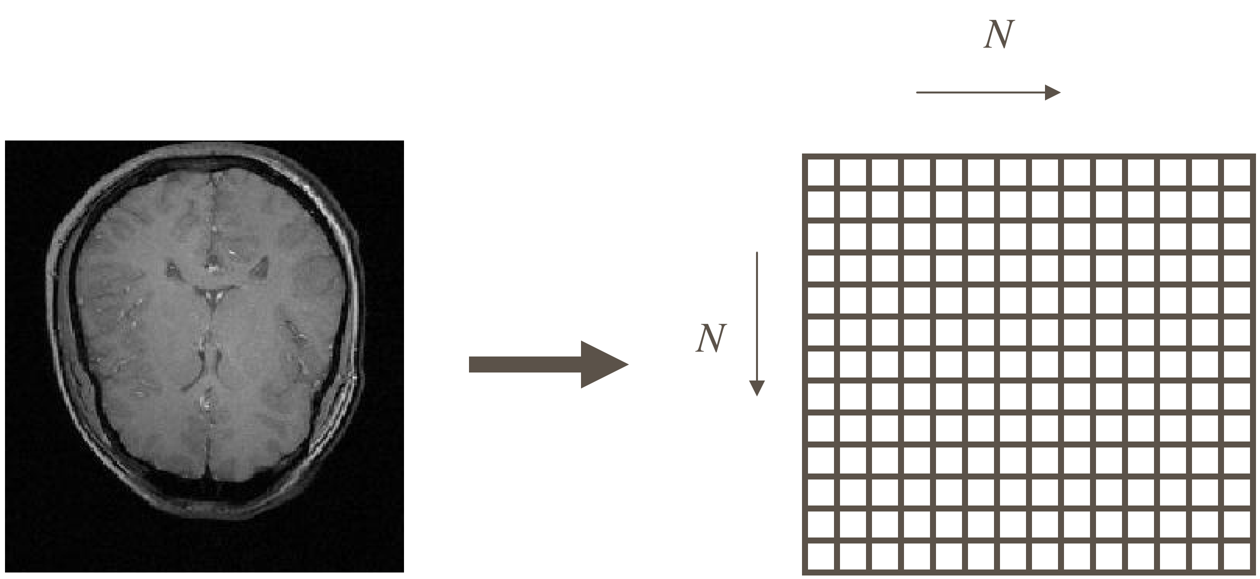
Medical Image Reconstruction

- A map representing a physical property of tissues that allows the physician to diagnose abnormalities or assess physiological function
  - Acoustic impedance in ultrasound imaging
  - X-ray attenuation in conventional x-ray imaging
  - Proton density, T_1, T_2, T_2^* in MRI



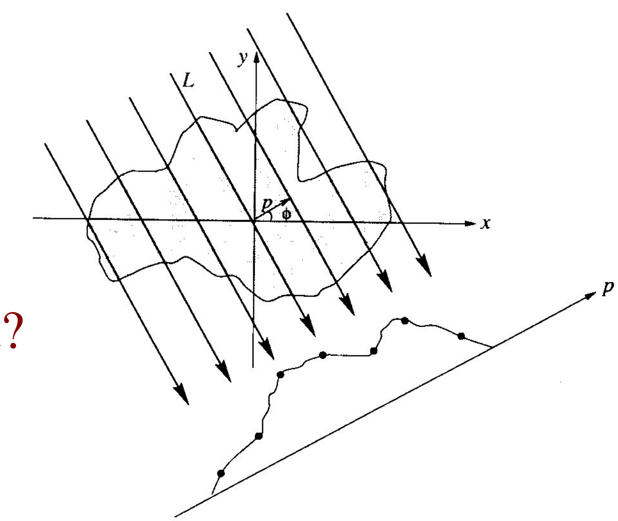
CT Image Reconstruction

- For an $N \times N$ image, we have N^2 unknowns to estimate
  - Sufficient equations must be available
  - In most cases the problem can be formulated as a linear system



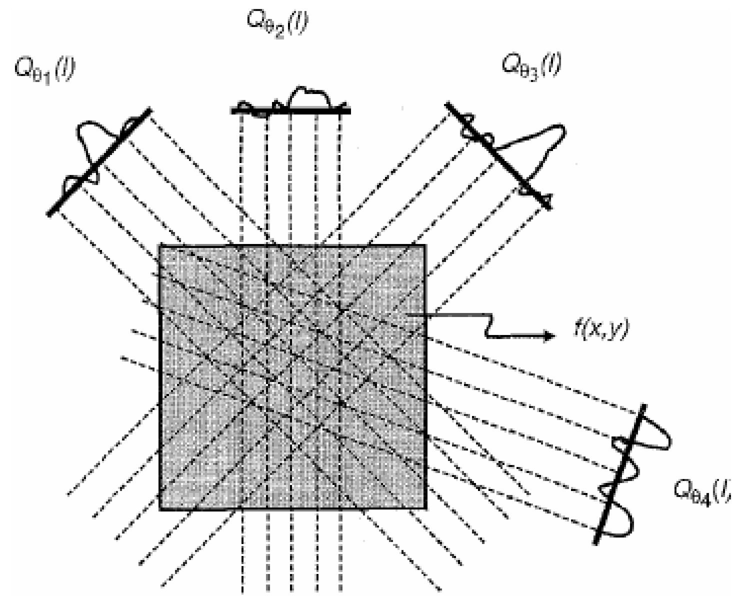
Conventional Projection Imaging

- The image points represents the line integral of the tissue property along the incident ray
  - Plain x-ray imaging
  - Difficulty to discern overlapping structures along projection ray
- Question: Can we gain more information by projecting in different directions?
  - Different directions provide different equations about the different parts of the image
  - How many directions are needed?



Computed Tomography: Reconstruction from Projections

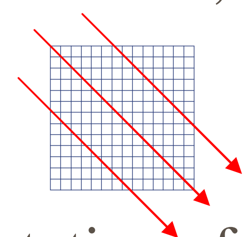
- Collects data from projections at different angles and attempts to construct a "cross-section" that resolves the location ambiguity present in projection images
- Example: X-ray computed tomography



Mathematical Formulation

- Let image points be expressed as I(x,y)
- The projection data at an angle θ can be expressed as,

$$P_\theta(\rho) = \sum_{x,y} \alpha_\rho^\theta(x,y) \cdot I(x,y)$$

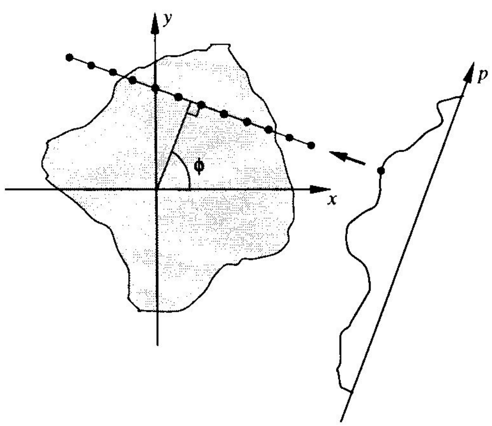


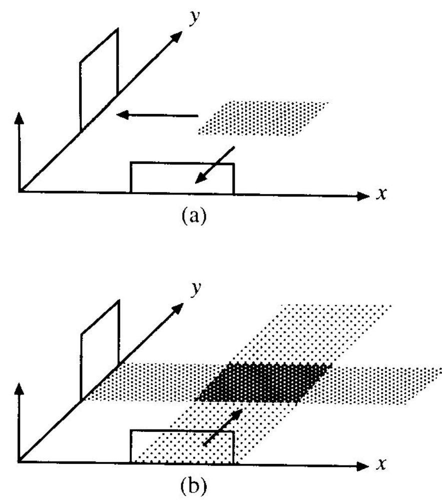
- From all projections, a linear system representation of the problem can be constructed as,

$$\begin{bmatrix} P_{\theta_1}(\rho_1) \\ P_{\theta_1}(\rho_2) \\ \vdots \\ P_{\theta_n}(\rho_m) \end{bmatrix} = \vec{P} = \begin{bmatrix} \alpha_{\theta_1}^{\rho_1}(x_1,y_1) & \alpha_{\theta_1}^{\rho_1}(x_1,y_2) & \cdots & \alpha_{\theta_1}^{\rho_1}(x_N,y_N) \\ \alpha_{\theta_1}^{\rho_2}(x_1,y_1) & \alpha_{\theta_1}^{\rho_2}(x_1,y_2) & \cdots & \alpha_{\theta_1}^{\rho_2}(x_N,y_N) \\ \vdots & \vdots & \cdots & \vdots \\ \alpha_{\theta_n}^{\rho_m}(x_1,y_1) & \alpha_{\theta_n}^{\rho_m}(x_1,y_2) & \cdots & \alpha_{\theta_n}^{\rho_m}(x_N,y_N) \end{bmatrix} \cdot \begin{bmatrix} I(x_1,y_1) \\ I(x_1,y_2) \\ \vdots \\ I(x_N,y_N) \end{bmatrix} = A \cdot \vec{I}$$

Back-Projection Method

- Start from a projection value and back-project a ray of equal pixel values that would sum to the same value
- Back-projected ray is added to the estimated image and the process is repeated for all projection points at all angles
- With sufficient projection angles, structures can be somewhat restored



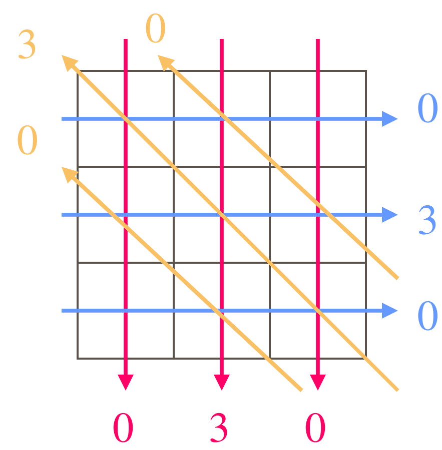


Back-Projection Example

True Image

0	0	0
0	3	0
0	0	0

Projection Data



0	0	0
0	0	0
0	0	0

Initial Solution

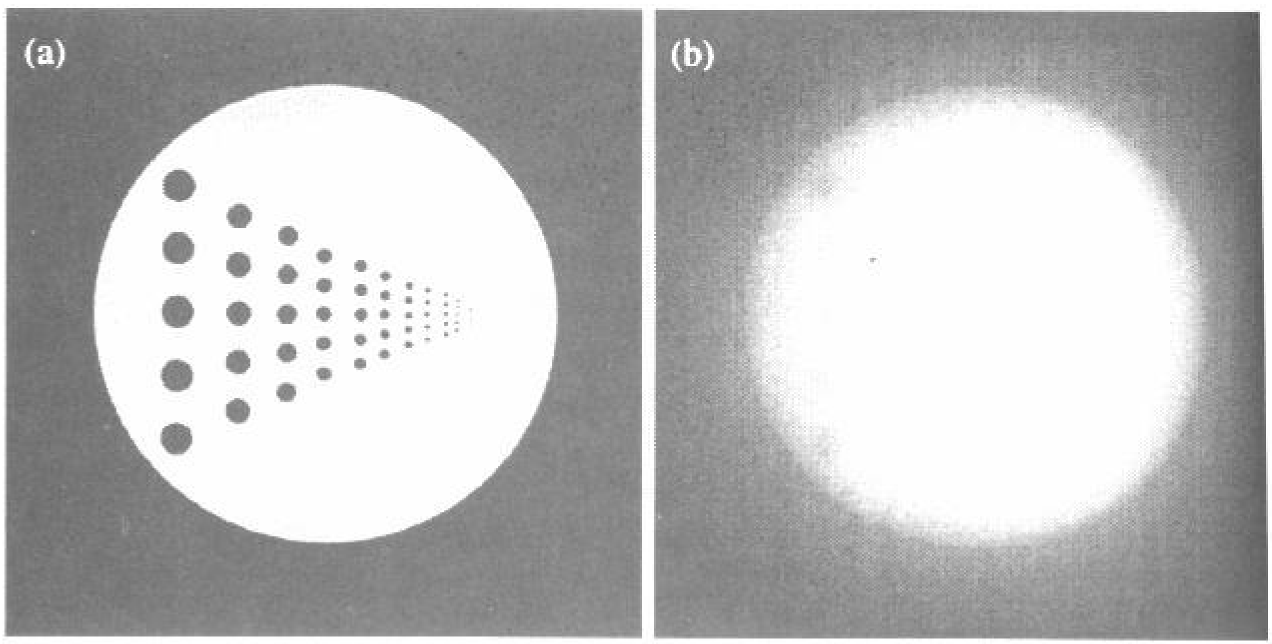
0	1	0
0	1	0
0	1	0

0	1	0
1	2	1
0	1	0

1	1	0
1	3	1
0	1	1

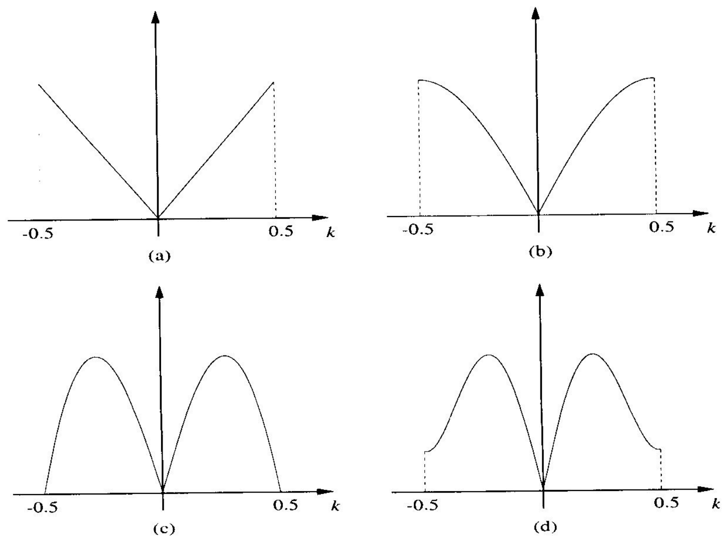
Iterate until sufficient accuracy is achieved

- Problems with back-projection include mainly severe blurring in the computed images

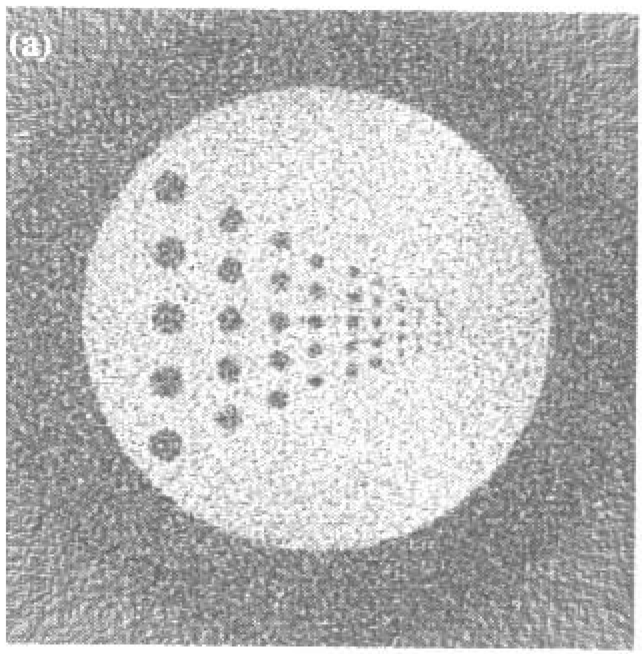


Filtered Back-Projection

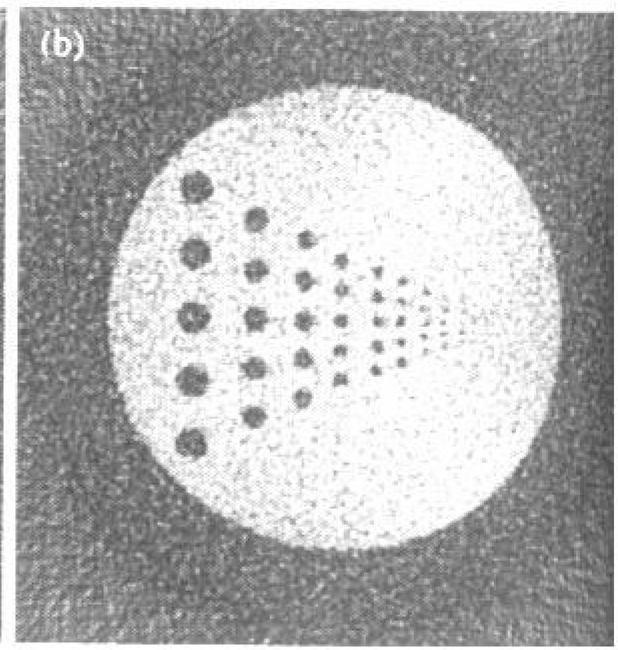
- From the analytical formulation inverse Radon transform, the derivative of the projections should be used instead of the original
  - Low-pass emphasis results in blurring of computed image
- Several types of high-pass filters can be used
  - Ram-Lak
  - Shepp-Logan
  - Cosine filter
  - Generalized Hamming



- Less blurring is observed with filtered back-projection



Ram-Lak Filter



Cosine Filter

Algebraic Reconstruction Technique (ART)

- A low-complexity iterative solver to the algebraic reconstruction problem
- Starts with an initial estimate and tries to push the estimate closer to the true solution
  - Instead of back-projecting the average ray value, the error between the projection computed from current estimate and the true is used

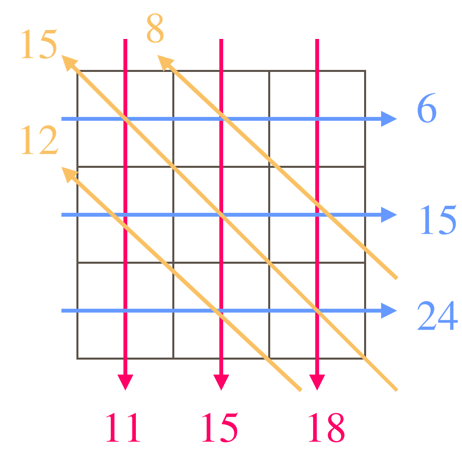
$$Update = P_\theta(\rho) - \sum_{x,y} \alpha_\rho^\theta(x,y) \cdot \hat{I}(x,y)$$

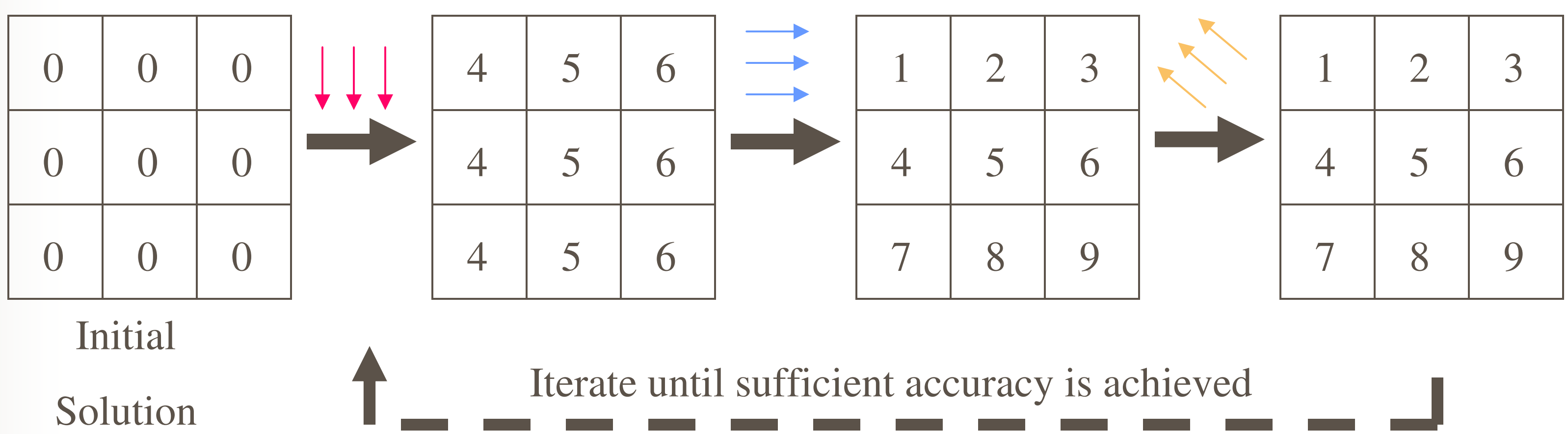
ART Example

True Image

1	2	3
4	5	6
7	8	9

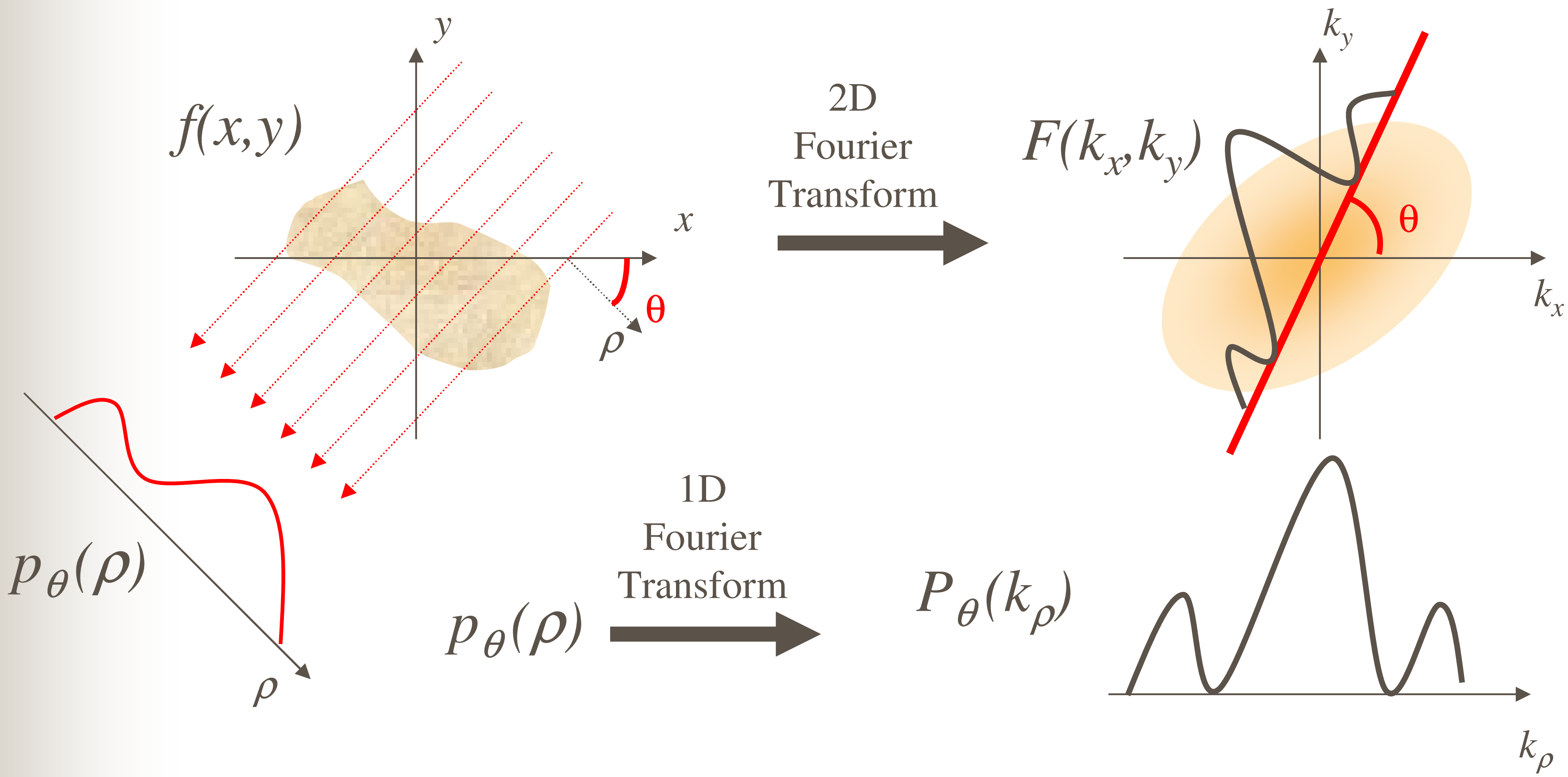
Projection Data





Transformation to Frequency Domain: Projection Slice Theorem

- A property of the Fourier transform
- Relates the projection data in the spatial domain to the frequency domain
- States that the 1D Fourier transform of the projection of an image at an angle θ is equal to the slice of the 2D Fourier transform at the same angle



f(x,y)
y
x
θ
ρ
$p_\theta(\rho)$
ρ
2D
Fourier
Transform
$F(k_x,k_y)$
k_y
k_x
θ
1D
Fourier
Transform
$p_\theta(\rho)$
$P_\theta(k_\rho)$
k_ρ

Mathematical Illustration

- 2D Fourier transformation:

$$F(k_x,k_y)=\iint f(x,y)\cdot e^{-j2\pi(k_x\cdot x+k_y\cdot y)}dxdy$$

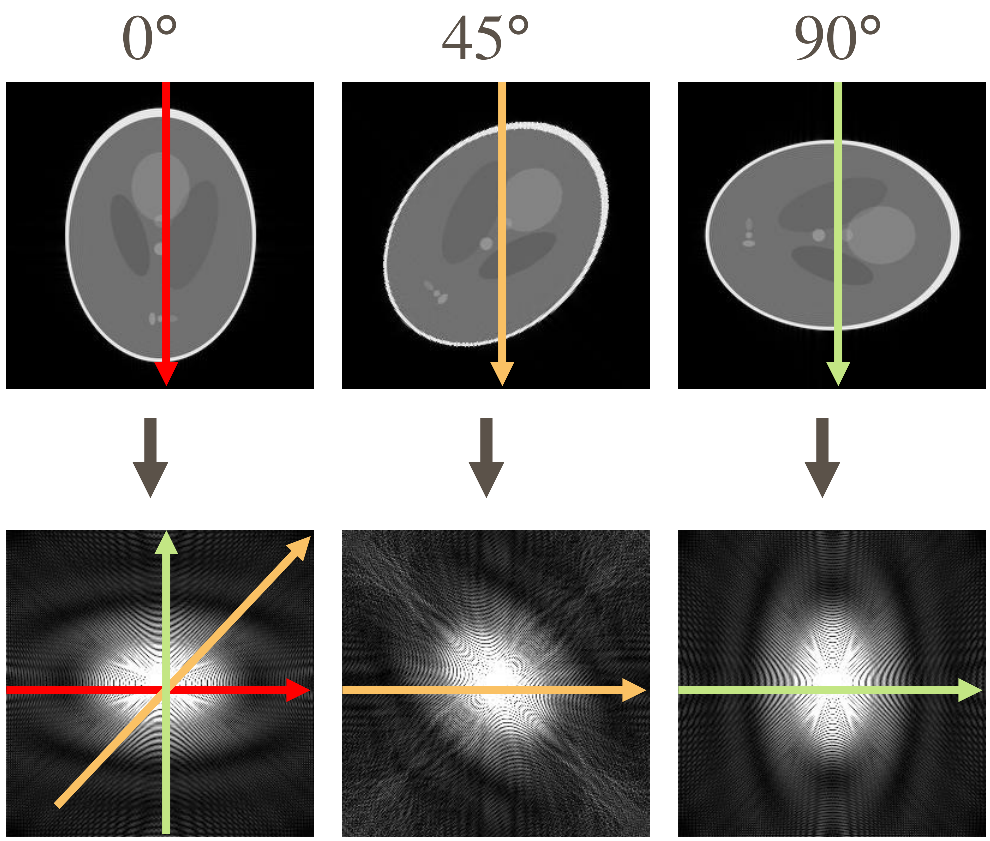
- The slice of the 2D Fourier transform at k_x=0 is given by:

$$F(0,k_y)=\int\left(\underline{\int f(x,y)dx}\right)\cdot e^{-j2\pi k_y\cdot y}dy$$

and at k_y=0 is given by

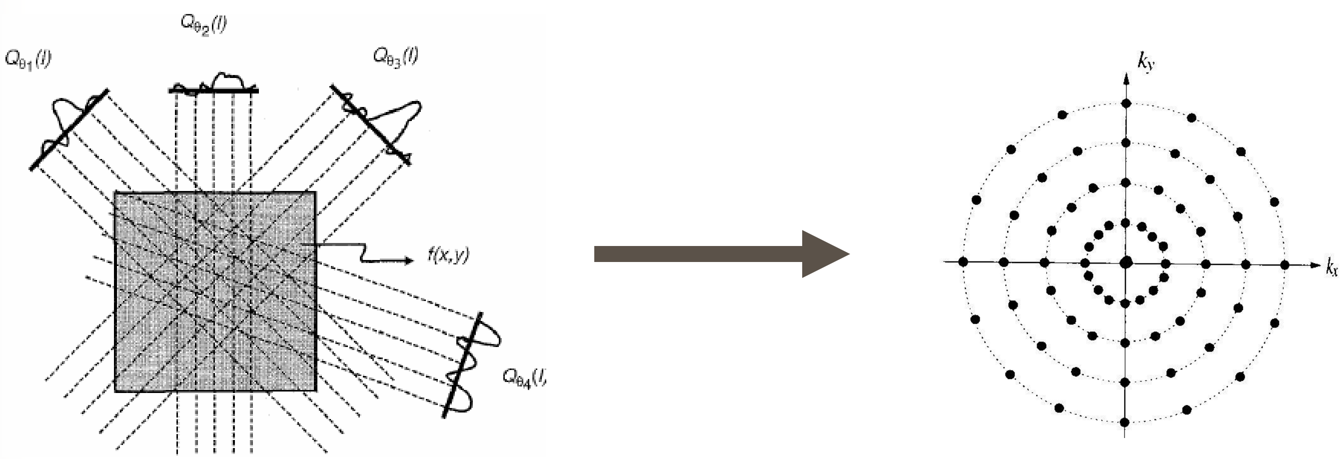
$$F(k_x,0)=\int\left(\underline{\int f(x,y)dy}\right)\cdot e^{-j2\pi k_x\cdot x}dx$$

- For a general angle, the rotation property of the Fourier transformation can be used to generalize the mathematical result for a vertical projection to any angle



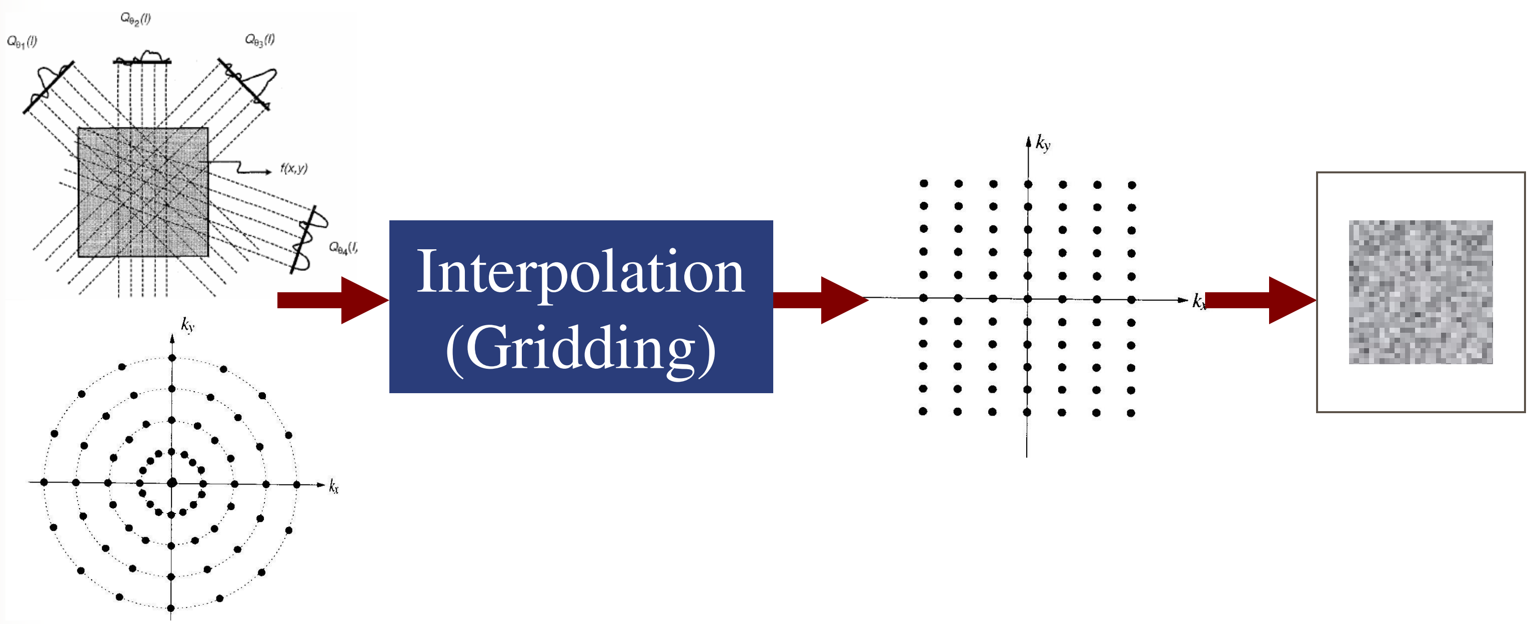
Reconstruction from Projections: Frequency Domain Perspective

- The projection data can be shown to correspond to radial sampling of the frequency domain
- It is not straightforward to numerically compute the image from this frequency domain representation
  - Limitation of the DFT to uniform sampled data



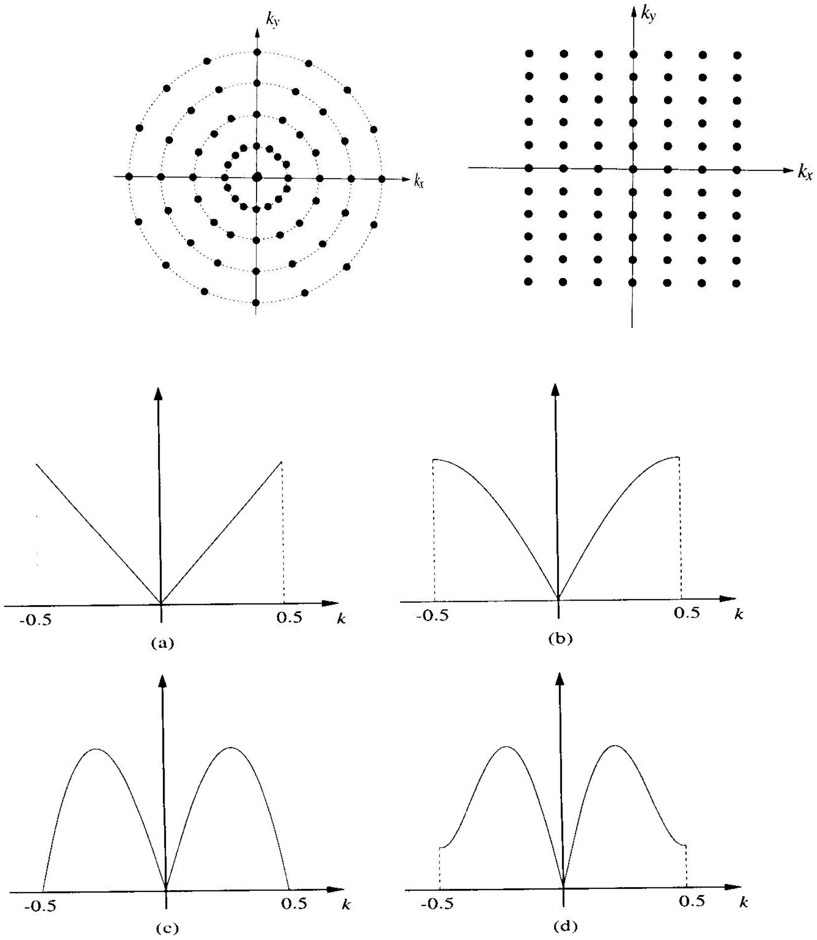
Reconstruction in the Frequency Domain

- Interpolation can be used in the frequency domain to re-grid the radial sampling to uniform sampling
- Inverse DFT can then be efficiently used to compute the image



Another Explanation of Filtered Back-Projection

- Sampling density is higher near the center of the frequency domain compared to uniform sampling
  - Direct addition of projections results in low-pass emphasis that causes blurring of computed image
- Density compensation has a very similar form to the filters used in this method

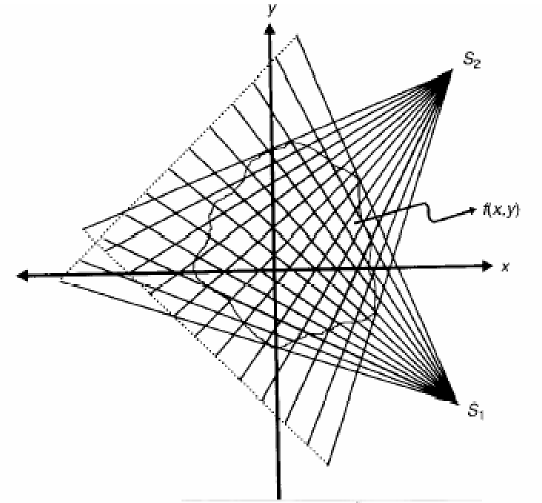


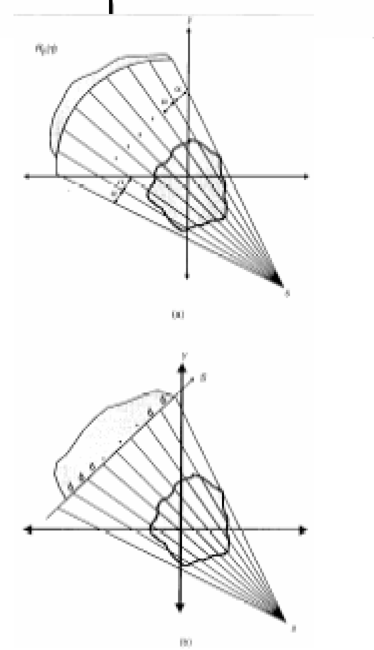
Relevant Issues

- Sampling density must meet the Nyquist criterion
  - Number of projections (i.e., number of angles)
  - Number of samples in each projection (i.e., number of detectors)
  - Visible artifacts arise when undersampling occurs
- Number of iterations
  - Accuracy vs. computation time

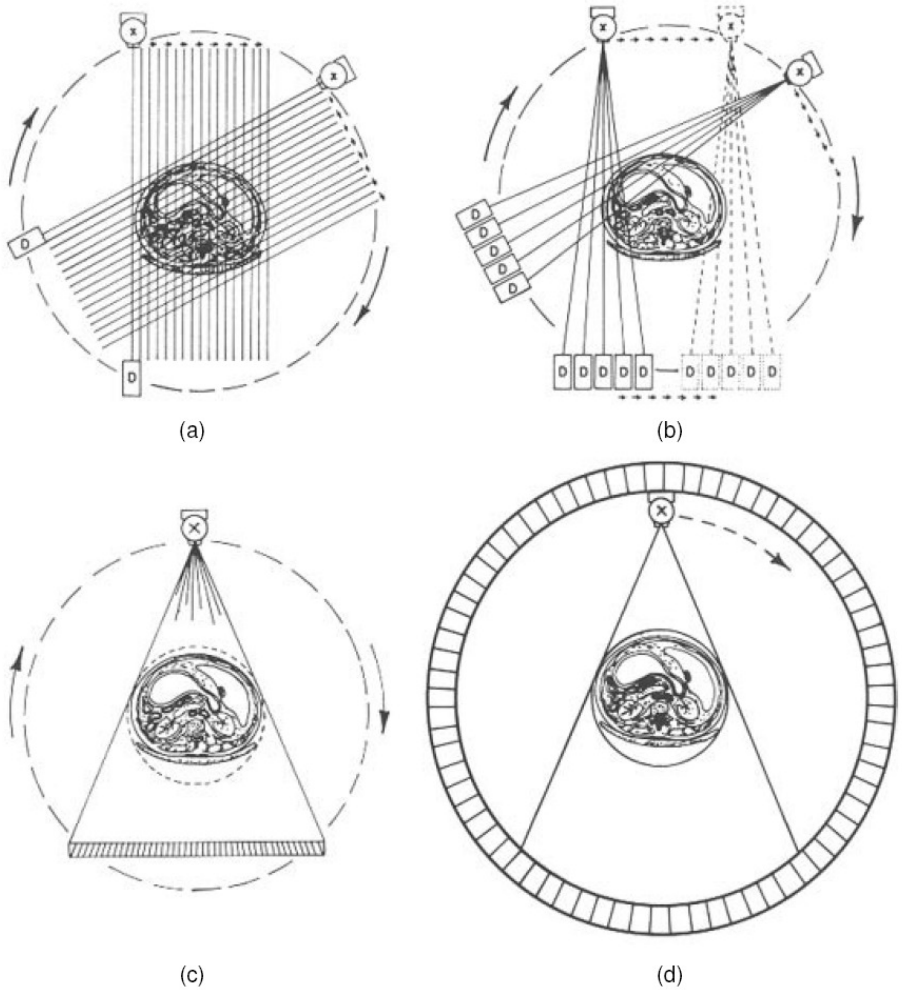
Problem Extensions: Fan Beam Problem

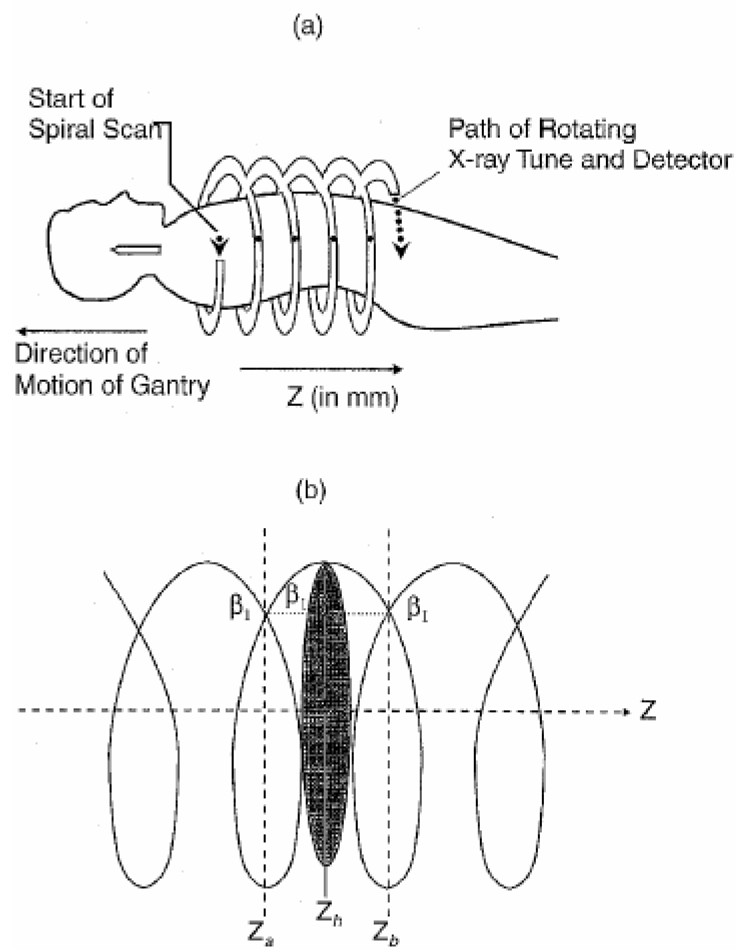
- In newer CT generations, fan beams are used to gain more efficiency in hardware implementation
- Detectors may be aligned on a line or a circular arc
- A modification of the algebraic reconstruction method is used to compute the image





CT Generations





CT Numbers

$$\text{CT number} = 1000\frac{(\mu - \mu_w)}{\mu_w}$$

- -1000 for air
- 0 for water
- +1000 for bone

- Image display adjustment

Tissue	M (cm^{-1})
Bone	0.528
Blood	0.208
Gray matter	0.212
White matter	0.213
CSF	0.207
Water	0.206
Fat	0.185
Air	0.0004

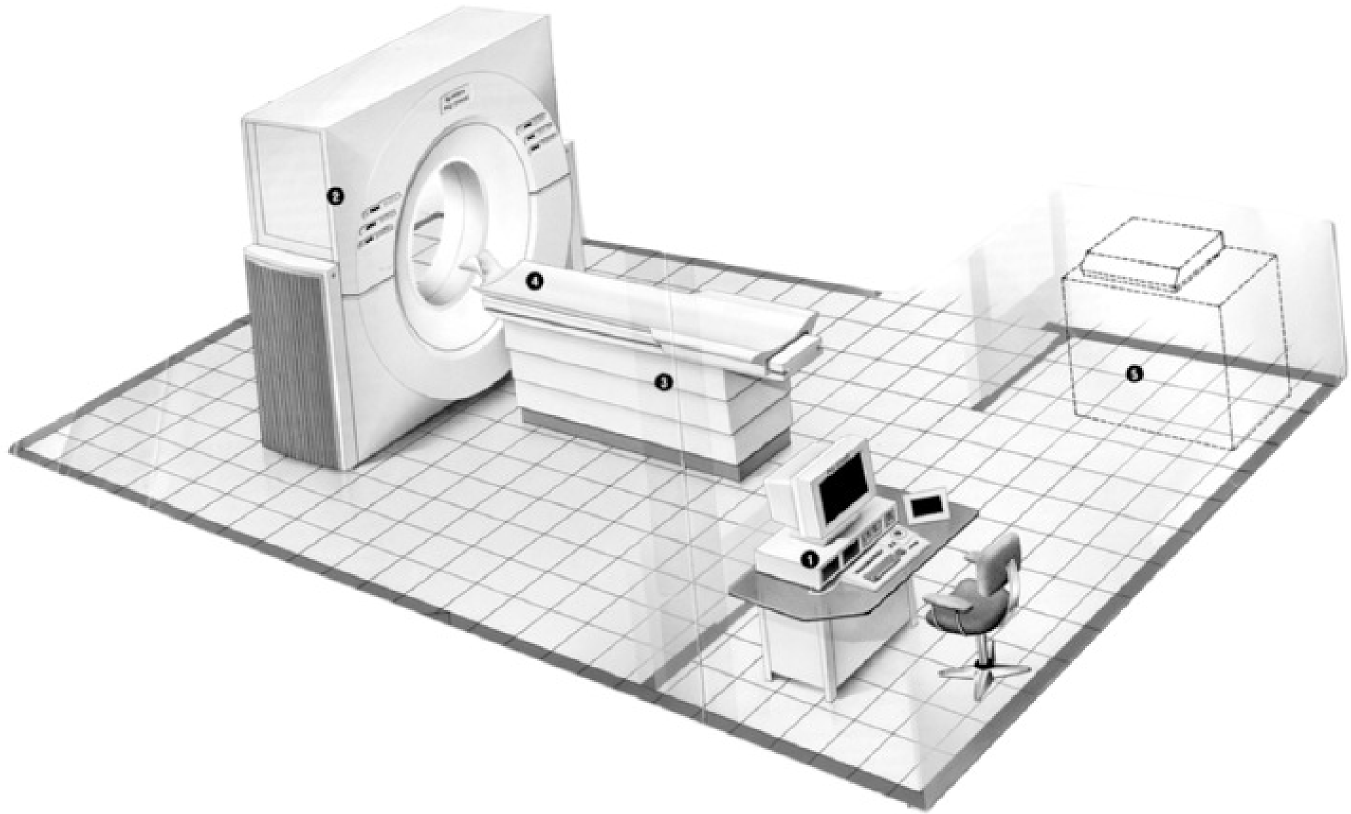


FIGURE 62.1 Schematic drawing of a typical CT scanner installation, consisting of (1) control console, (2) gantry stand, (3) patient table, (4) head holder, and (5) laser imager. (Courtesy of Picker International, Inc.)

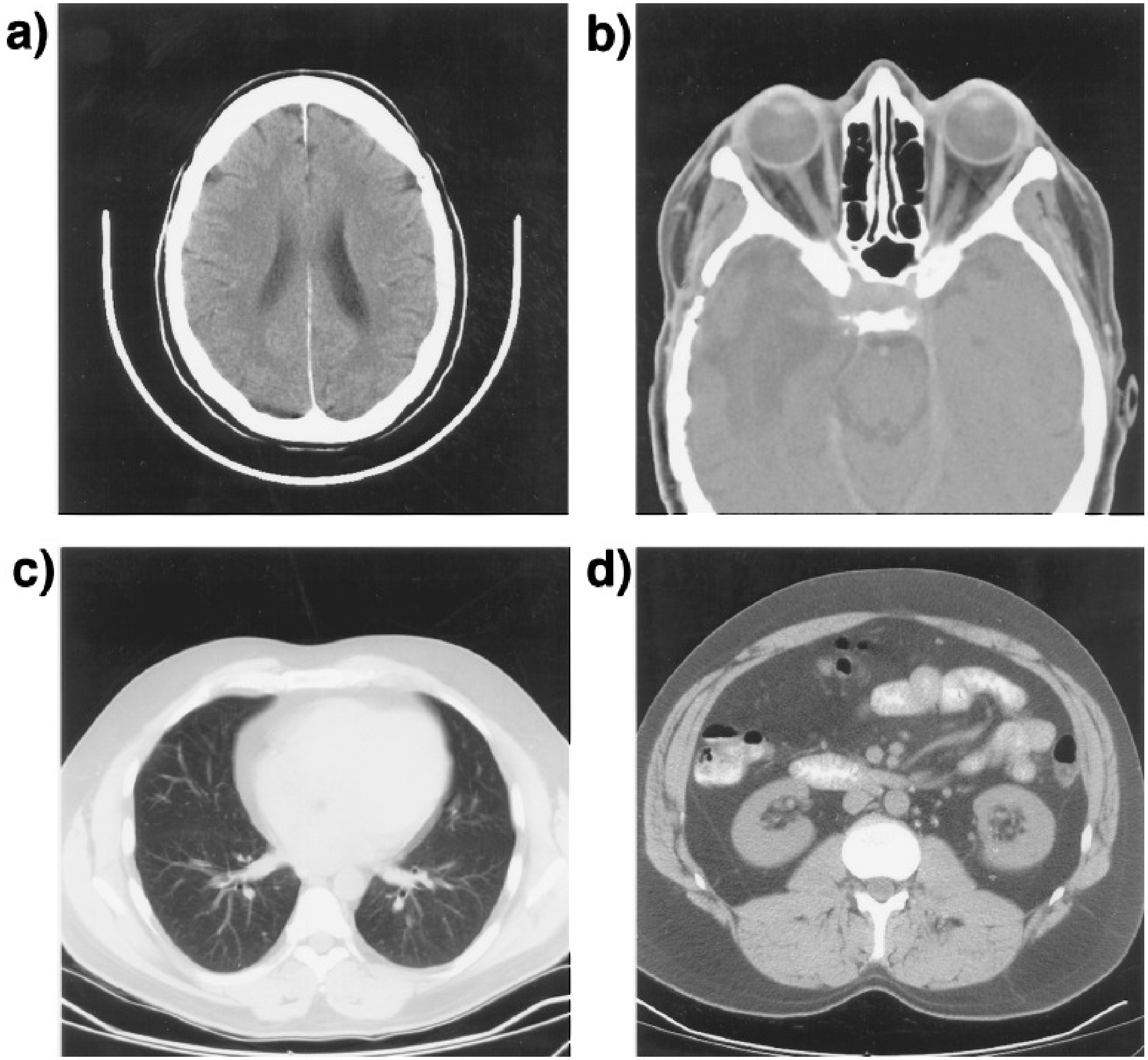


FIGURE 62.2 Typical CT images of (*a*) brain, (*b*) head showing orbits, (*c*) chest showing lungs, and (*d*) abdomen.