



**Digital Signal Processing Practice Exam**  
May 2012

**Solve As Much As You Can – Maximum Grade: 40 Points**

**Q1.** [2 Points] Determine the output of the systems described by the following difference equation with input and initial conditions as specified:

$$y[n] = x[n] - 0.5 y[n-1], \quad x[n] = u[n-1], \quad y[-1] = 1$$

**Q2.** [4 Point Each] Determine whether the following signals are periodic, and for those which are, find the fundamental period:

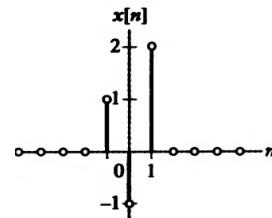
$$x[n] = \cos\left(\frac{8}{15} \pi n\right)$$

$$x[n] = \cos\left(\frac{7}{15} \pi n\right)$$

**Q3.** [4 Points] For the shown a discrete  $x(n)$ , sketch the following signals derived from  $x(n)$ :

(a)  $y_1(n) = x(n+2)$

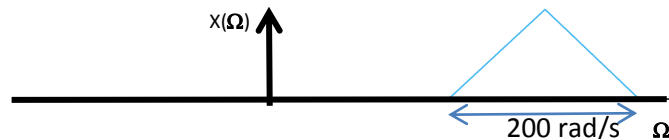
(b)  $y_2(n) = x(2n)$



**Q4.** [4 Points] Determine if this system is (a) linear, (b) time invariant, (c) causal, and (d) recursive:

$$y(n) = x(n + 1) + 2n$$

**Q5.** [3 Points] Determine the maximum sampling period that produces no aliasing for this signal:



**Q6.** [4 Points] A signal  $f(t) = e^{-j(100\pi t)}$  was sampled with an ideal pulse train. Sketch the continuous-time Fourier transformation for sampling rate of 10k Samples/s and estimate the reconstructed signal using a lowpass filter with cutoff frequency of  $\Omega_s/2$ .

**Q7.** [2 Points] Determine the z-transform of the following signal:

$$x[n] = 2e^{-2n}u[n]$$

**Q8.** [2 Points] Determine the inverse z-transform of the following function:

$$F(z) = \frac{z + 1}{z^2(z - 1)}$$

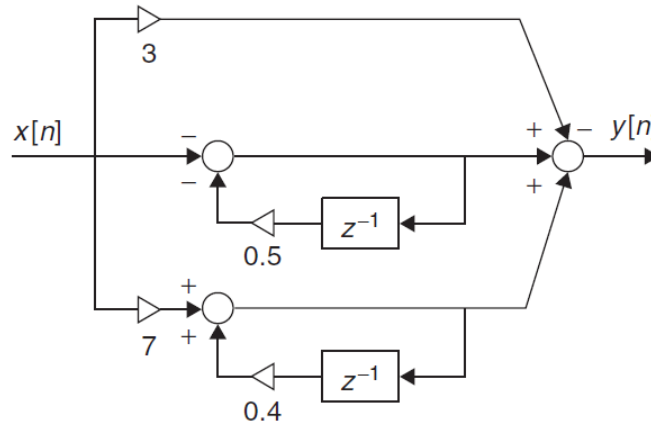
**Q9.** [4 Points] Convert the following filter to discrete-time filter with  $f_s = 1k$  samples/s

$$H(s) = \frac{s^2}{s^2 + \sqrt{2}s + 1}$$

Q10. [2 Points] Draw the realization of the following filter: (page 696)

$$H(z) = \frac{1 + 1.2z^{-1} + 0.2z^{-2}}{1 - 0.4z^{-1} + z^{-2} - 0.4z^{-3}}$$

Q11. [2 Points] Derive the filter transfer function  $X(z)$  of the following realization: (page 699)



Q12. (page 685)

Design a low-pass FIR filter with  $N = 21$  to be used in filtering analog signals and that approximates the following ideal frequency response:

$$H_d(e^{j\omega}) = \begin{cases} 1 & 0 \leq f \leq 125 \text{ Hz} \\ 0 & \text{elsewhere in } 0 \leq f \leq f_s/2 \end{cases}$$

where  $\omega = 2\pi f/f_s$  and  $f_s = 1000$  Hz is the sampling rate. Use first a rectangular window, and then a Hamming window. Compare the designed filters.