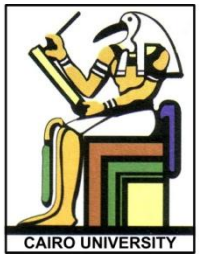


Medical Image Reconstruction
Term II – 2012

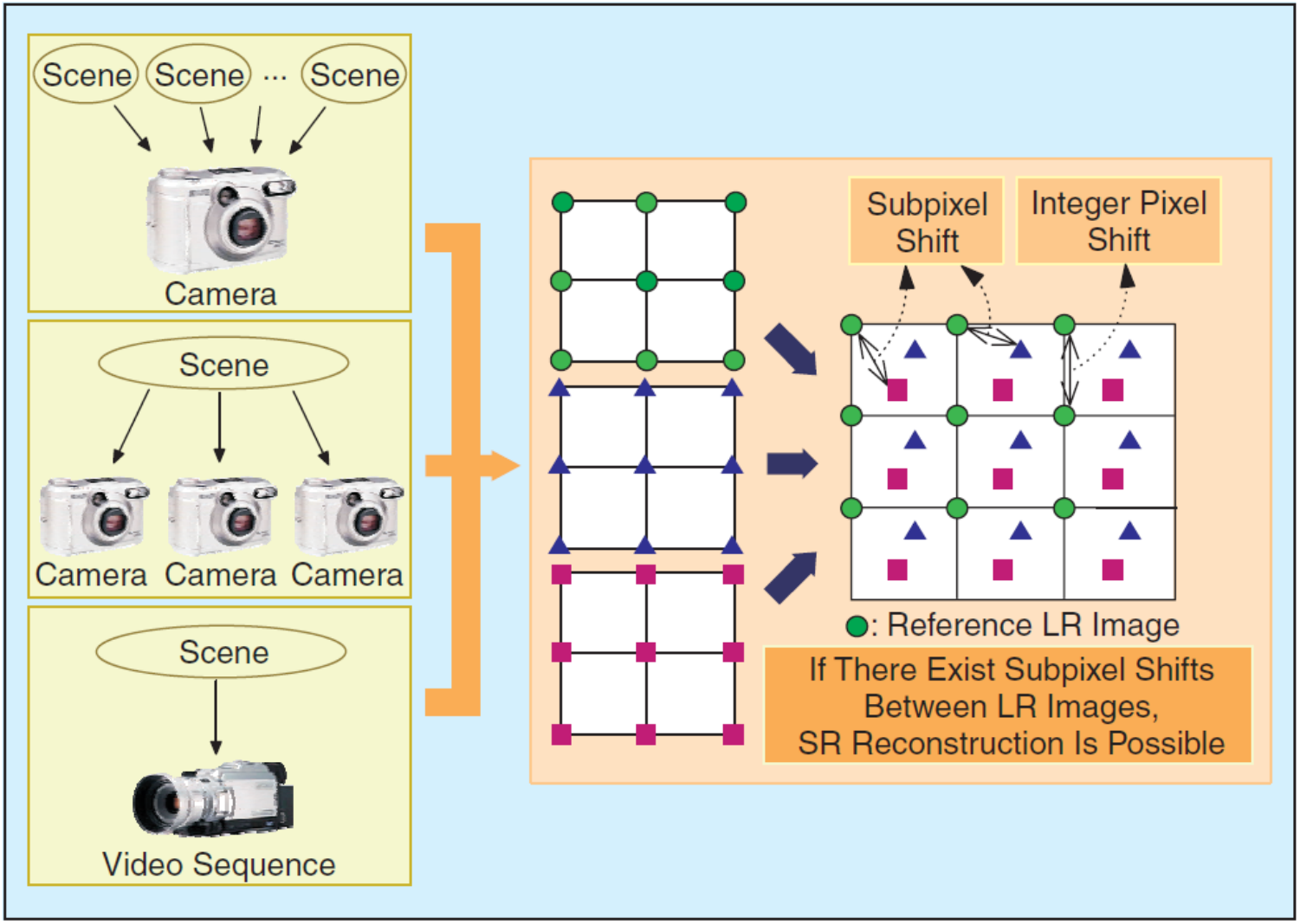
Topic 7:
Super-Resolution Image Reconstruction

Professor Yasser Mostafa Kadah

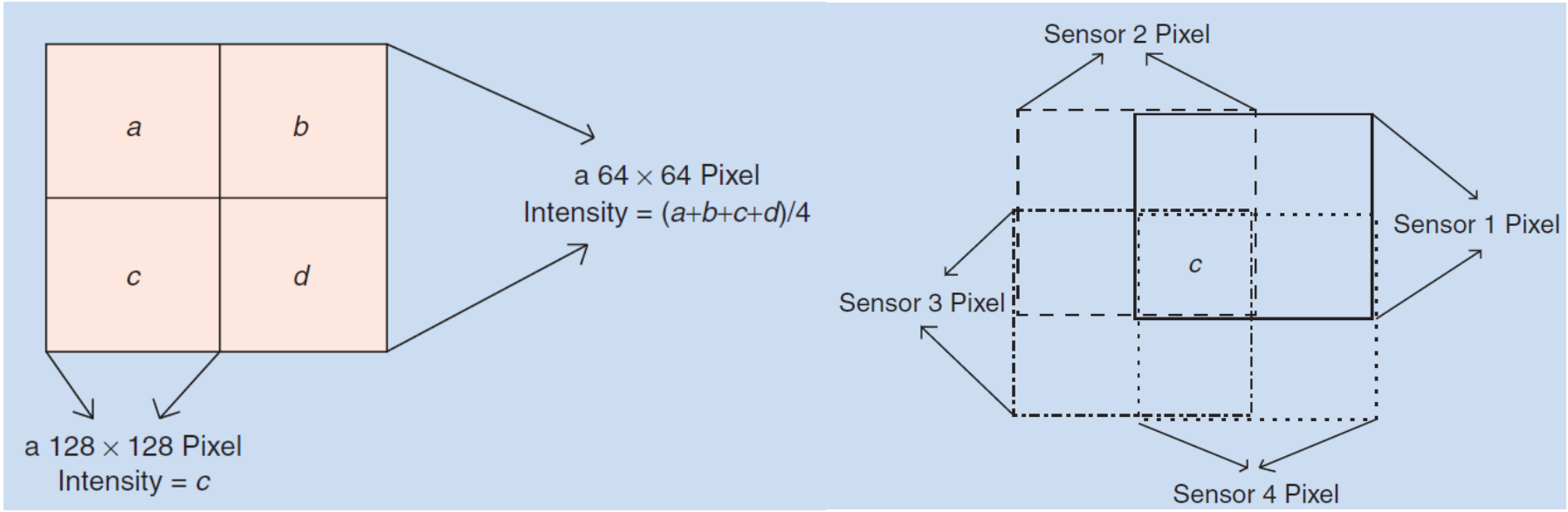
Super Resolution Reconstruction (SRR)

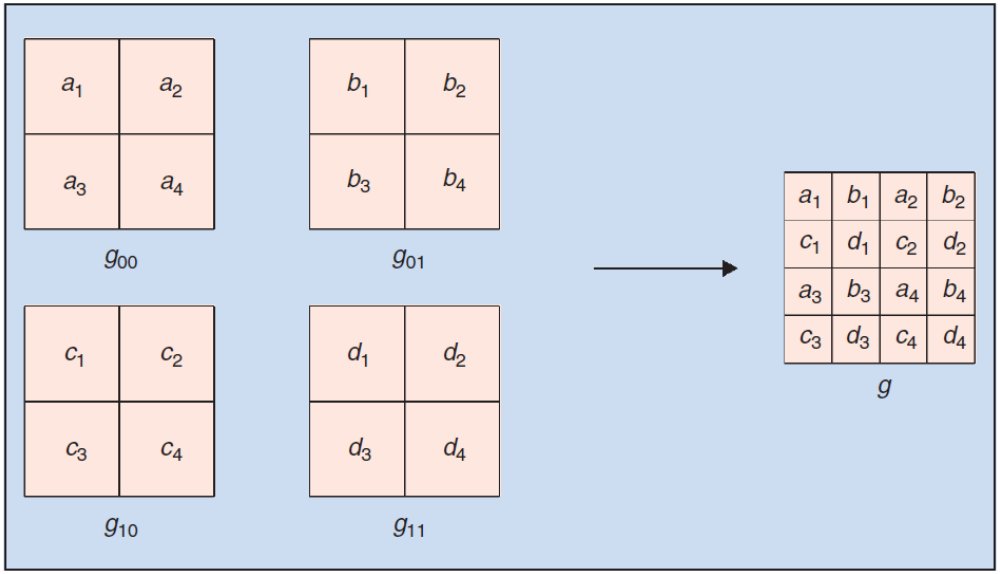
- Obtain higher resolution image reconstructions that are much better than what the classical approaches allow for the same amount of data
  - Combine multiple low-resolution images to get high resolution reconstruction
  - Exploit motion in image sequences to enhance resolution
  - Exploit a priori knowledge about image to form models and estimate model parameters
  - Compressed sensing using random sampling

Model for SRR from Multiple LR Images



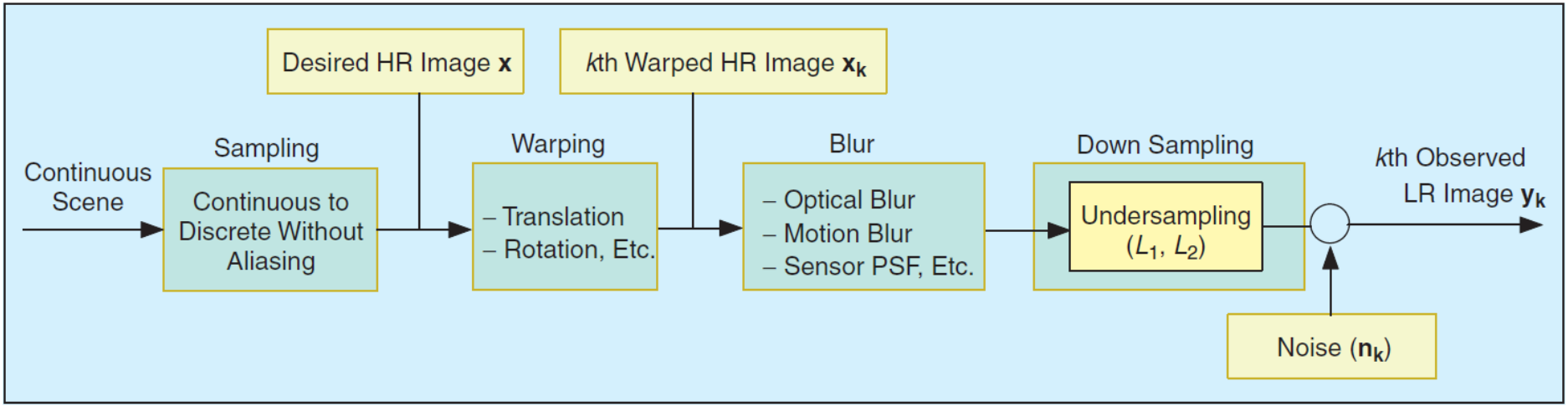
Example of Blur Operator



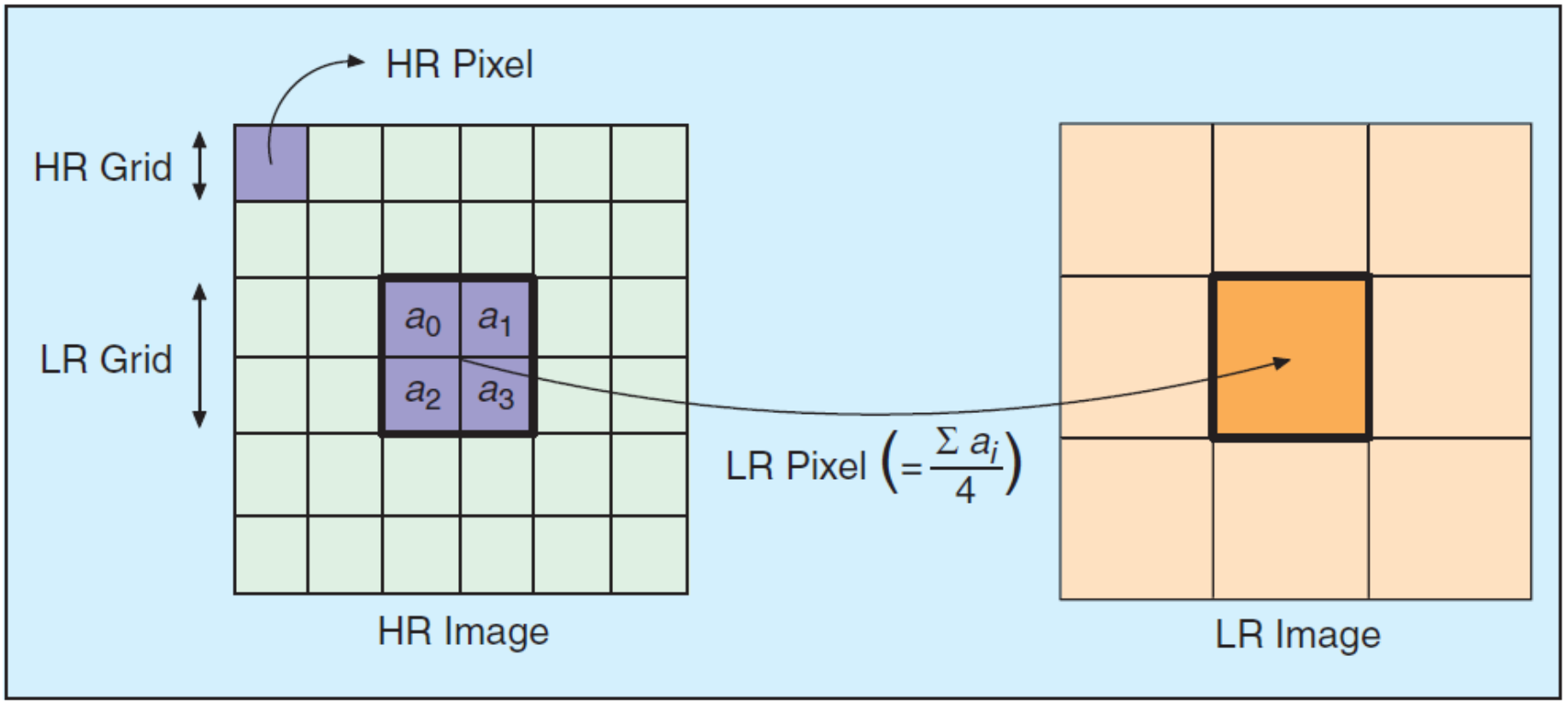


General Model

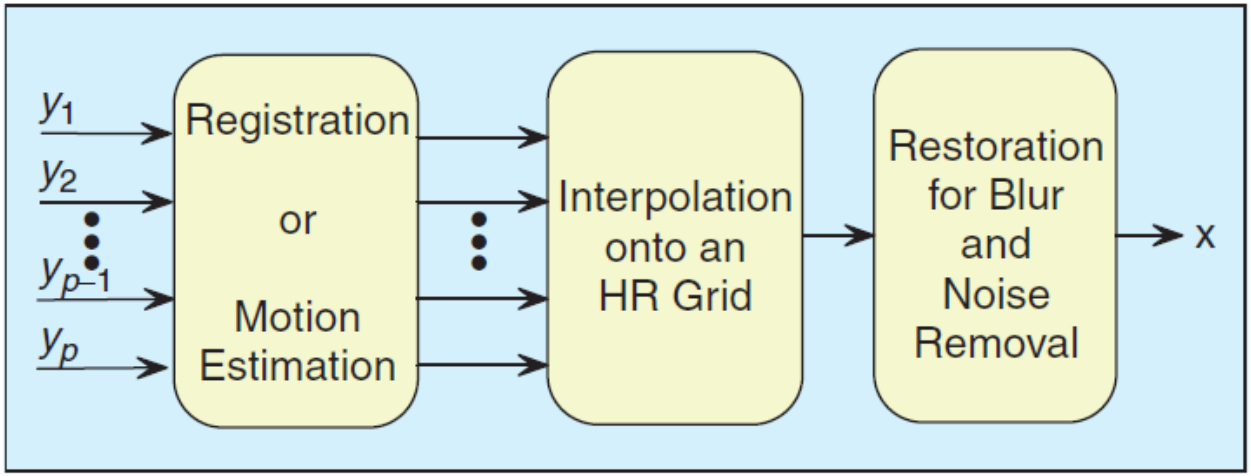
- Observation model relating LR images to HR images

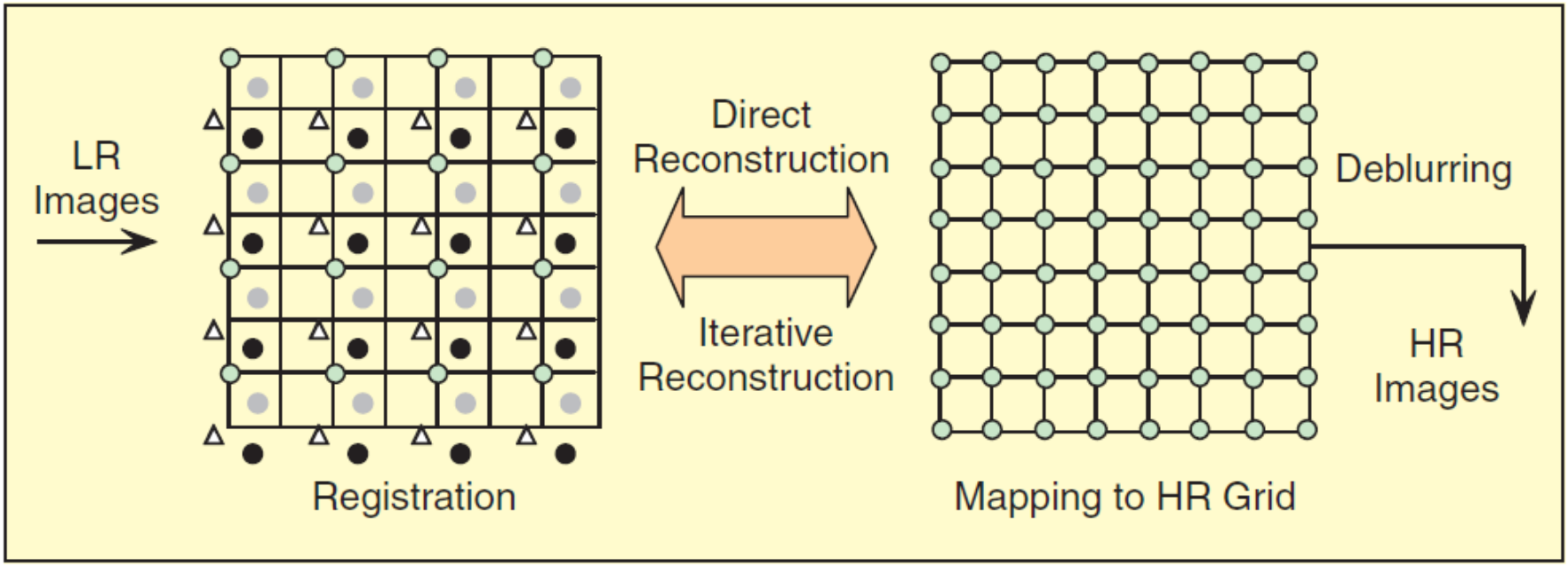


Example: Low-Resolution Sensor PSF

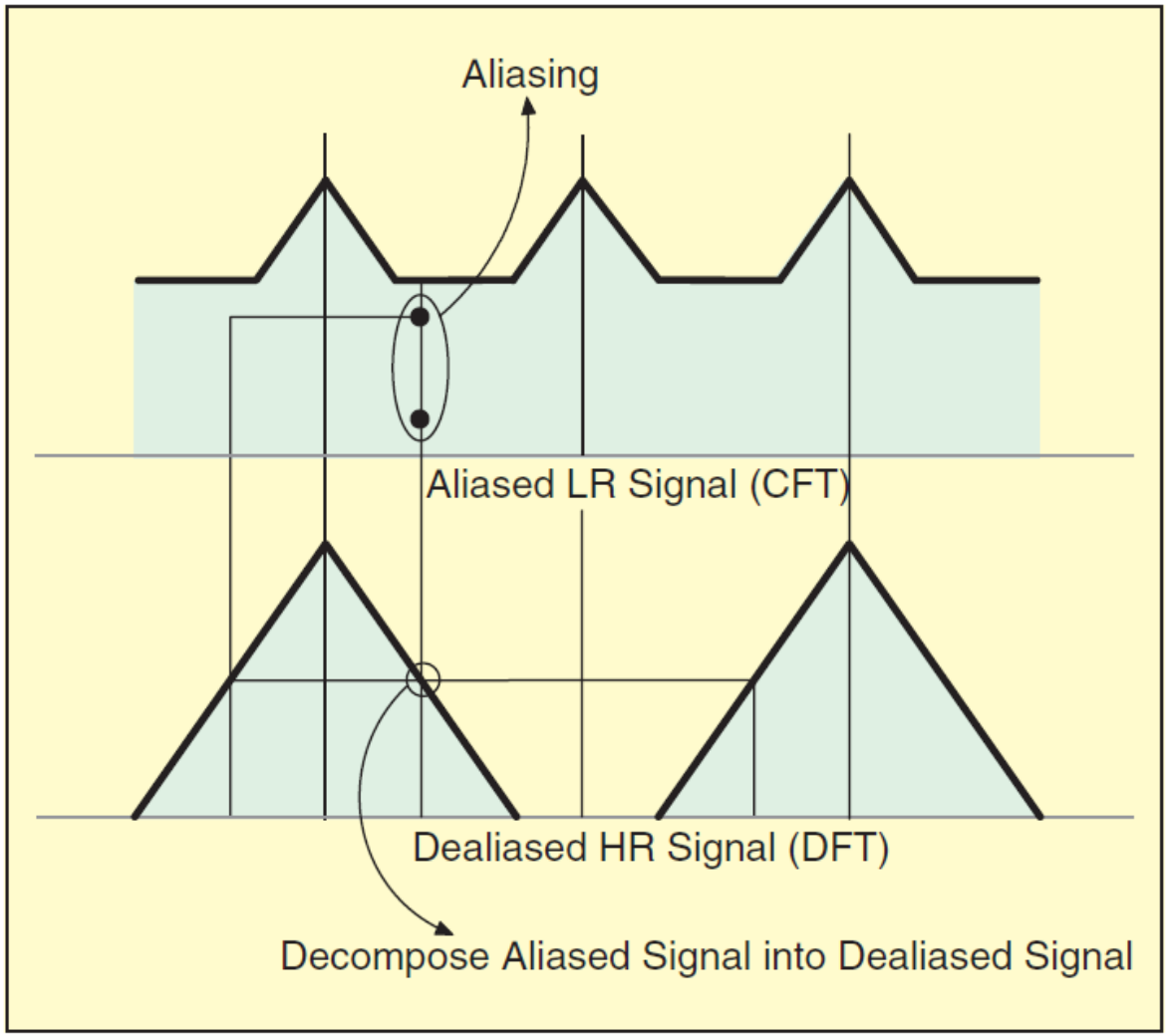


Registration-Interpolation-Based SRR

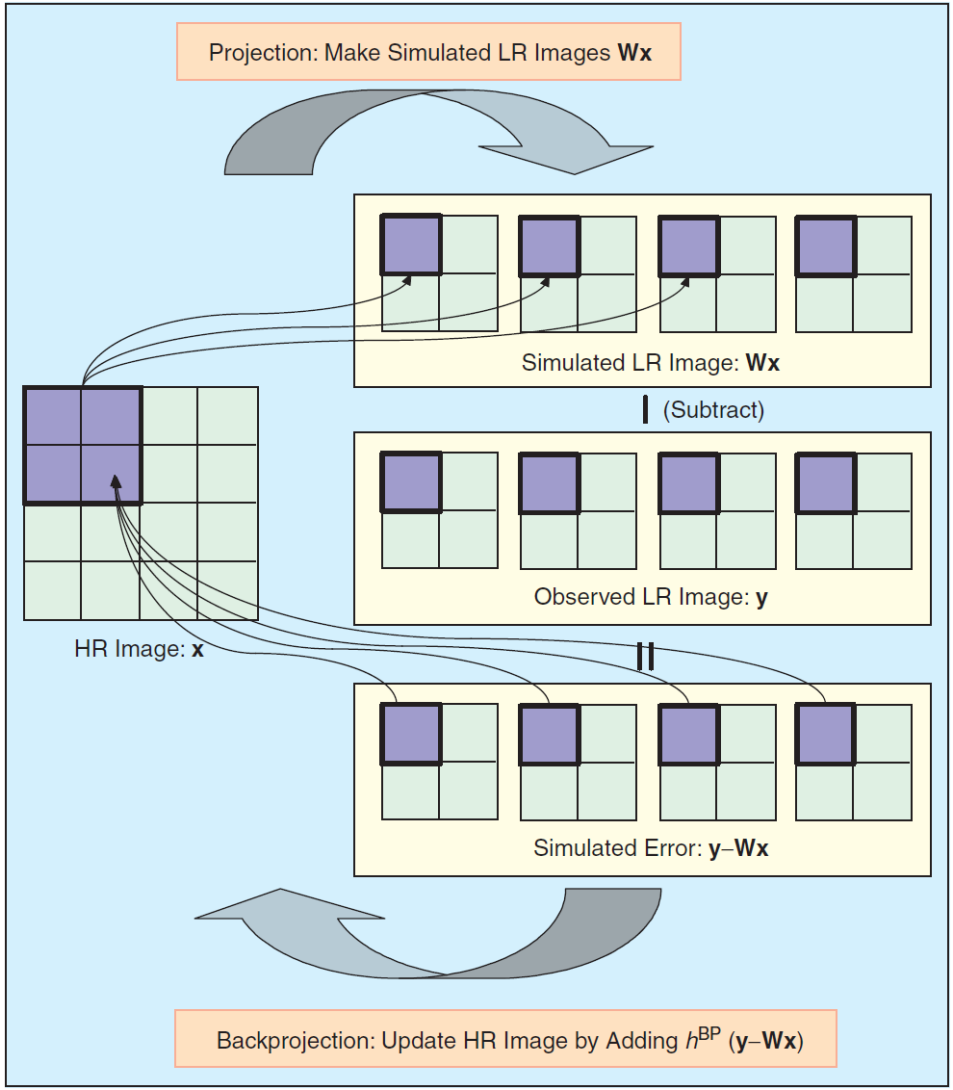




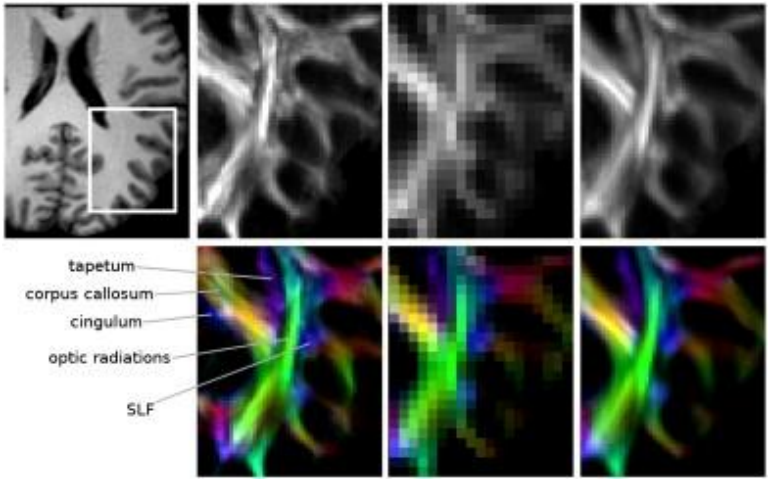
Frequency Domain Approach

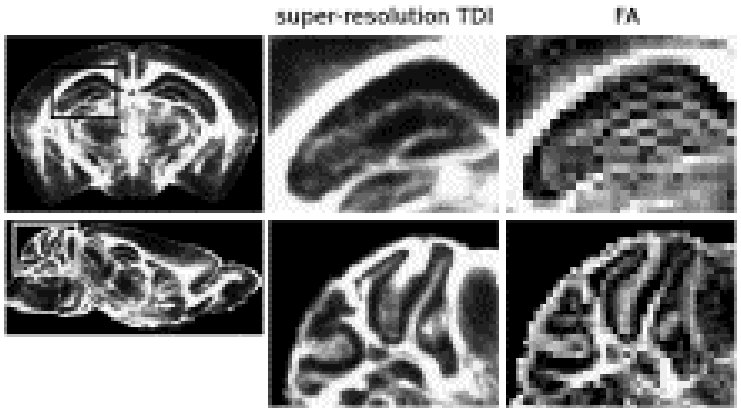


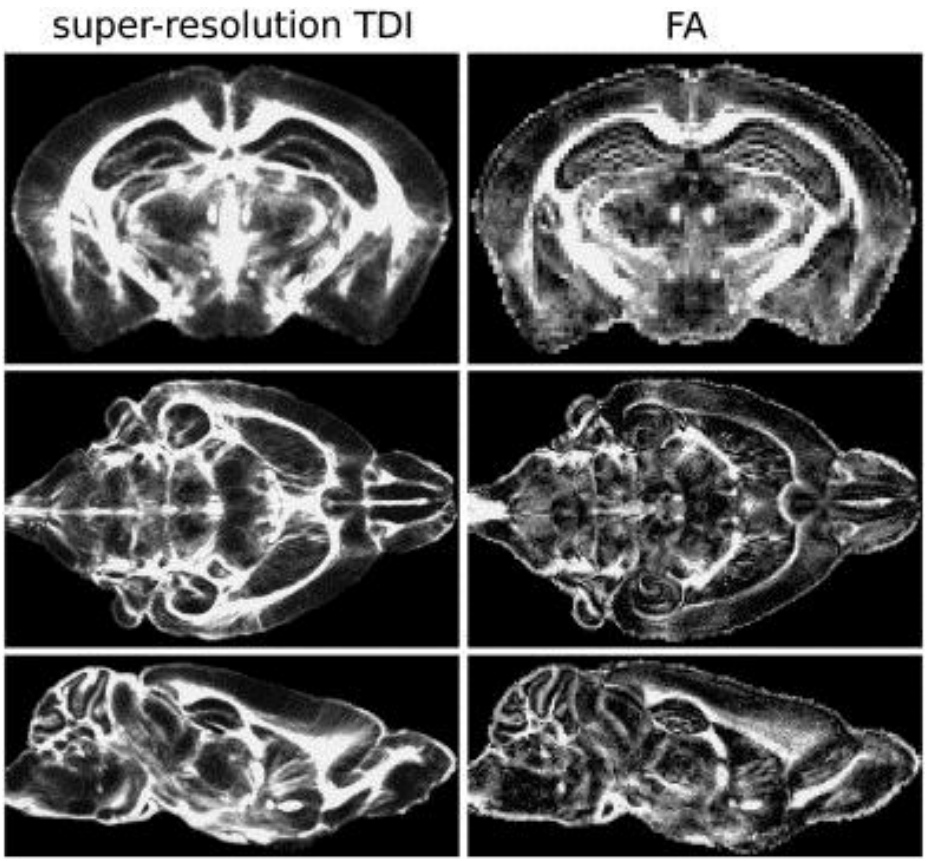
Iterative Back-Projection Approach



Examples of SRR

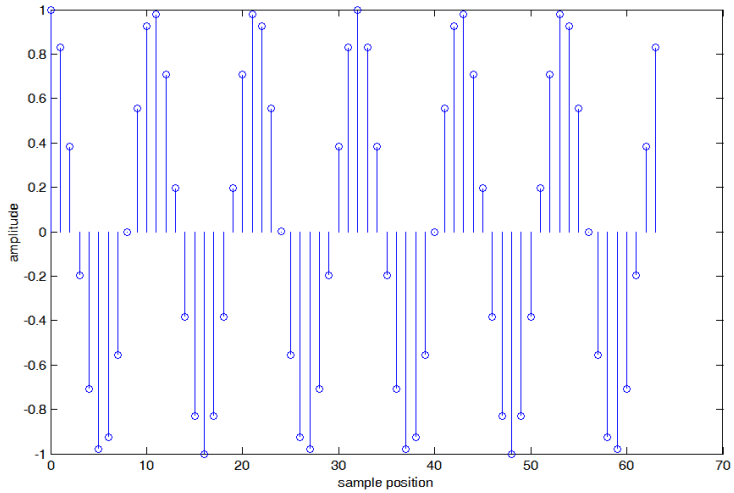




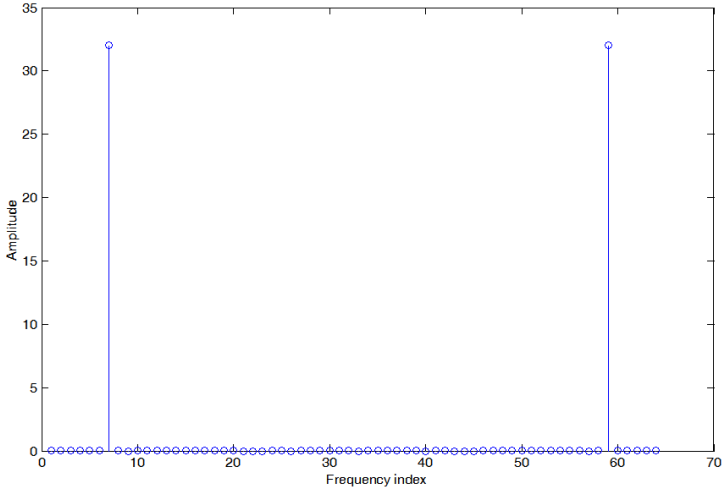


Sampling Issues

- Uniform vs. random sub-sampling



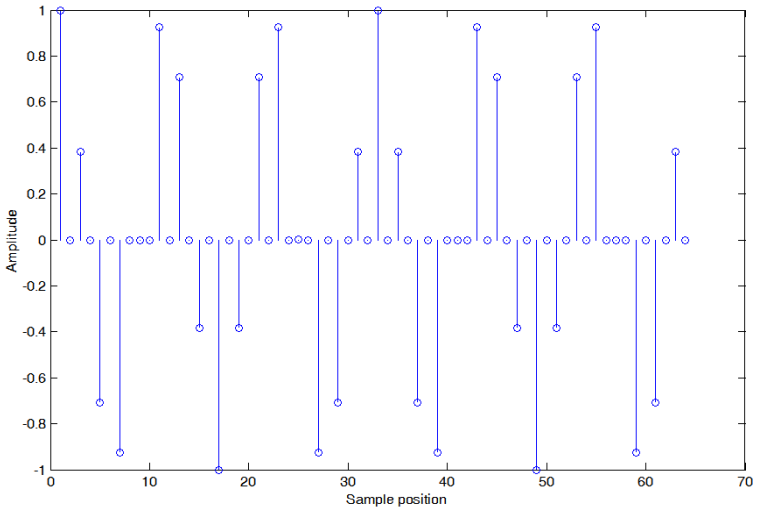
Time signal

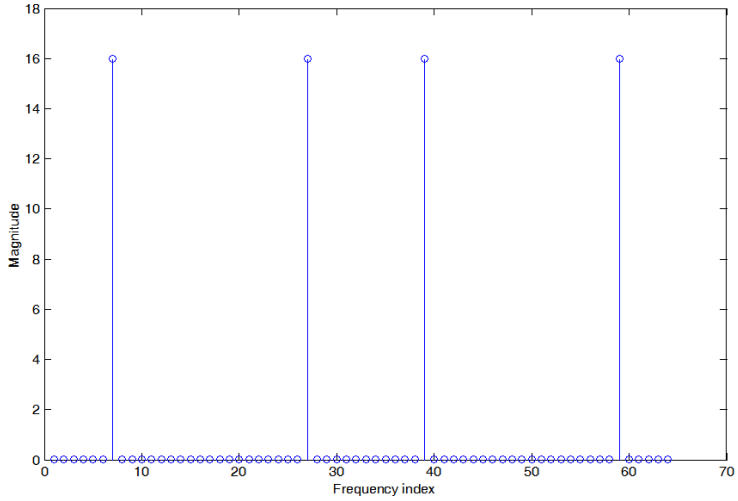


DFT

Sampling Issues

- Uniform vs. random sub-sampling



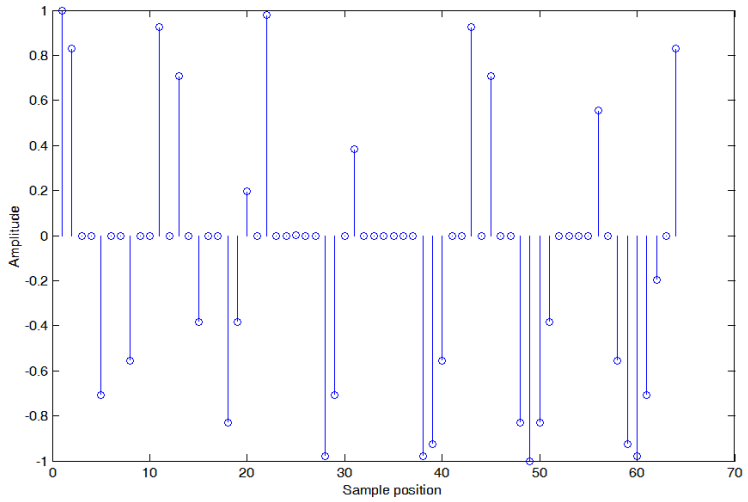


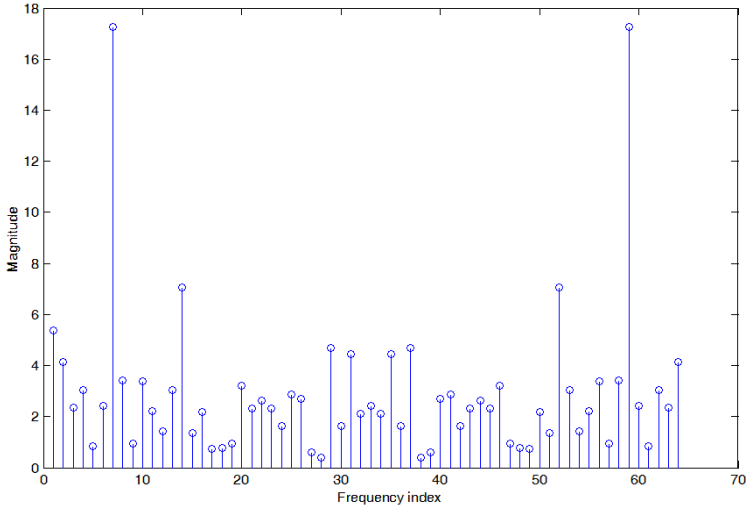
Time signal DFT

Ambiguity with uniform sub-sampling!

Sampling Issues

- Uniform vs. random sub-sampling



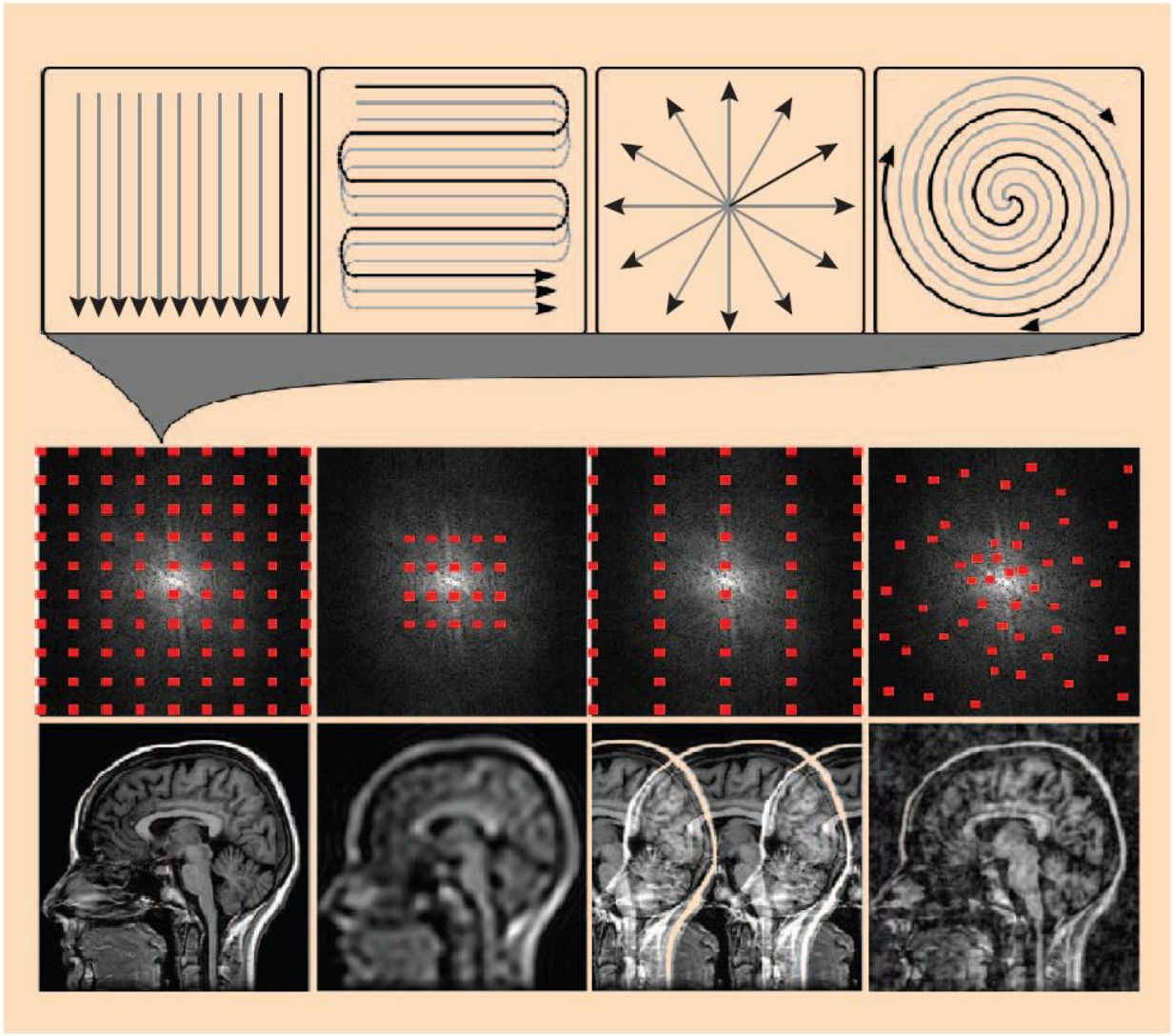


Time signal

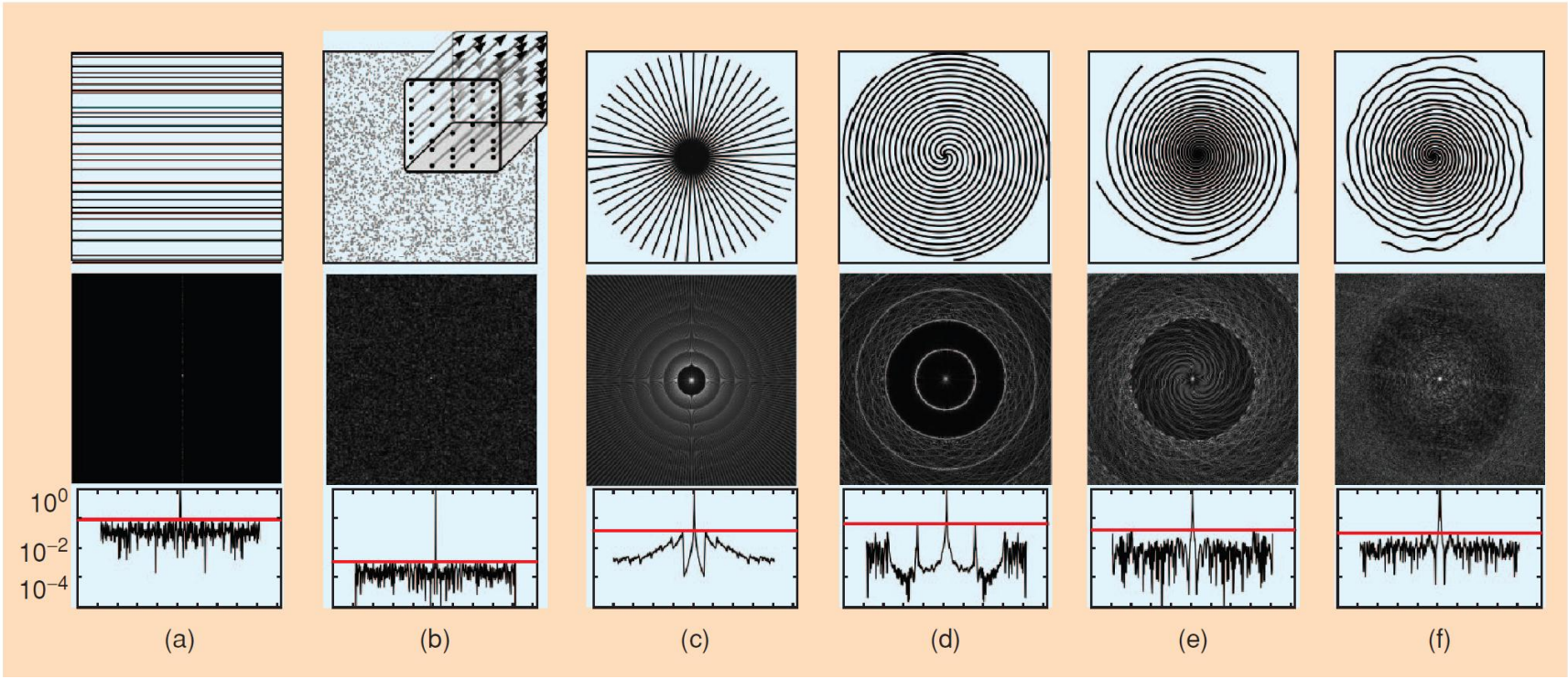
DFT

No ambiguity with non-uniform sub-sampling!

Undersampling Artifacts



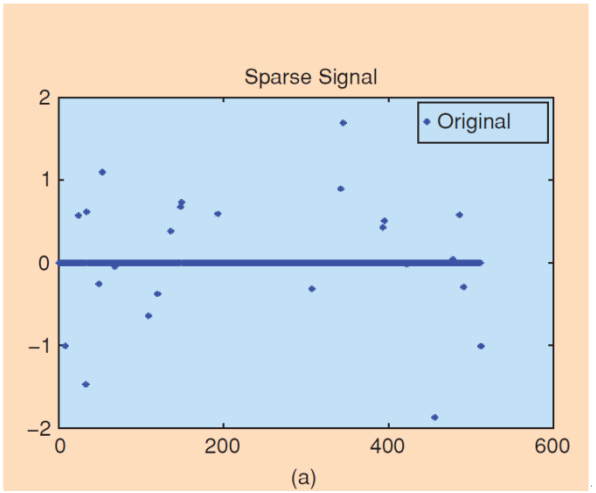
Undersampling Artifacts

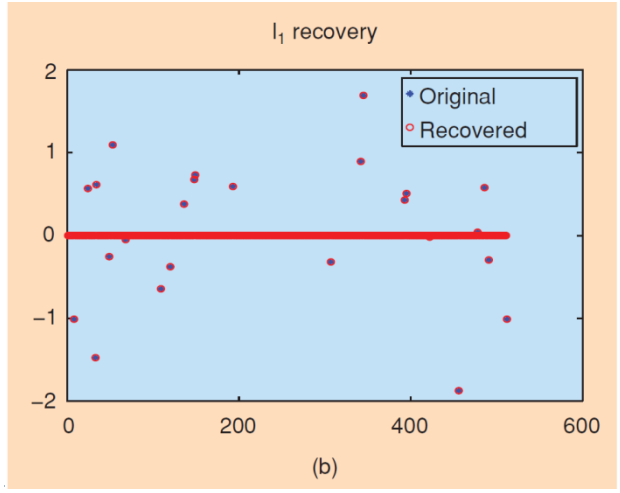


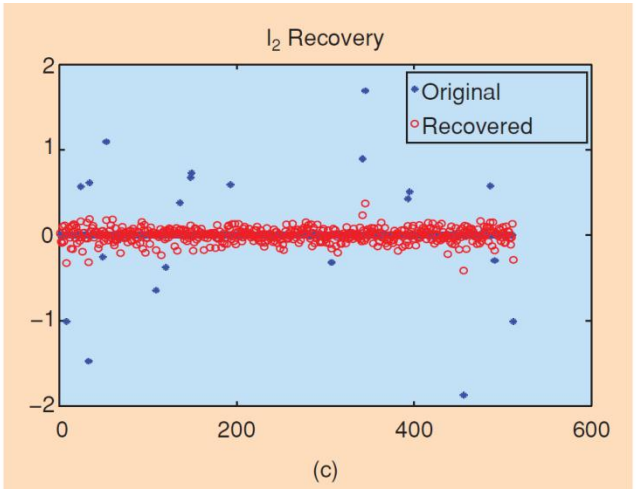
Compressed Sensing

$$\min_{\tilde{x}\in\mathbb{R}^n} \|\tilde{x}\|_{\ell_1} \qquad \text{subject to} \qquad A\tilde{x} = y\ (= Ax).$$

[FIG2] (a) A sparse real valued signal and (b) its reconstruction from 60 (complex valued) Fourier coefficients by ℓ_1 minimization. The reconstruction is exact. (c) The minimum energy reconstruction obtained by substituting the ℓ_1 norm with the ℓ_2 norm; ℓ_1 and ℓ_2 give wildly different answers. The ℓ_2 solution does not provide a reasonable approximation to the original signal.

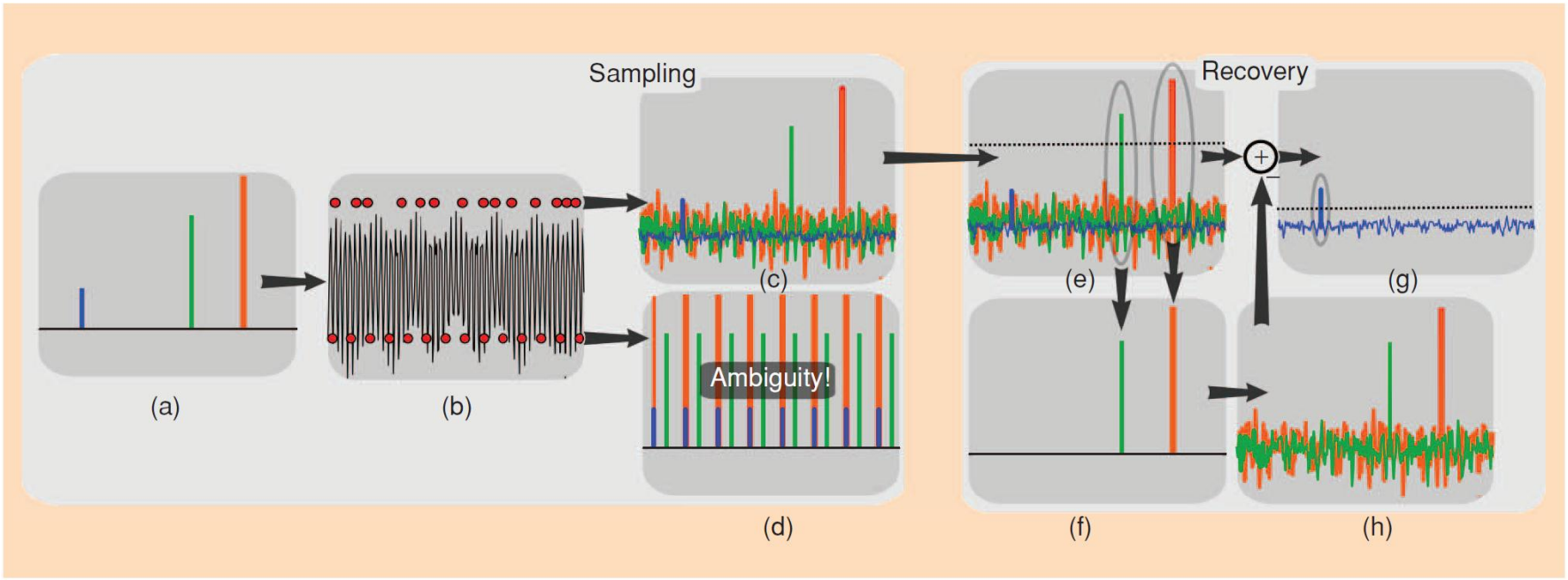






Undersampled Signal Recovery Procedure

- A sparse signal (a) is 8-fold undersampled in its 1-D *k-space* domain (b). Equispaced undersampling results in signal aliasing (d) preventing recovery. Pseudo-random undersampling results in incoherent interference (c). Some strong signal components stick above the interference level, are detected and recovered by thresholding (e) and (f). The interference of these components is computed (g) and subtracted (h), thus lowering the total interference level and enabling recovery of weaker components.



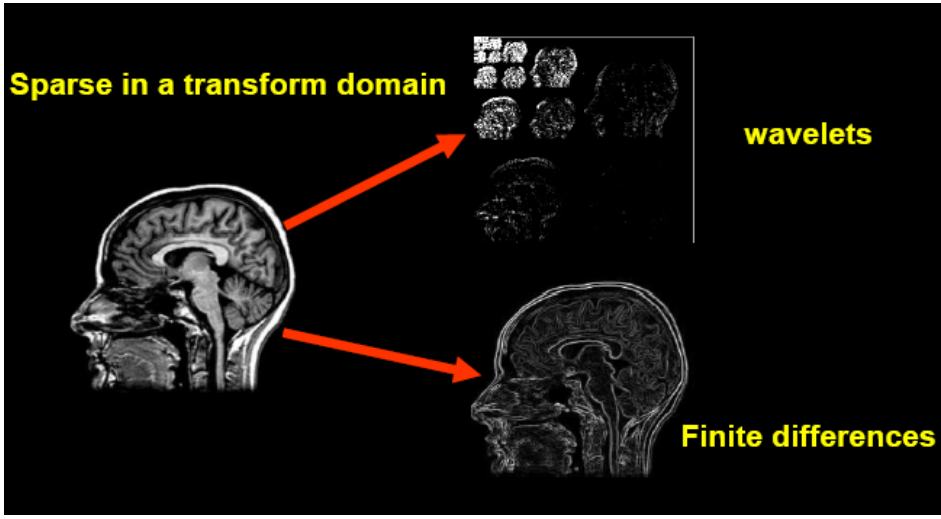
Compressed Sensing in MRI

- Sparse image reconstruction
  (Candes and Donoho 2003-2004)

$$\min \quad \|Tm\|_1$$

$$s.t. \quad \|Fm - y\|_2 \leq \varepsilon$$

  - *m* image
  - *y* measured k-space data
  - *T* transform that sparsifies image
  - *F* undersampled Fourier matrix

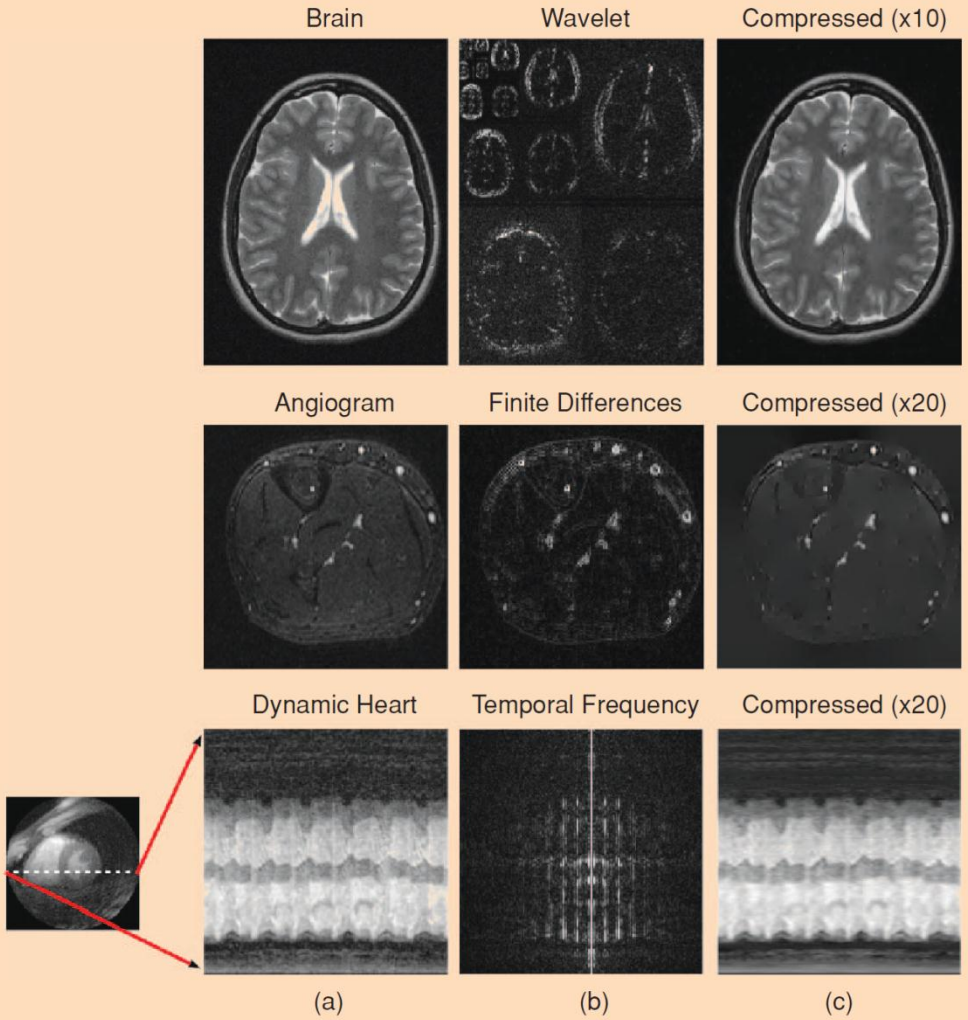


After Lustig et al 2005

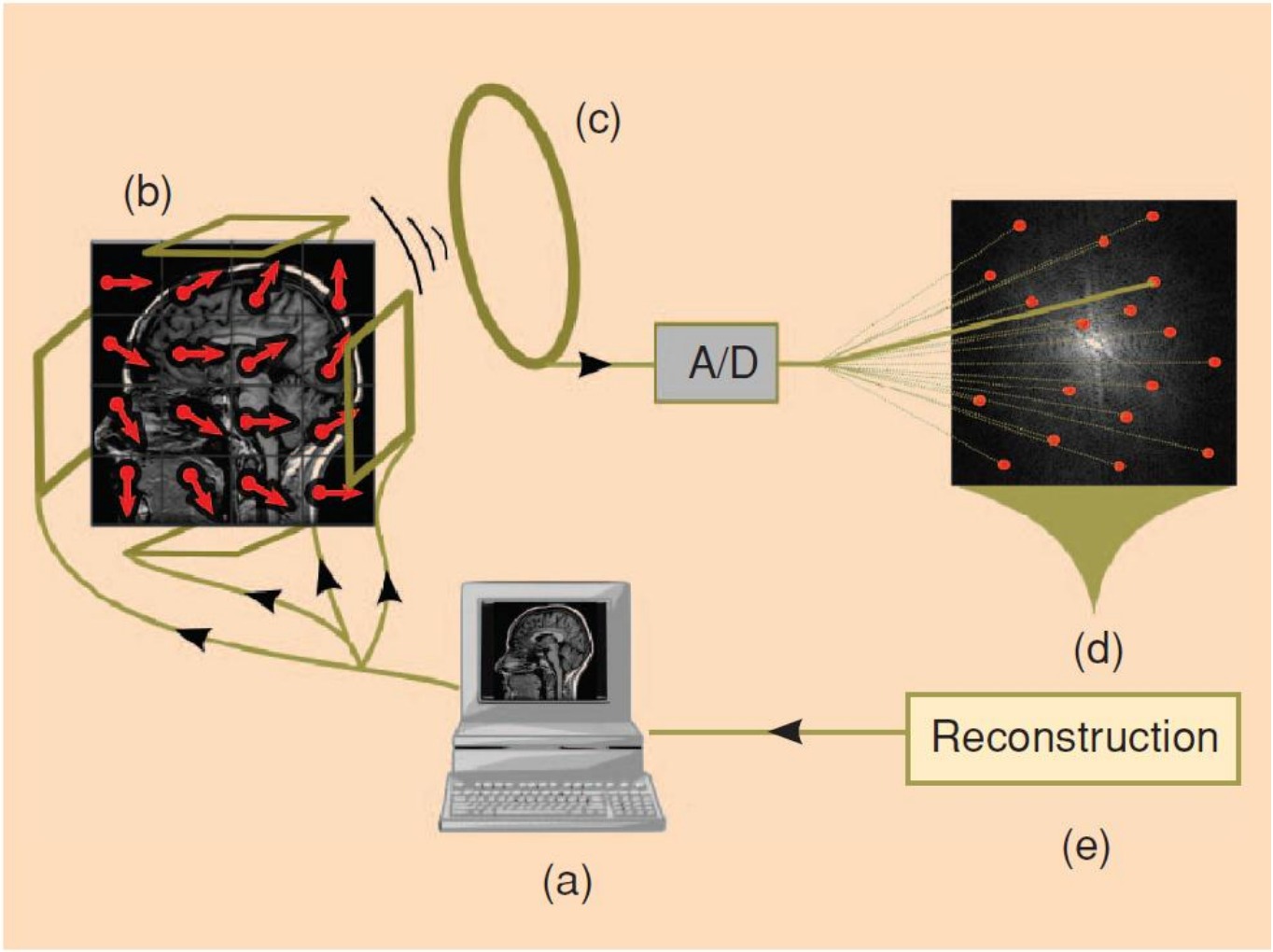
Transform Sparsity of MRI

MANY NATURAL SIGNALS ARE SPARSE OR COMPRESSIBLE IN THE SENSE THAT THEY HAVE CONCISE REPRESENTATIONS WHEN EXPRESSED IN THE PROPER BASIS.

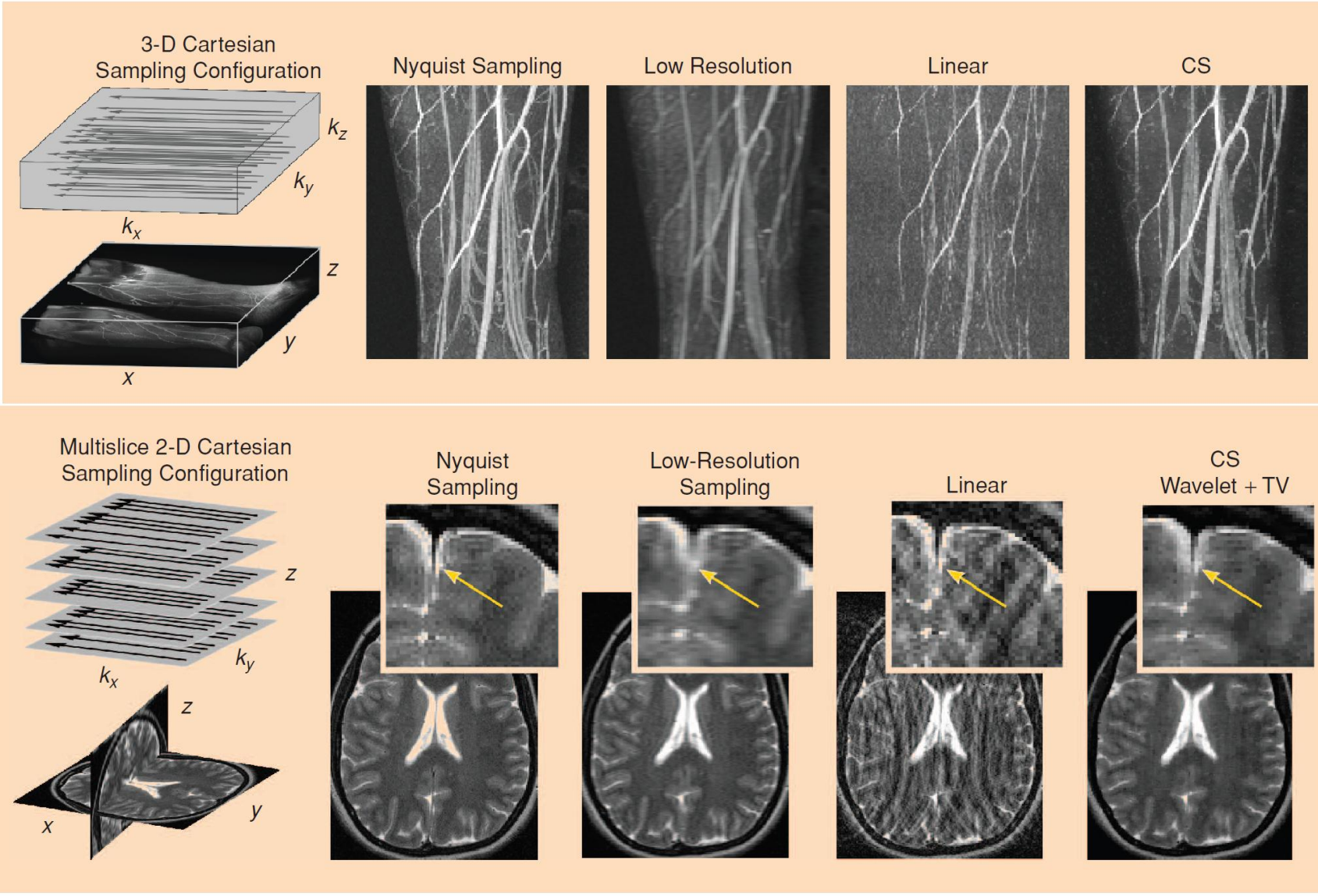
WHAT IS MOST REMARKABLE ABOUT THESE SAMPLING PROTOCOLS IS THAT THEY ALLOW A SENSOR TO VERY EFFICIENTLY CAPTURE THE INFORMATION IN A SPARSE SIGNAL WITHOUT TRYING TO COMPREHEND THAT SIGNAL.



Compressed Sensing MRI



Compressed Sensing Examples



Exercise

- 1. Design an experiment to use compressed sensing to estimate a 1D sparse signal from its random samples. Use different degrees of undersampling and comment on the quality of your results
- 2. Do a literature search about the topics related to SRR in medical imaging applications and come up with a list of references that should be the starting point for a biomedical engineering researcher in this area.