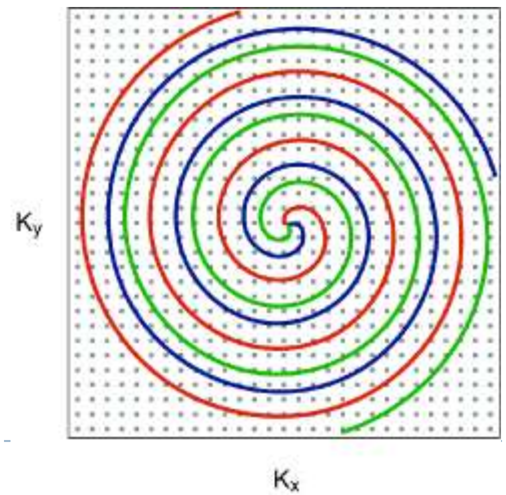
Medical Image Reconstruction
Term II – 2010

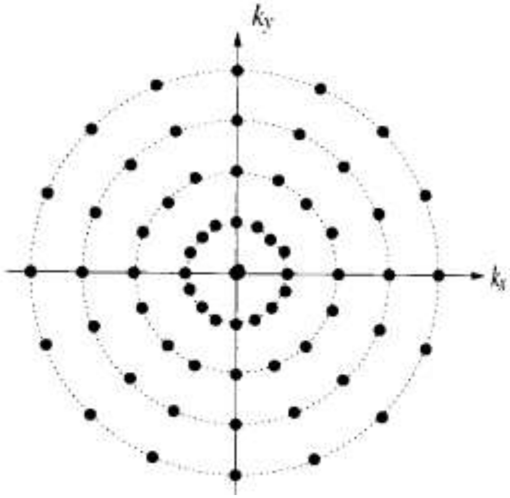
Topic 2: Reconstruction from Nonuniformly Sampled k-Space

Professor Yasser Mostafa Kadah

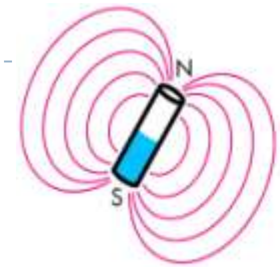
Reconstruction from k-Space Samples

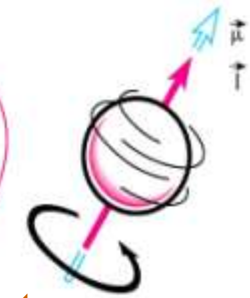
- Practical application in several imaging modalities
  - Computed Tomography (CT): radial sampling
  - Magnetic Resonance Imaging (MRI): several nonuniform sampling strategies are used such as radial, spiral, and random sampling
- Main problem: how to compute Inverse 2D DFT to compute the image!





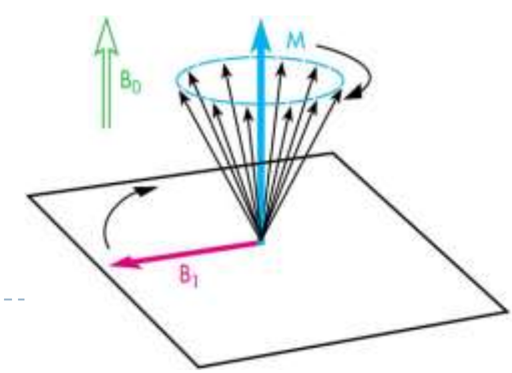
Nuclear Magnetic Resonance





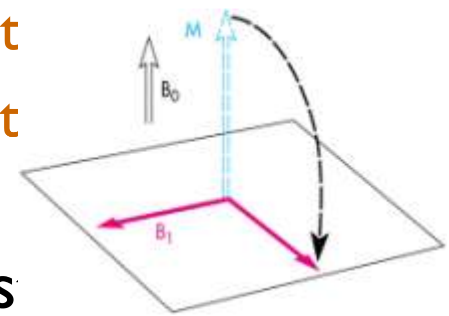
- Magnetic resonance phenomenon
  - Nucleus acts as a tiny bar magnet
  - In the presence of external magnetic field, such tiny magnets are arranged randomly
  - Net magnetization at steady state = 0
- In the presence of a strong magnet field, tiny magnets tend to align along the field
  - Probability can be computed from Boltzmann ratio
  - Nonzero net magnetization results
- Precession frequency given by Larmor Eqn.

$$\omega = \gamma B_0$$

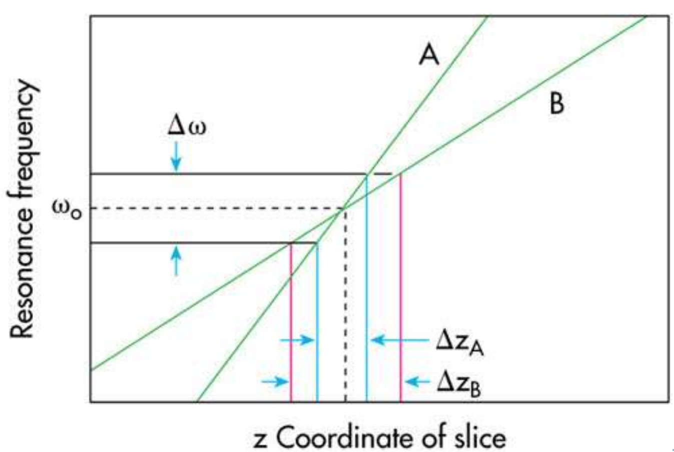


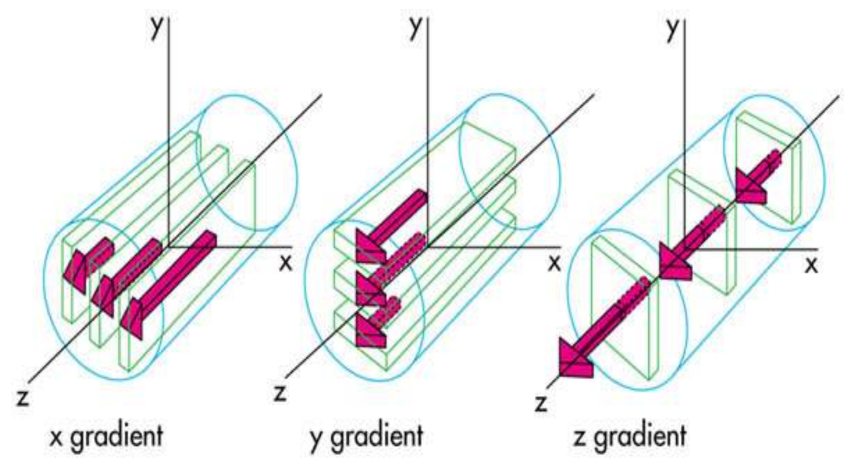
Magnetic Resonance Imaging

- RF energy applied at the Larmor frequency cause nutation of the net magnetization
  - Make it possible to measure the MR net magnet
  - Relaxation to steady state magnetization after it induces RF signal in reception coils



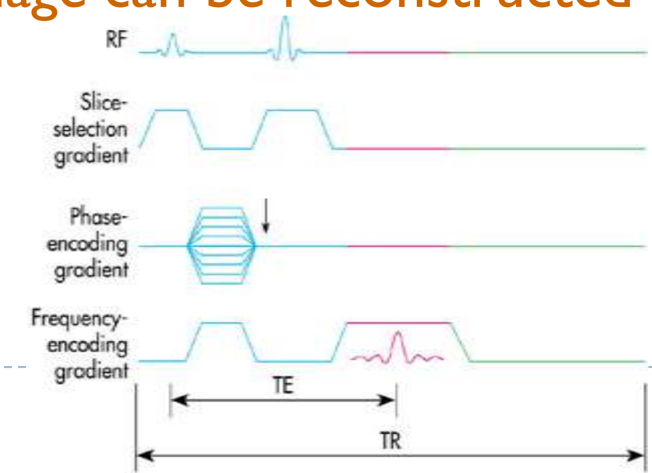
- Using spatial encoding, an image can be cons the received signal

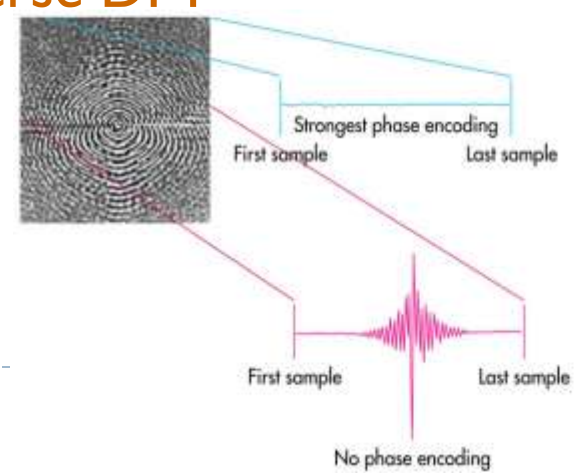




MRI: Data Collection

- MR Imaging sequences
  - B_0 is applied all the time
  - RF pulse to excite the spins
  - Magnetic field gradients for spatial encoding
  - Data sampling
- The simplest case is when the sampling is rectilinear
  - k-space trajectory is zeroth gradient moment
  - k-space is uniformly sampled 2D FT of the image
  - Image can be reconstructed using inverse DFT





General Reconstruction Problem

k-space Data

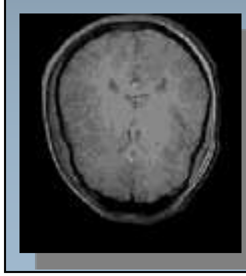
Imaging Conditions

- Reception Coils
- Sampling pattern
- Stimulus
- Diffusion weightings
- A priori information
- Model

Image Reconstruction System

MR Image

- Anatomical image
- Functional image



Specific Problems

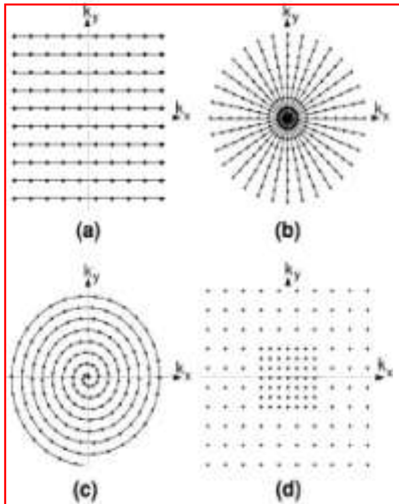
- Anatomical Imaging
  - Reconstruction from nonuniformly sampled k-space (Gridding)
  - Reconstruction from partial Fourier data (Partial Fourier)
  - Reconstruction from k-space magnitude-only data (Motion correction)
  - Reconstruction for parallel imaging (Parallel imaging)
- Functional Imaging
  - Reconstruction of connectivity maps using diffusion tensor imaging (DTI)

Gridding Problem

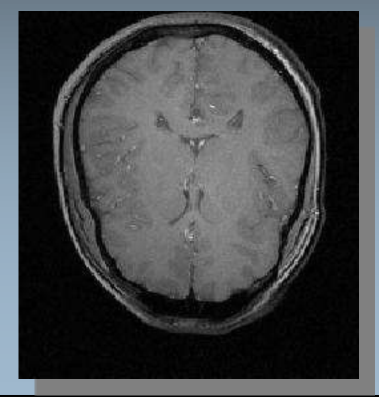
Sampled data

Image Reconstruction System

MR Image

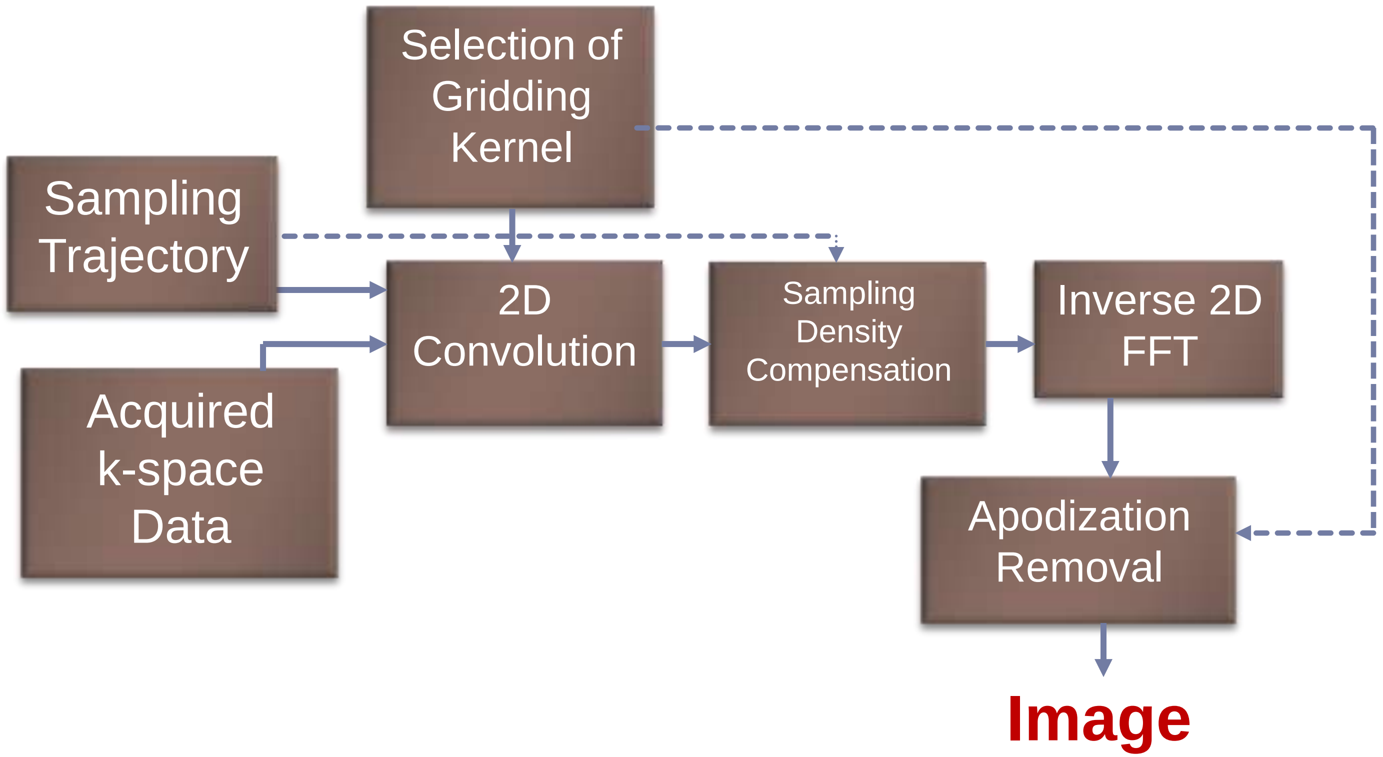


Sampling Pattern



Conventional Gridding

- Conventional gridding through convolution with interpolation kernel (O'Sullivan, 1985; Jackson et al., 1991; Meyer et al., 1992)



Meyer Gridding Algorithm

Fast Spiral Coronary Artery Imaging

CRAIG H. MEYER,* BOB S. HU,† DWIGHT G. NISHIMURA,
AND ALBERT MACOVSKI‡

4. Convolve the data into the 2D array. This is done by multiplying each density-compensated data point by a Kaiser–Bessel window a few grid points in width, evaluating the result at each grid point within the window, and adding the result into the array. An auxiliary array is used to keep track of the amount of energy put into each grid point, which is the product of the density-compensation factor and the Kaiser–Bessel function evaluation.

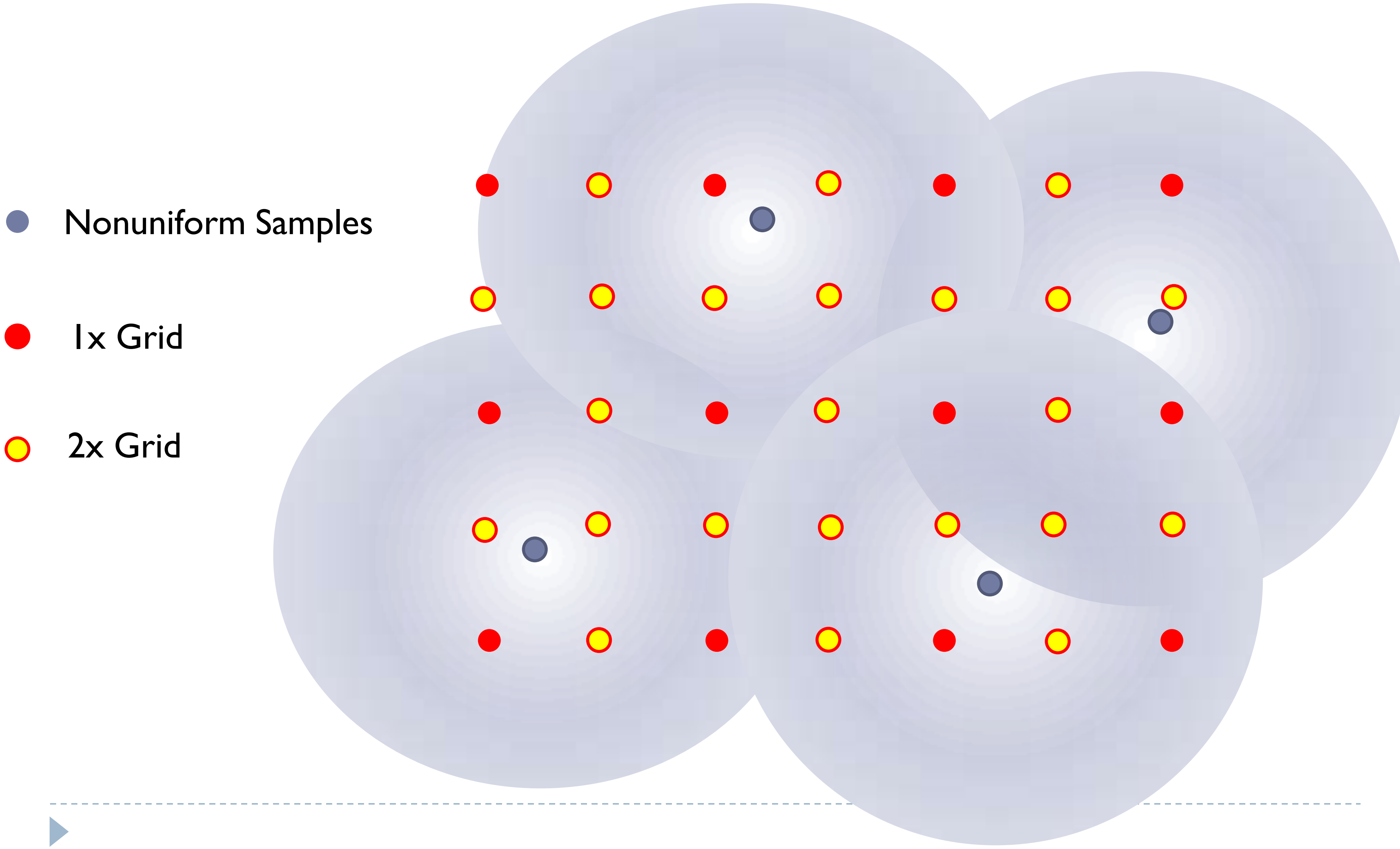
5. Normalize the energy in each grid point by dividing by the auxiliary array. If the density compensation of step 3 is done properly, this step results in only a minor correction.

6. Perform a complex 2D FFT.

7. Divide by the transform of the Kaiser–Bessel window to remove the apodization resulting from the convolution of step 4.

8. Take the magnitude of the result.

Meyer Gridding Algorithm



Jackson Gridding Window Selection

Selection of a Convolution Function for Fourier Inversion Using Gridding

John I. Jackson, Craig H. Meyer, Dwight G. Nishimura, *Member, IEEE*, and Albert Macovski, *Fellow, IEEE*

TABLE I
THE PARAMETER VALUES FOR EACH FUNCTION TYPE THAT PROVIDE THE LEAST RELATIVE ALIASED ENERGY WHEN GRIDDING ONTO A REGULAR GRID

Window Width	Two-Term cos	Three-Term cos		Gaussian	Kaiser-Bessel
	α	α	β	σ	β
1.5	0.7600	0.8701	0.2311	0.4241	1.9980
2.0	0.7146	0.8099	0.3108	0.4927	2.3934
2.5	0.6185	0.6932	0.4176	0.4839	3.3800
3.0	0.5534	0.5995	0.4675	0.5063	4.2054
3.5	0.5185	0.5383	0.4831	0.5516	4.9107
4.0		0.4998	0.4891	0.5695	5.7567
4.5		0.4653	0.4972	0.5682	6.6291
5.0		0.4463	0.4985	0.5974	7.4302

Jackson's Gridding Window Selection

TABLE II

THE PARAMETER VALUES FOR EACH FUNCTION TYPE THAT PROVIDE THE LEAST RELATIVE ALIASED ENERGY WHEN GRIDDING ONTO A 2× SUBSAMPLED GRID

Window Width	Two-Term cos	Three-Term cos		Gaussian	Kaiser-Bessel
	α	α	β	σ	β
1.5	0.5273	0.4715	0.4917	0.2120	6.6875
2.0	0.5125	0.4149	0.4990	0.2432	9.1375
2.5	0.5076	0.4011	0.4996	0.2691	11.5250
3.0	0.5068	0.3954	0.4997	0.2920	13.9086
3.5	0.5051	0.3897	0.4999	0.3145	16.2734
4.0		0.3850	0.5000	0.3363	18.5547
4.5		0.3833	0.5000	0.3557	
5.0		0.3823	0.5000	0.3737	

Exercise

- Do a literature search on the problem of nonuniform sampling in 2D and summarize your findings about the sampling criteria to avoid aliasing in less than 500 words (in addition to a list of references). [1 Point]
- (a) Write a program to generate radial k-space sampling of the Shepp-Logan phantom. (b) Write a program to perform gridding on the generated radial k-space data to compute an image. [Parts (a)+(b) together = 2 Points]
- In less than 500 words, describe how one can compare the quality of different reconstruction methods and/or parameters based on measurements from the generated images. [1 Point]