# Medical Image Reconstruction
# Term II – 2010

# Topic 3: Partial Fourier
#          Reconstruction methods

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# Partial Fourier Reconsruction

▸ PF reconstruction is based on the fact that if the object is real in image space, its Fourier transform is Hermitian.

  ▸ One-half of the *k-space is needed to* reconstruct a real image

▸ In reality, however, the reconstructed images are complex.
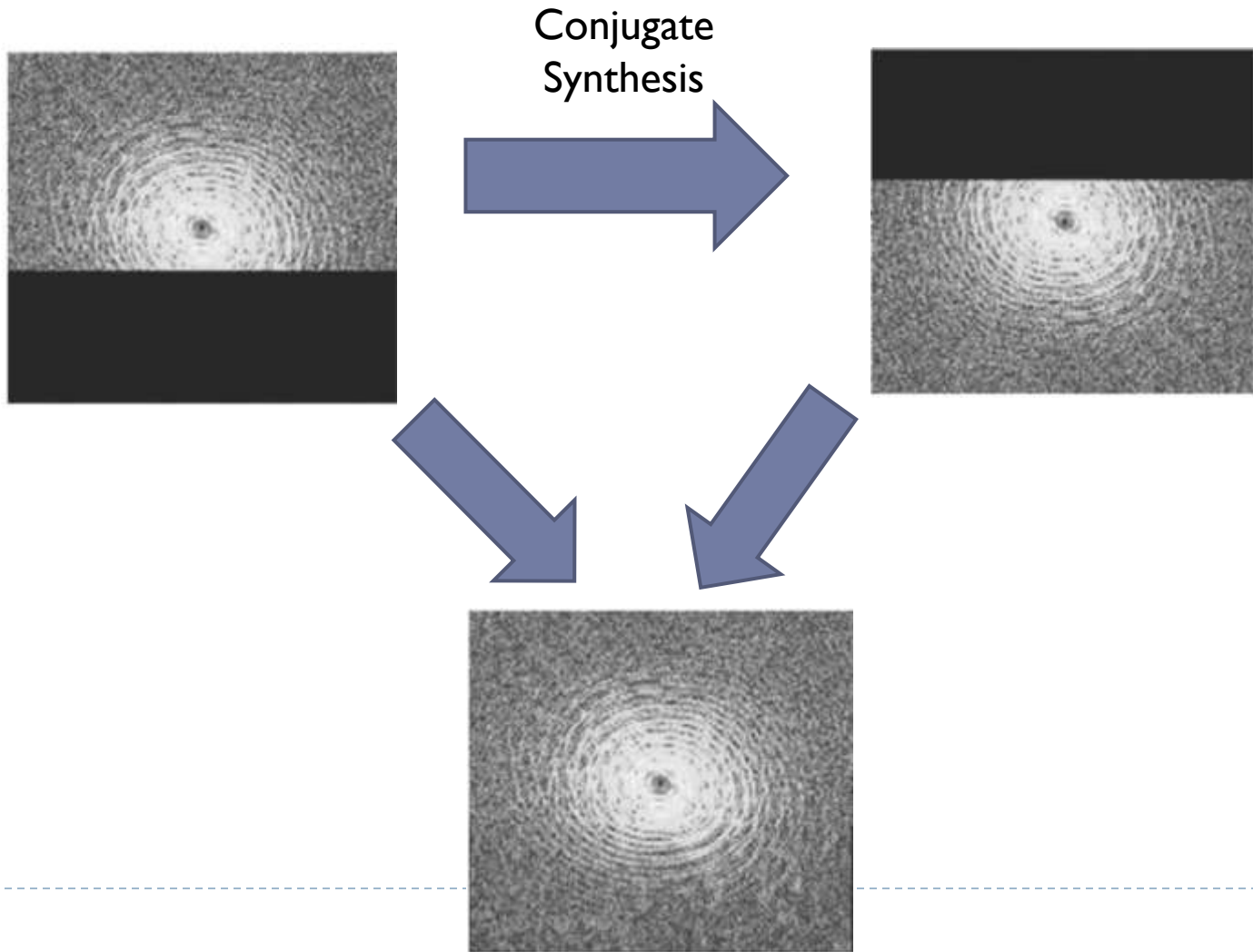


Partial Fourier
Reconsruction
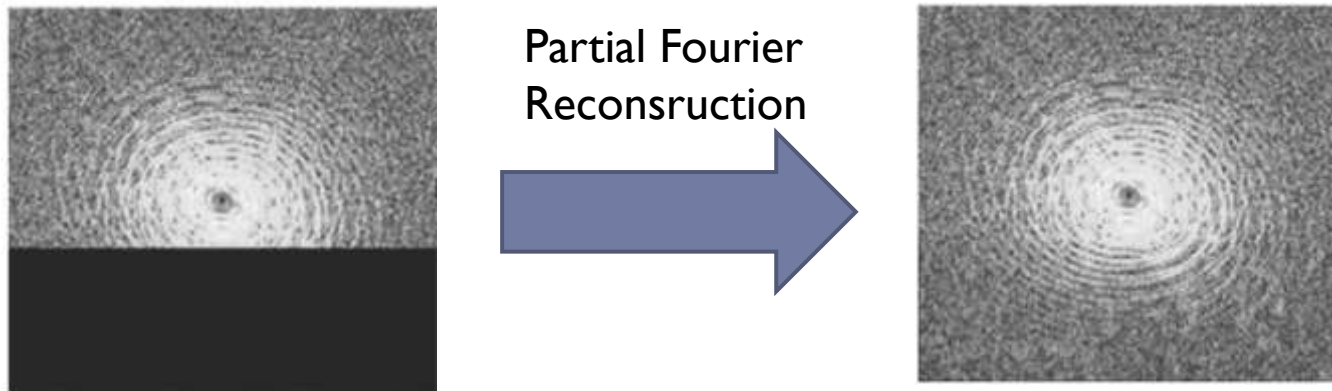
**Quantitative Evaluation of Several Partial Fourier Reconstruction Algorithms Used in MRI**
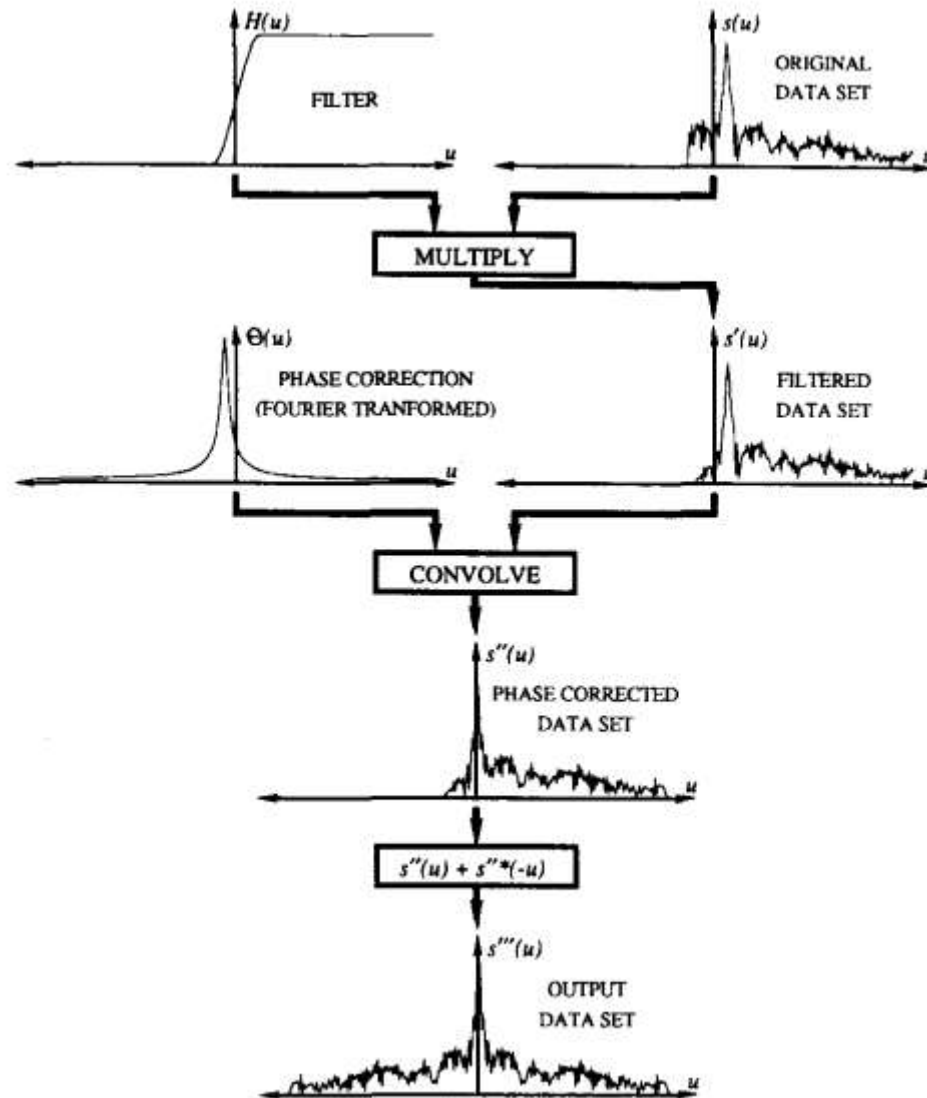
G. McGibney, M. R. Smith, S. T. Nichols, A. Crawley
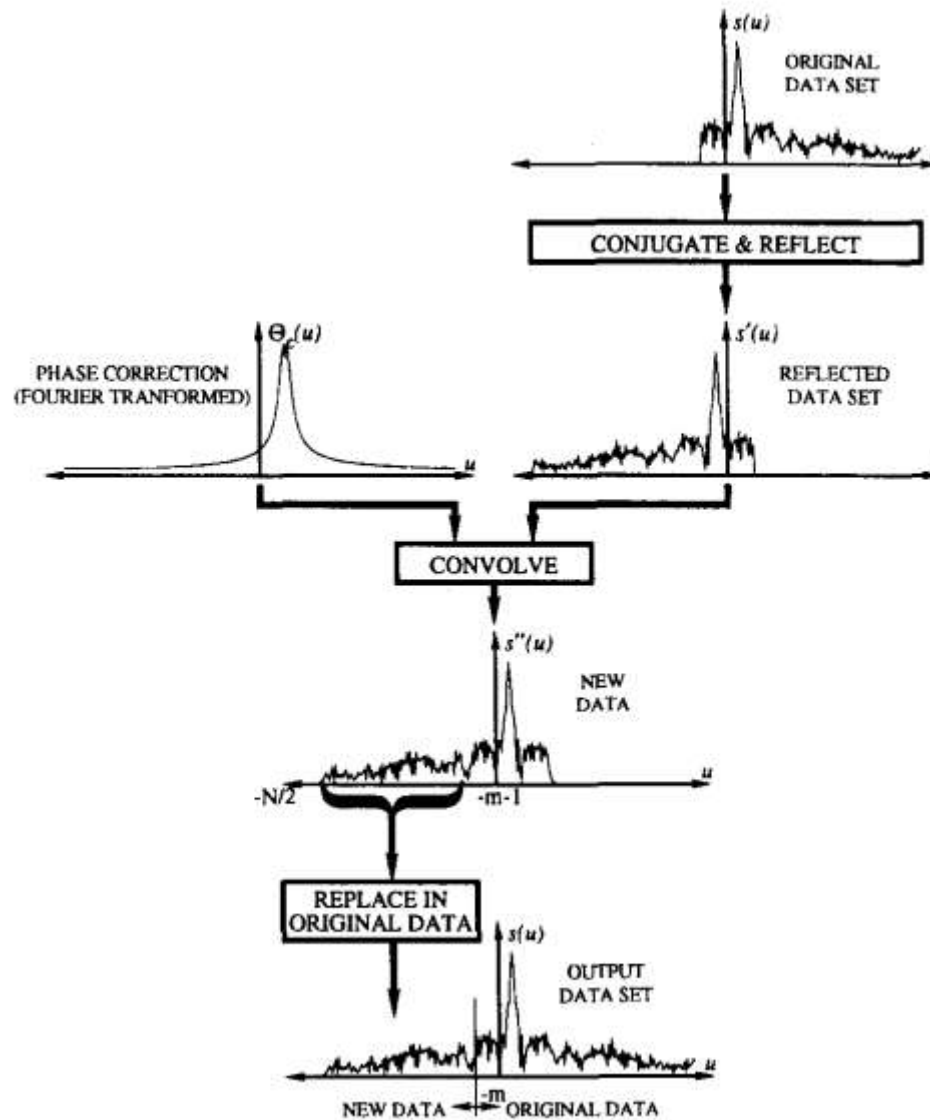
MRM 30:51–59 (1993)
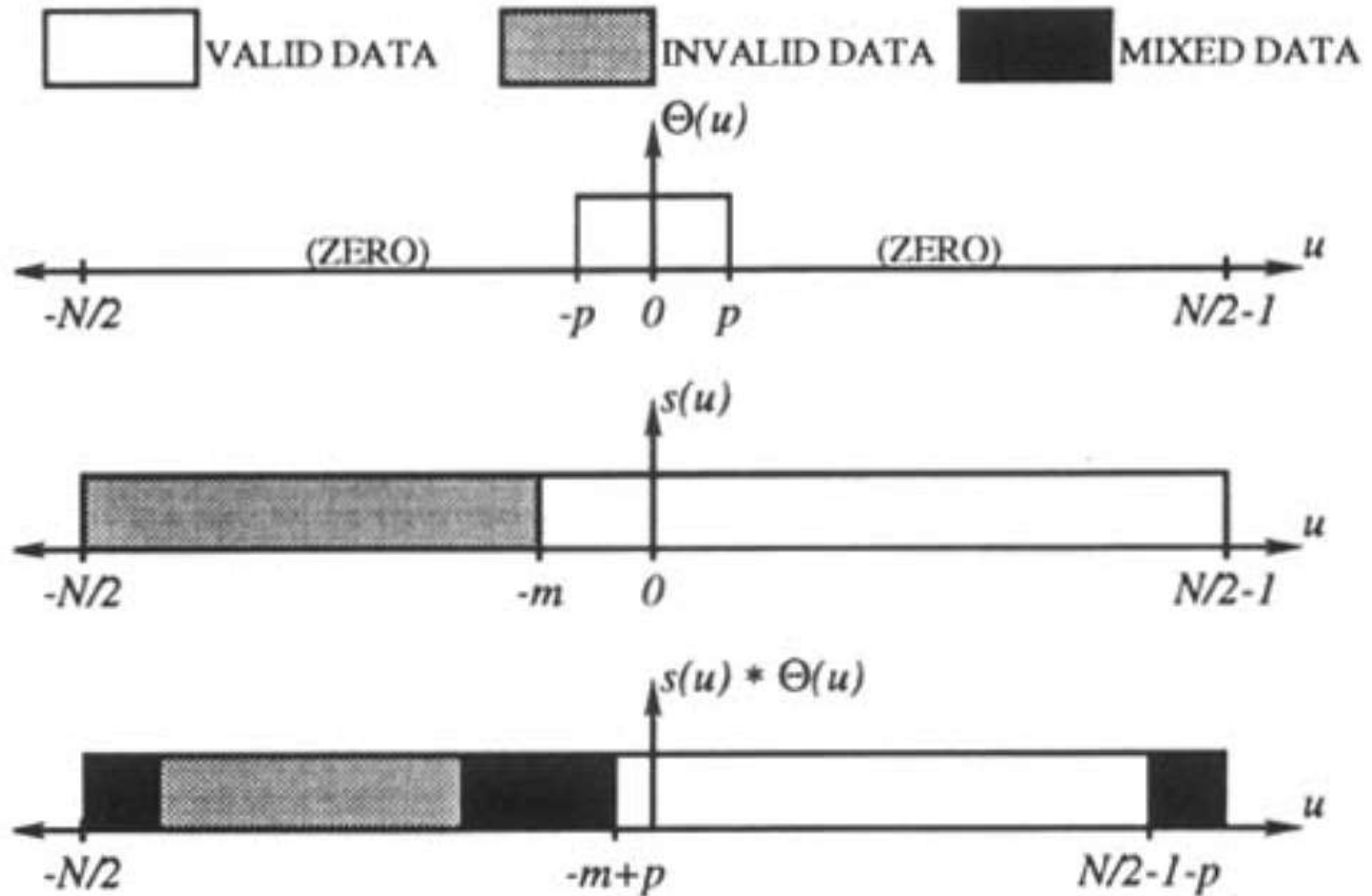
# Conjugate Synthesis

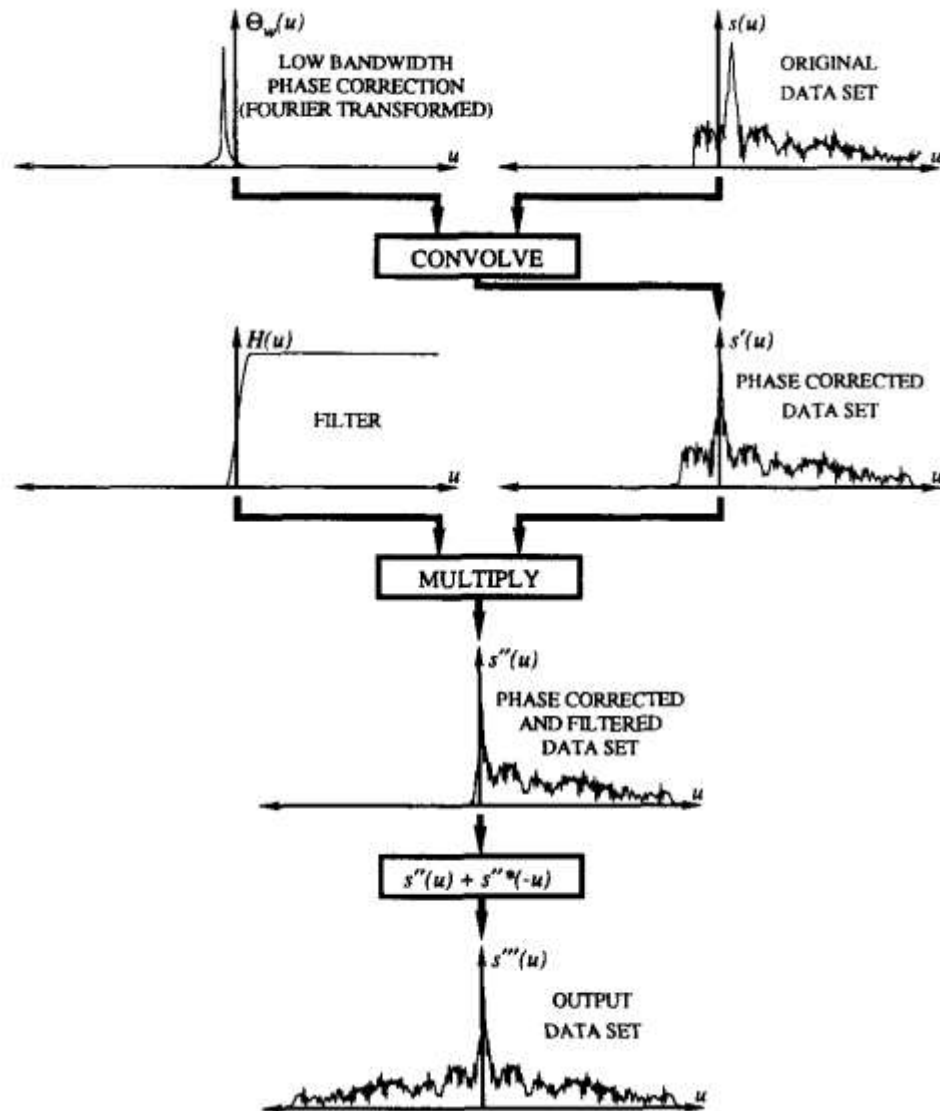▸ Assume image is purely real



Conjugate
Synthesis
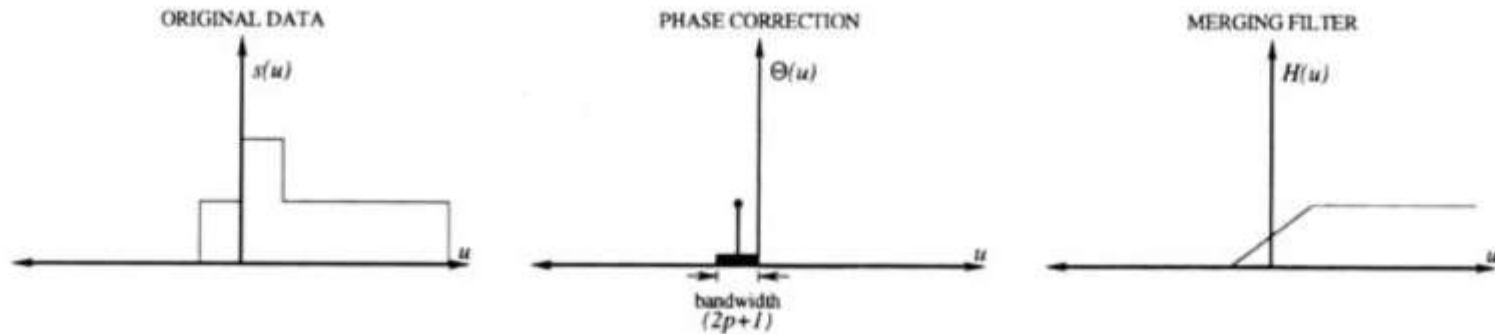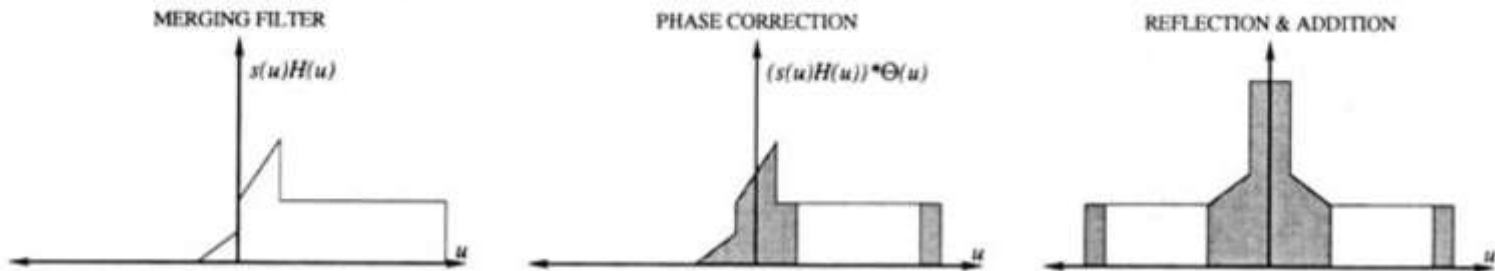
# Margosian Method

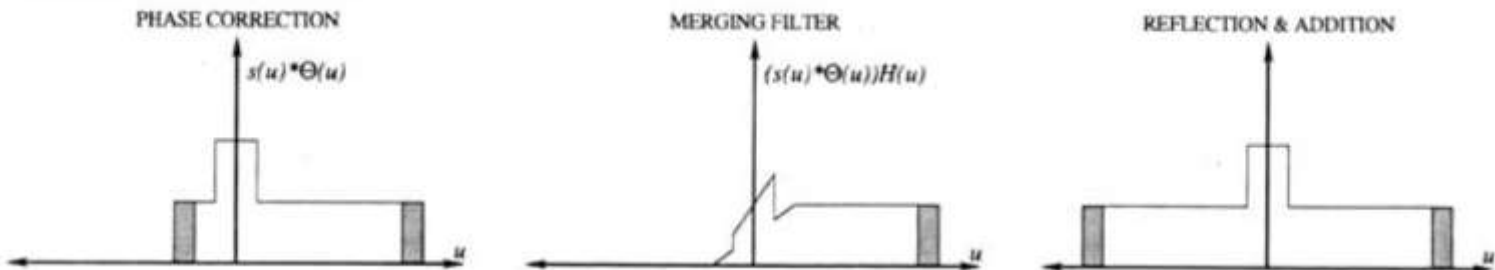# Cuppen/POCS Method

# Phase Correction Effects

# FIR and MoFIR Methods

# Comparison of Distortions in Margosian and MoFIR Methods



ORIGINAL DATA — $s(u)$

PHASE CORRECTION — $\Theta(u)$ — bandwidth $(2p+1)$

MERGING FILTER — $H(u)$

**MARGOSIAN RECONSTRUCTION**

MERGING FILTER — $s(u)H(u)$

PHASE CORRECTION — $(s(u)H(u))*\Theta(u)$

REFLECTION & ADDITION

**MoFIR RECONSTRUCTION**

PHASE CORRECTION — $s(u)*\Theta(u)$

MERGING FILTER — $(s(u)*\Theta(u))H(u)$

REFLECTION & ADDITION
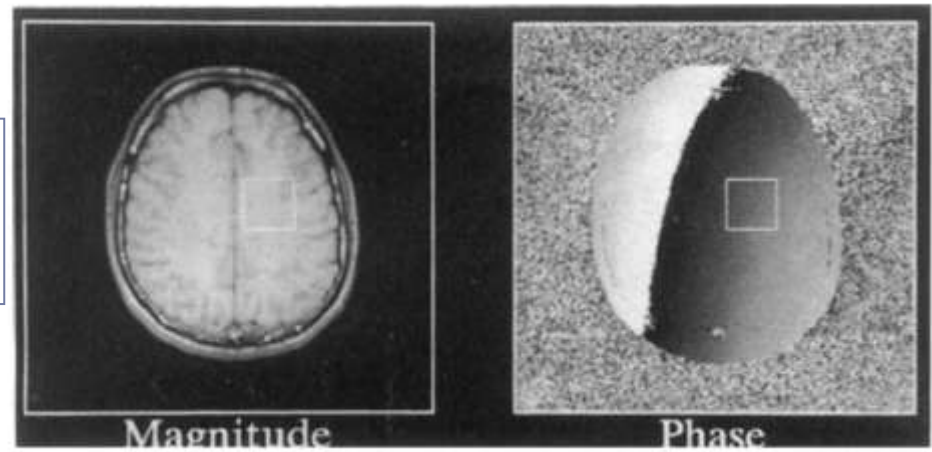
# Experimental Verification: Simulated Box and Real Data
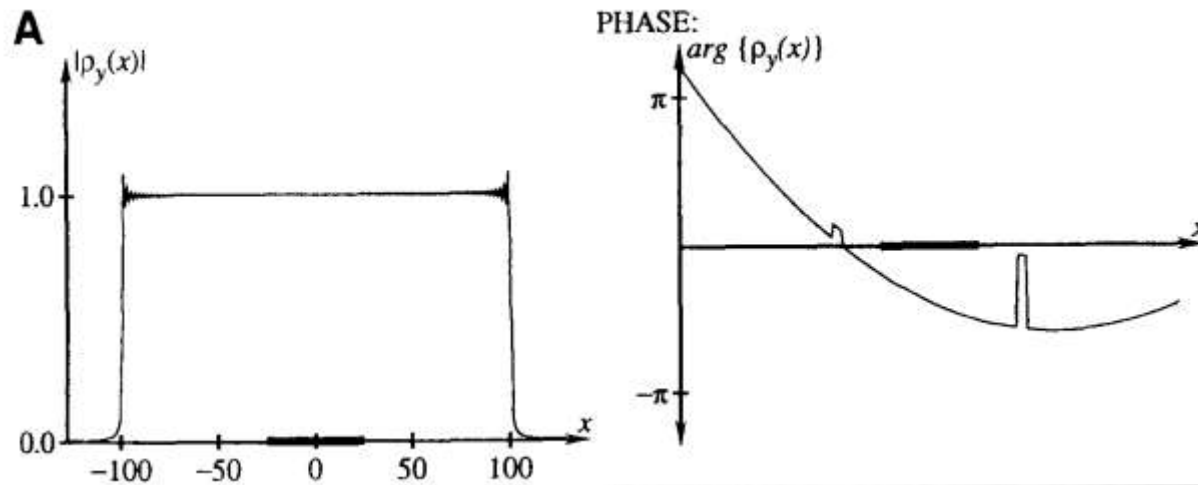


$$error = \frac{\sum_{(x,y)\epsilon R}(|\rho_{partial}| - |\rho_{full}|)^2}{\sum_{(x,y)\epsilon R}(|\rho_{full}|)^2} \cdot 100\%$$

# Results

CONJUGATE SYNTHESIS

$|\rho_y(x)|$

GE = GLOBAL ERROR
LE = LOCAL ERROR
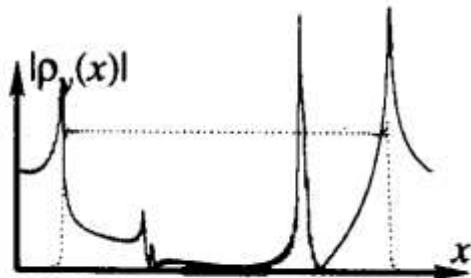
GE=89.3% LE=92.7%

EXACT ESTIMATE    INCORRECT ESTIMATE

MARGOSIAN

$|\rho_y(x)|$

$|\rho_y(x)|$
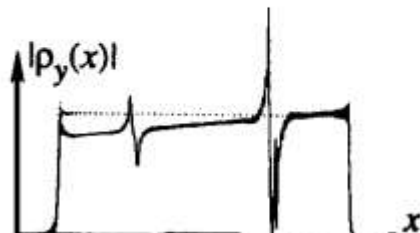
GE=2.79% LE=0.640%    GE=3.48% LE=0.650%
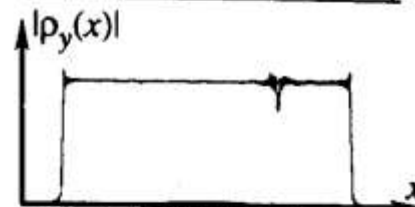
CUPPEN (4 ITERATIONS)

$|\rho_y(x)|$

$|\rho_y(x)|$

GE=.026% LE=0.001%    GE=0.986% LE=0.067%

POCS (4 ITERATIONS)

$|\rho_y(x)|$

$|\rho_y(x)|$

GE=0.072% LE=0.001%    GE=1.148% LE=0.019%

FIR

$|\rho_y(x)|$

$|\rho_y(x)|$
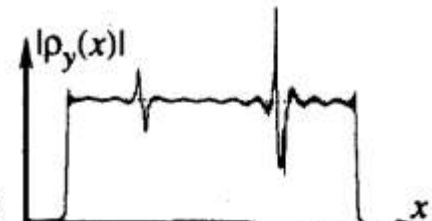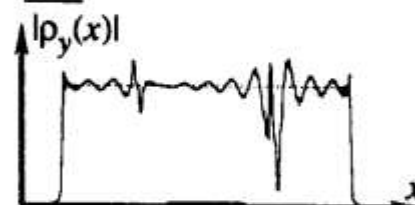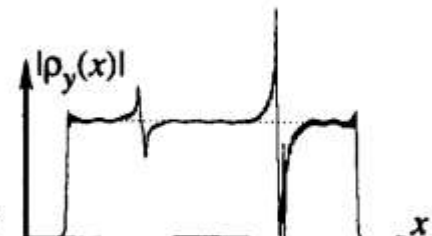
GE=1.767% LE=0.097%    GE=2.722% LE=0.004%

MoFIR

$|\rho_y(x)|$

$|\rho_y(x)|$
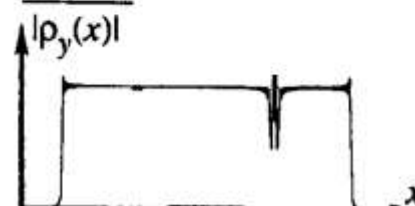
GE=0.437% LE=0.001%    GE=2.734% LE=0.003%

# Results

# Results

Comparison of the Times of the Partial Fourier Reconstruction Algorithms Both Individually and in Conjunction with the Phase Estimation Algorithms

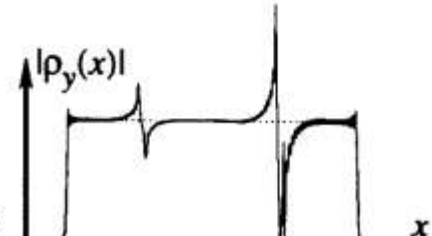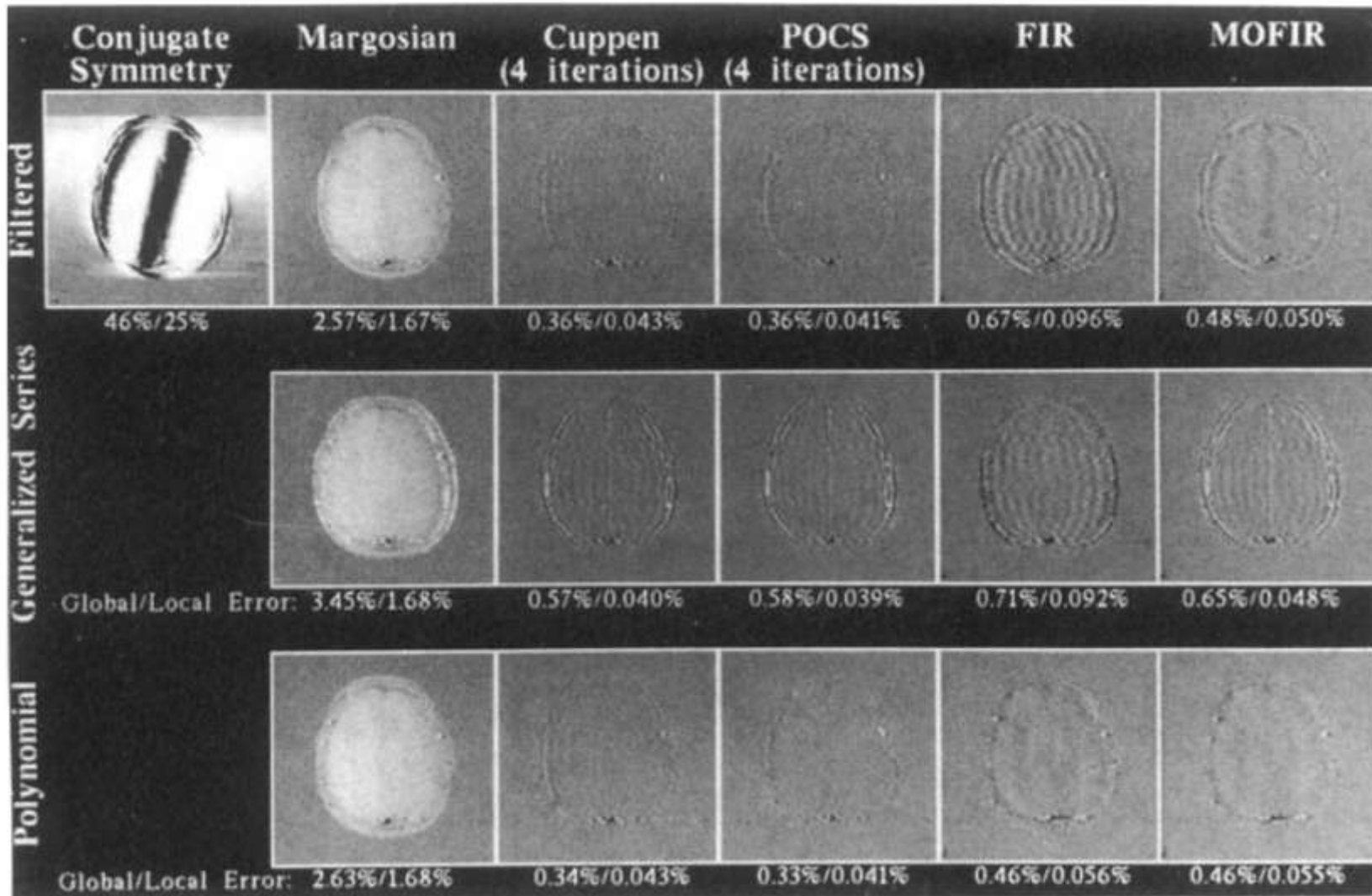| | Partial Fourier only (s) | Partial Fourier plus filtered estimate (s) | Partial Fourier plus generalized series estimate (s) | Partial Fourier plus polynomial estimate (s) |
|---|---|---|---|---|
| Conjugate-symmetry | 2.4 | 7.1 | 464 | 7.1 |
| Margosian-homodyne | 2.9 | 7.6 | 464 | 7.6 |
| Cuppen (4 iterations) | 12.5 | 17.2 | 474 | 17.2 |
| POCS (4 iterations) | 14.6 | 19.3 | 476 | 19.3 |
| *FIR* (direct) | 9.6 | 14.3 | 471 | 14.3 |
| *FIR* (circular) | 5.3 | 10.0 | 467 | 10.0 |
| *MoFIR* (circular) | 5.3 | 10.0 | 467 | 10.0 |

# Results

- 256x256 image, 16 Lines
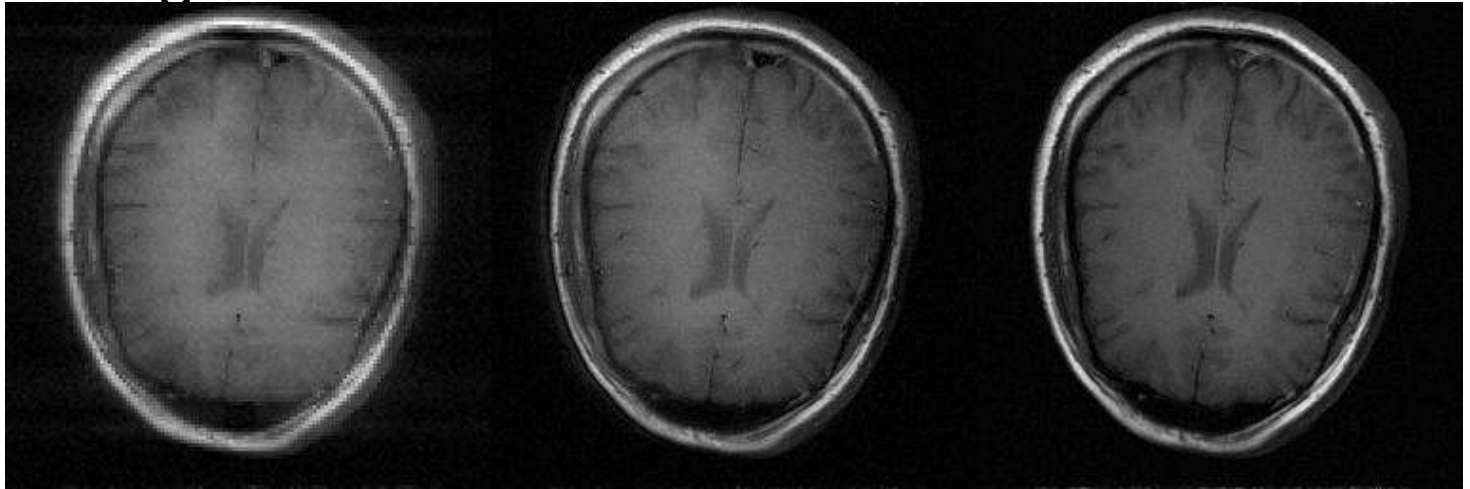


- 8 Lines
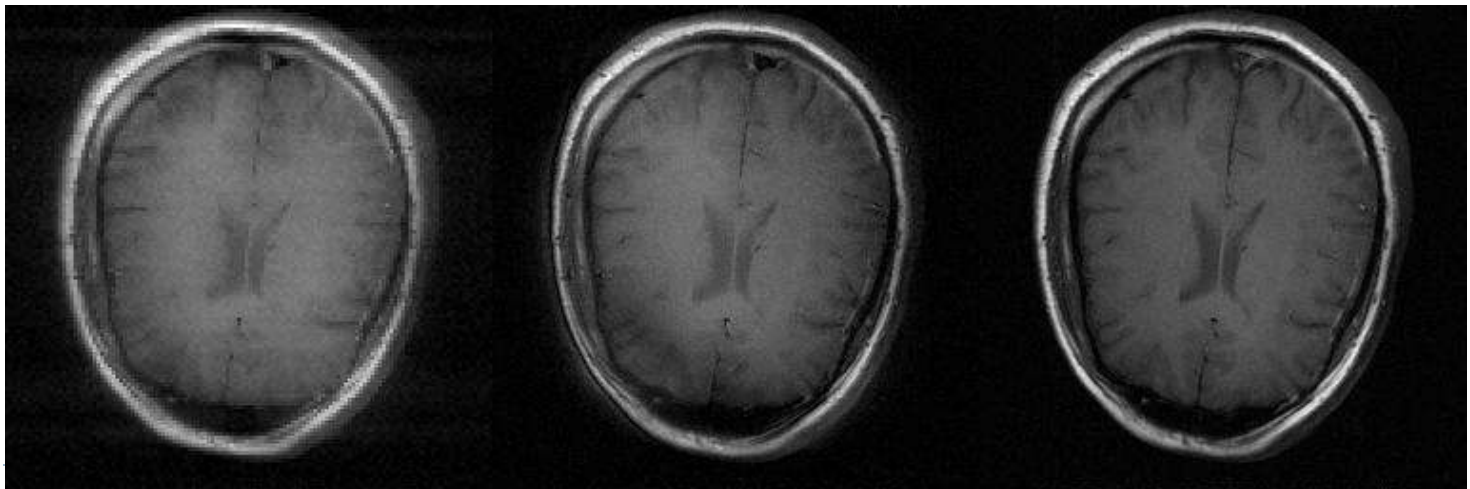
# Exercise

▸ Write a short paragraph (less than 500 words) on which partial Fourier reconstruction method you prefer and why [1 Point].

▸ Use the data set on the class web site to implement one of the methods of partial Fourier recostruction. The data set provided is for full k-space for you to have a gold standard to your reconstruction. You should use only part of it as an input to your reconstruction (say half + 16 lines). [2 Points]

▸ Do a literature search on the topic of partial Fourier reconstruction and come up with a list of all references related to the subject. [1 Point]

▸