Medical Image Reconstruction Term II – 2010

Topic 6: Compressed Sensing

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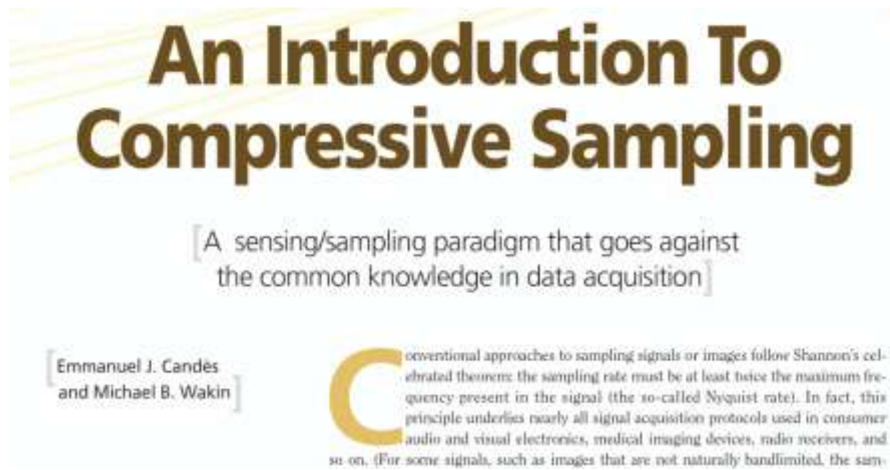
- Sampling Issues
- Compressed Sensing
- Application

Compressed Sensing MRI

[A look at how CS can improve on current imaging techniques]

[Michael Lustig, David L. Donoho, Juan M. Santos, and John M. Pauly]

Compressed sensing (CS) aims to reconstruct signals and images from significantly fewer measurements than were traditionally thought necessary. Magnetic resonance imaging (MRI) is an essential medical imaging tool with an inherently slow data acquisition process. Applying CS to MRI offers potentially significant scan time reductions, with benefits for patients and health care economics.



An Introduction To Compressive Sampling

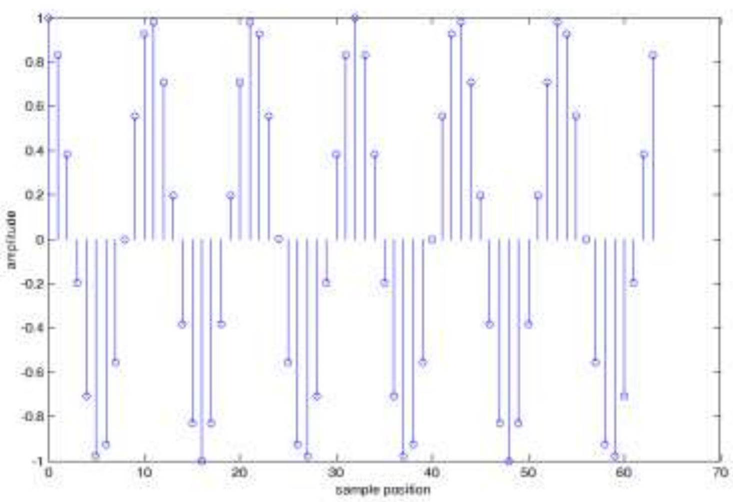
[A sensing/sampling paradigm that goes against the common knowledge in data acquisition]

[Emmanuel J. Candès and Michael B. Wakin]

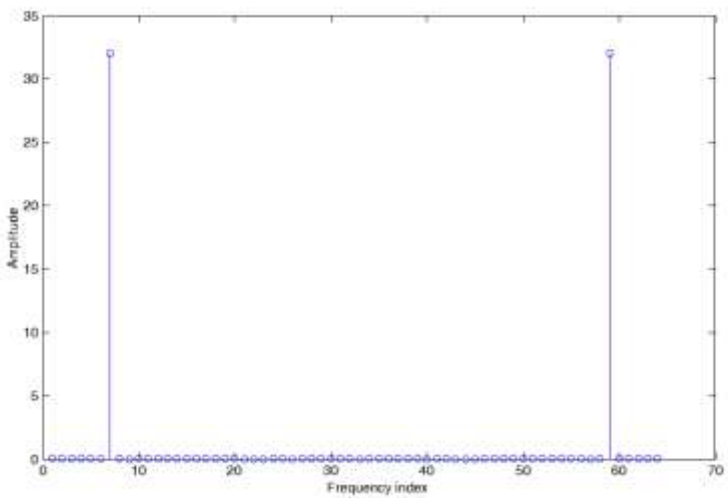
Conventional approaches to sampling signals or images follow Shannon's celebrated theorem: the sampling rate must be at least twice the maximum frequency present in the signal (the so-called Nyquist rate). In fact, this principle underlies nearly all signal acquisition protocols used in consumer audio and visual electronics, medical imaging devices, radio receivers, and so on. (For some signals, such as images that are not naturally bandlimited, the sam-

Sampling Issues

- Uniform vs. random sub-sampling



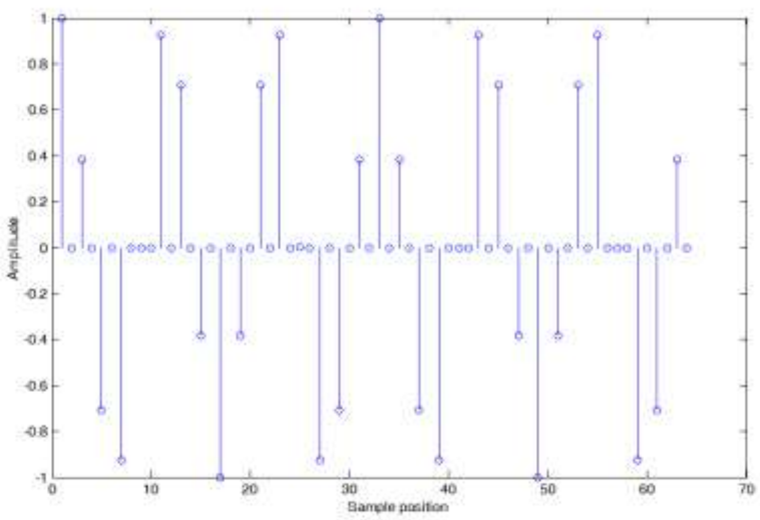
Time signal

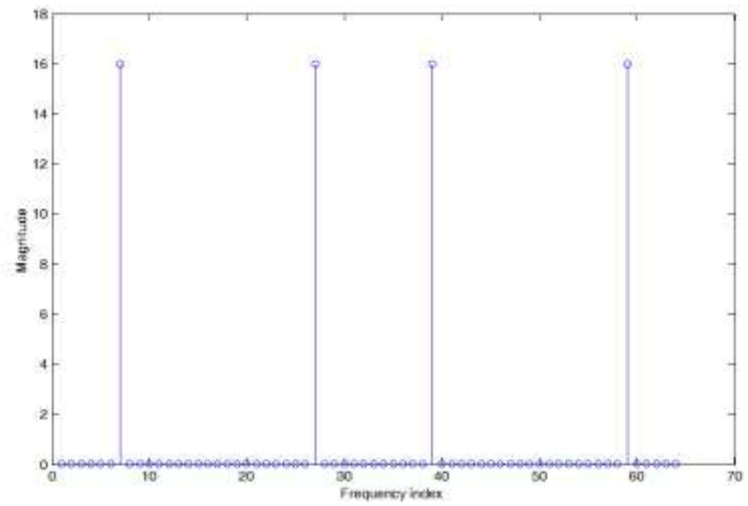


DFT

Sampling Issues

- Uniform vs. random sub-sampling





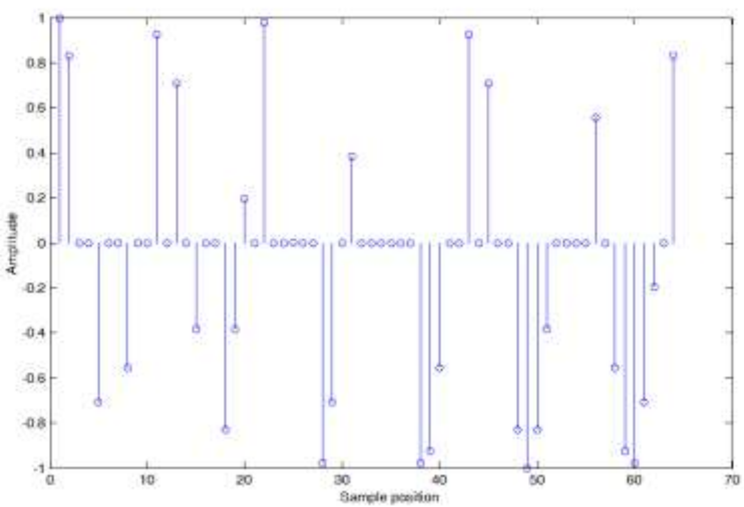
Time signal

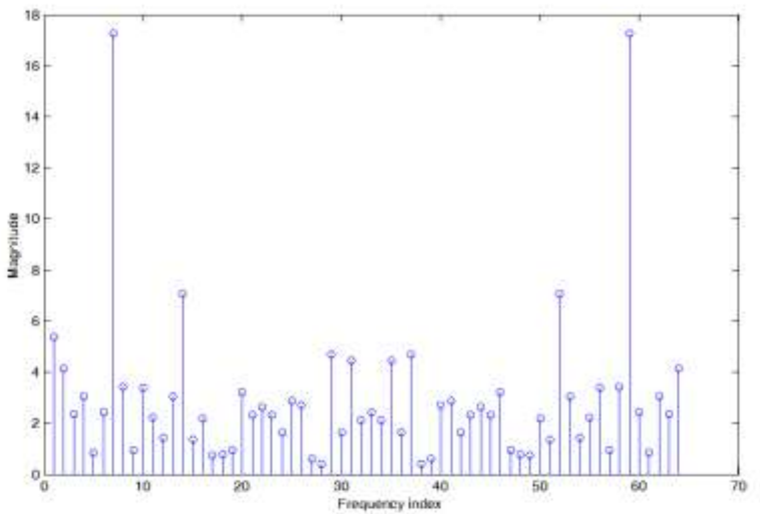
DFT

Ambiguity with uniform sub-sampling!

Sampling Issues

- Uniform vs. random sub-sampling



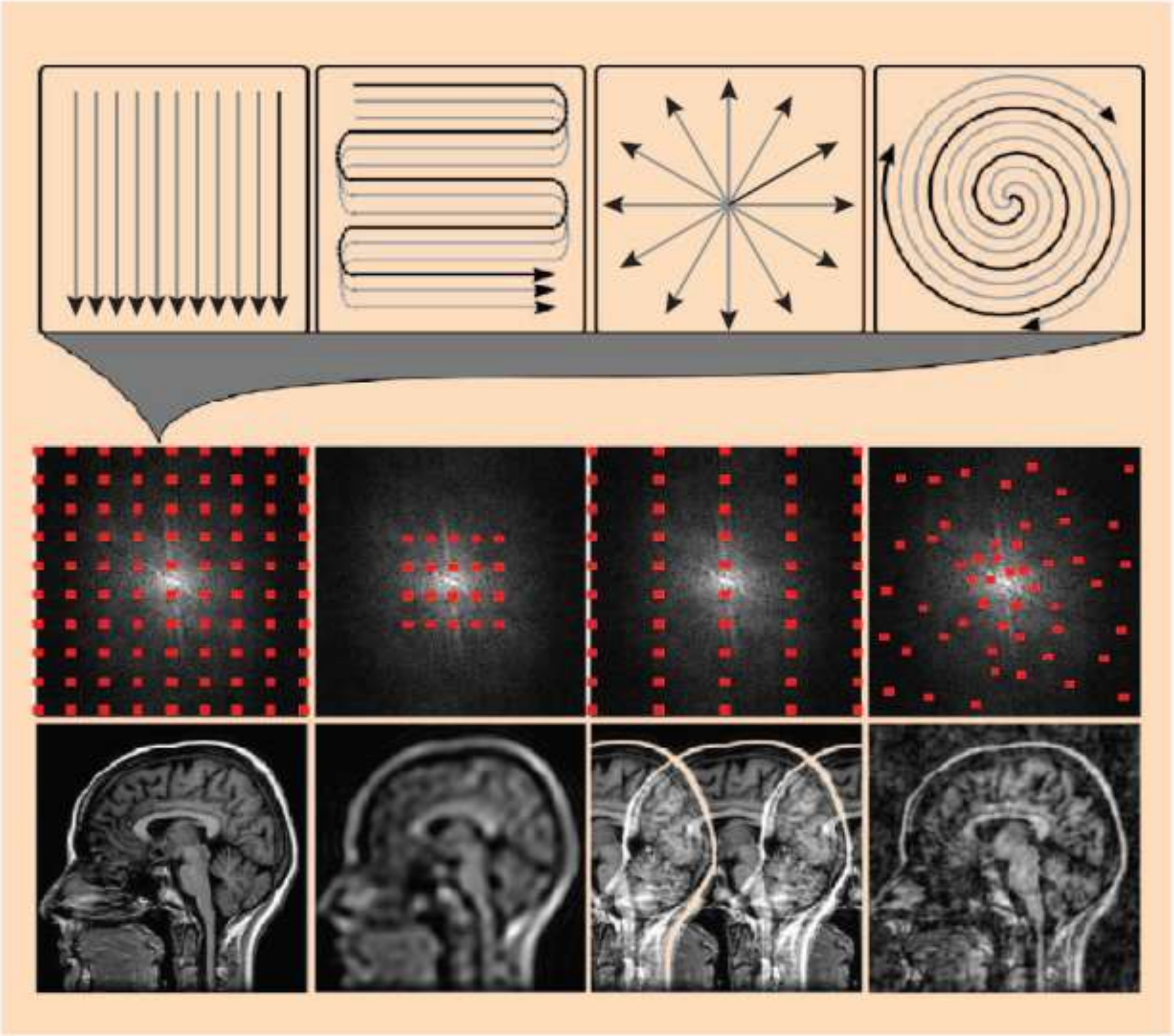


Time signal

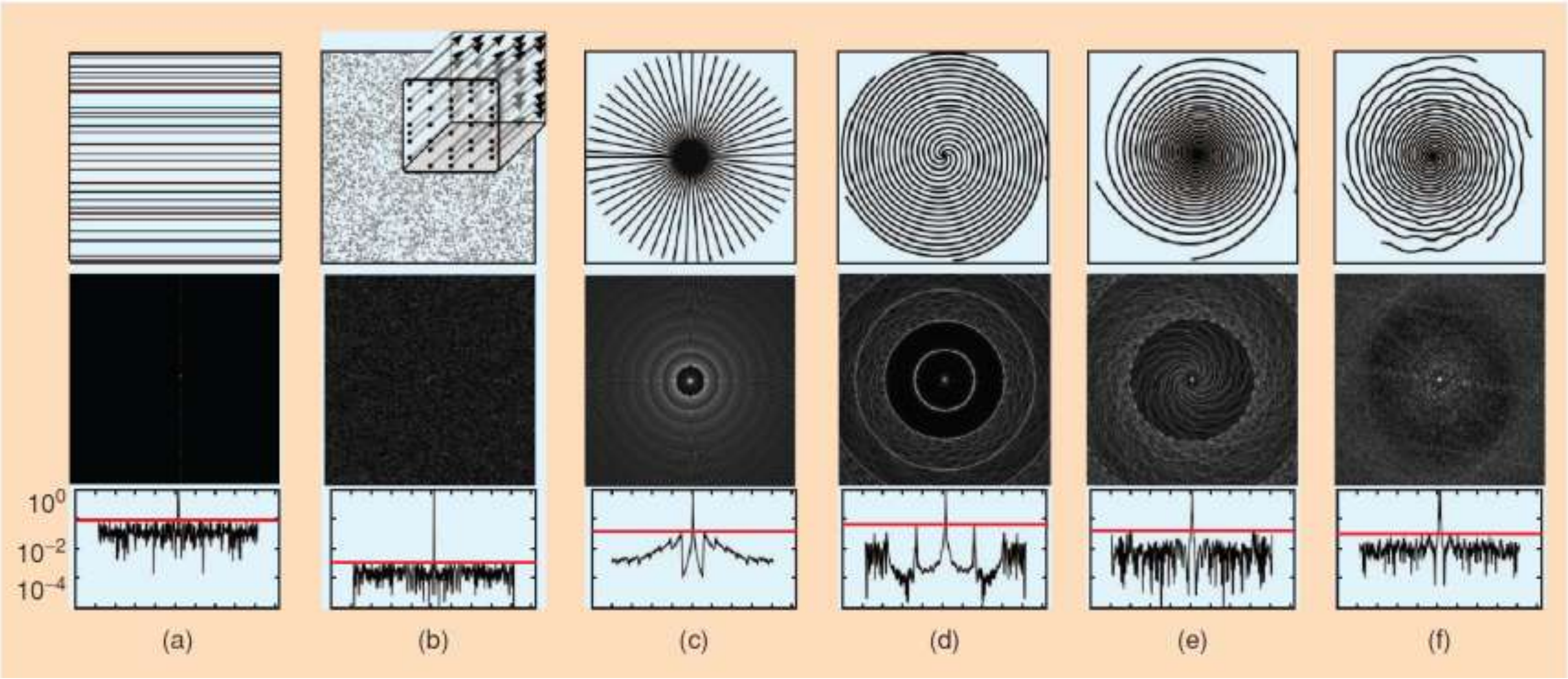
DFT

No ambiguity with non-uniform sub-sampling!

Undersampling Artifacts



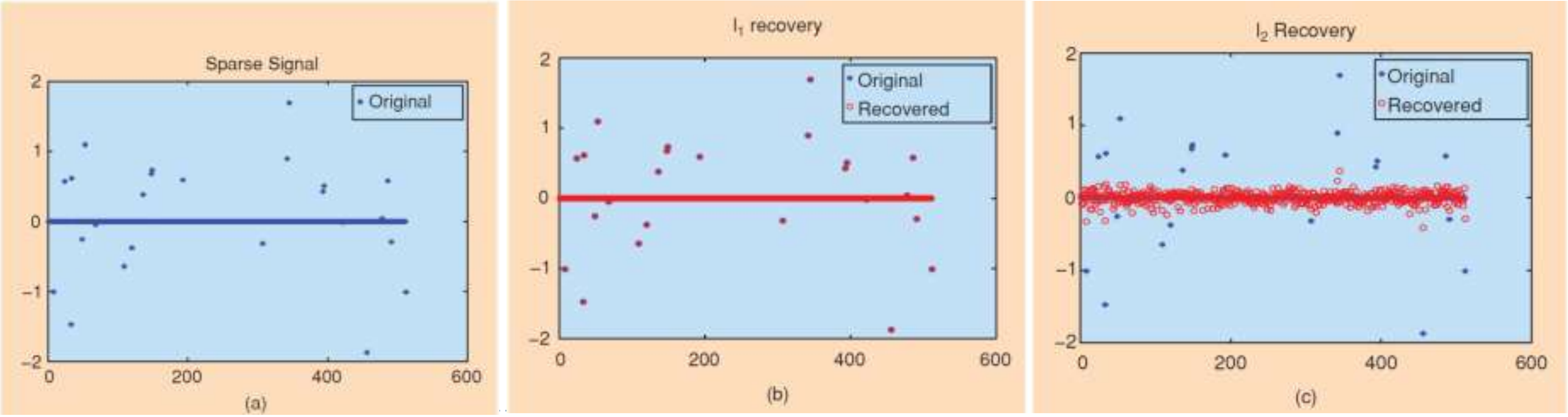
Undersampling Artifacts



Compressed Sensing

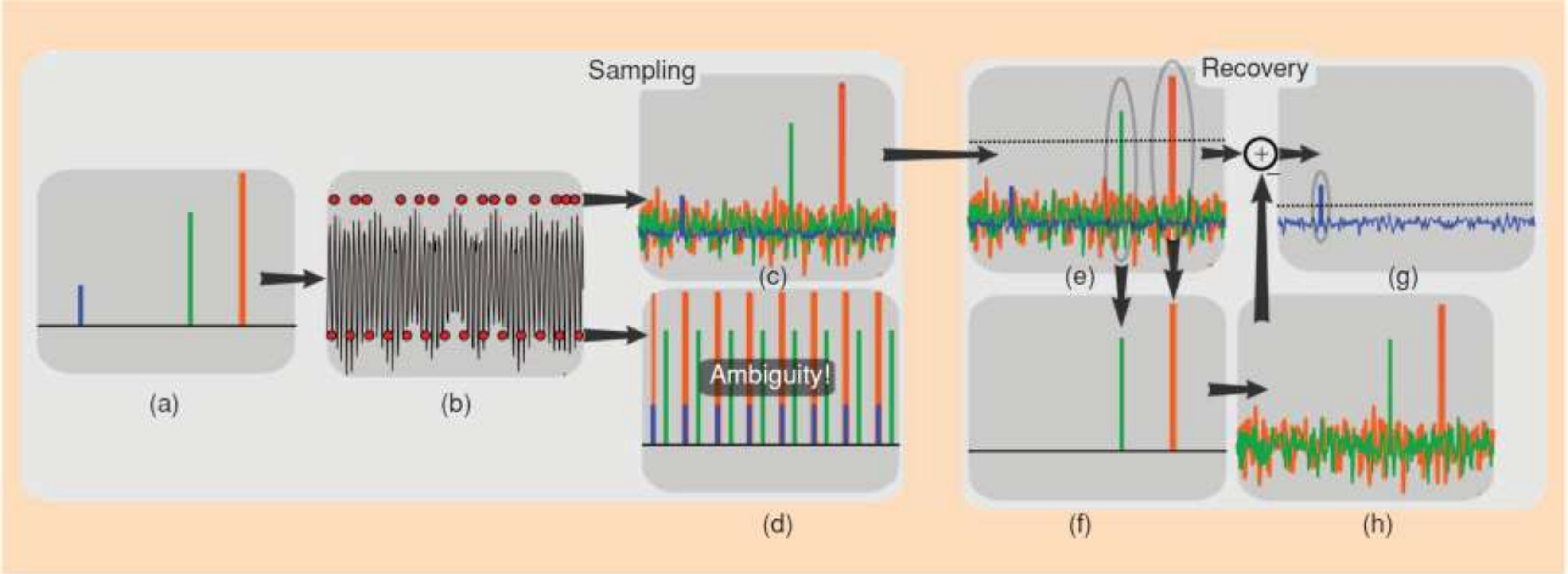
$$\min_{\tilde{x} \in \mathbb{R}^n} \|\tilde{x}\|_{\ell_1} \quad \text{subject to} \quad A\tilde{x} = y \ (= Ax).$$

[FIG2] (a) A sparse real valued signal and (b) its reconstruction from 60 (complex valued) Fourier coefficients by ℓ_1 minimization. The reconstruction is exact. (c) The minimum energy reconstruction obtained by substituting the ℓ_1 norm with the ℓ_2 norm; ℓ_1 and ℓ_2 give wildly different answers. The ℓ_2 solution does not provide a reasonable approximation to the original signal.



Undersampled Signal Recovery Procedure

- A sparse signal (a) is 8-fold undersampled in its 1-D *k-space* domain (b). Equispaced undersampling results in signal aliasing (d) preventing recovery. Pseudo-random undersampling results in incoherent interference (c). Some strong signal components stick above the interference level, are detected and recovered by thresholding (e) and (f). The interference of these components is computed (g) and subtracted (h), thus lowering the total interference level and enabling recovery of weaker components.

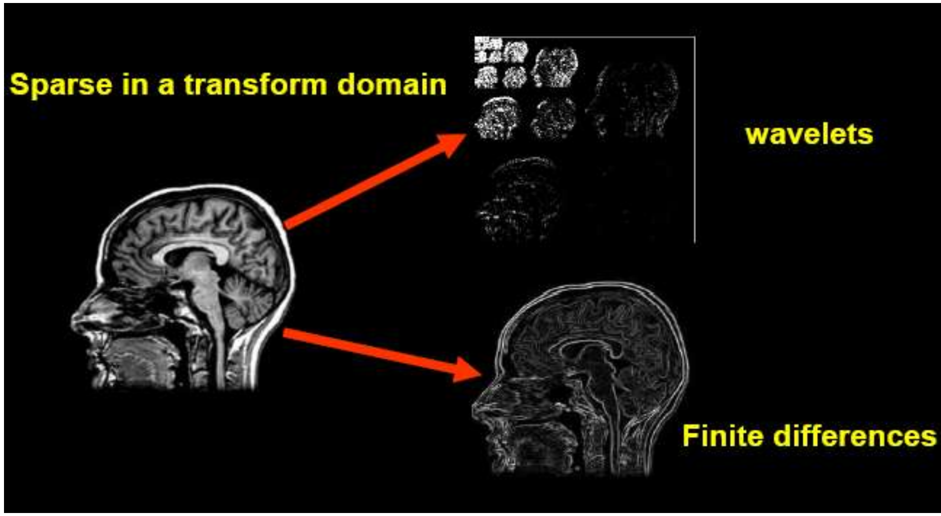


Compressed Sensing in MRI

- Sparse image reconstruction
  (Candes and Donoho 2003-2004)

$$\min \quad \|Tm\|_1$$
$$s.t. \quad \|Fm - y\|_2 \leq \varepsilon$$

  - *m* image
  - *y* measured k-space data
  - *T* transform that sparsifies image
  - *F* undersampled Fourier matrix

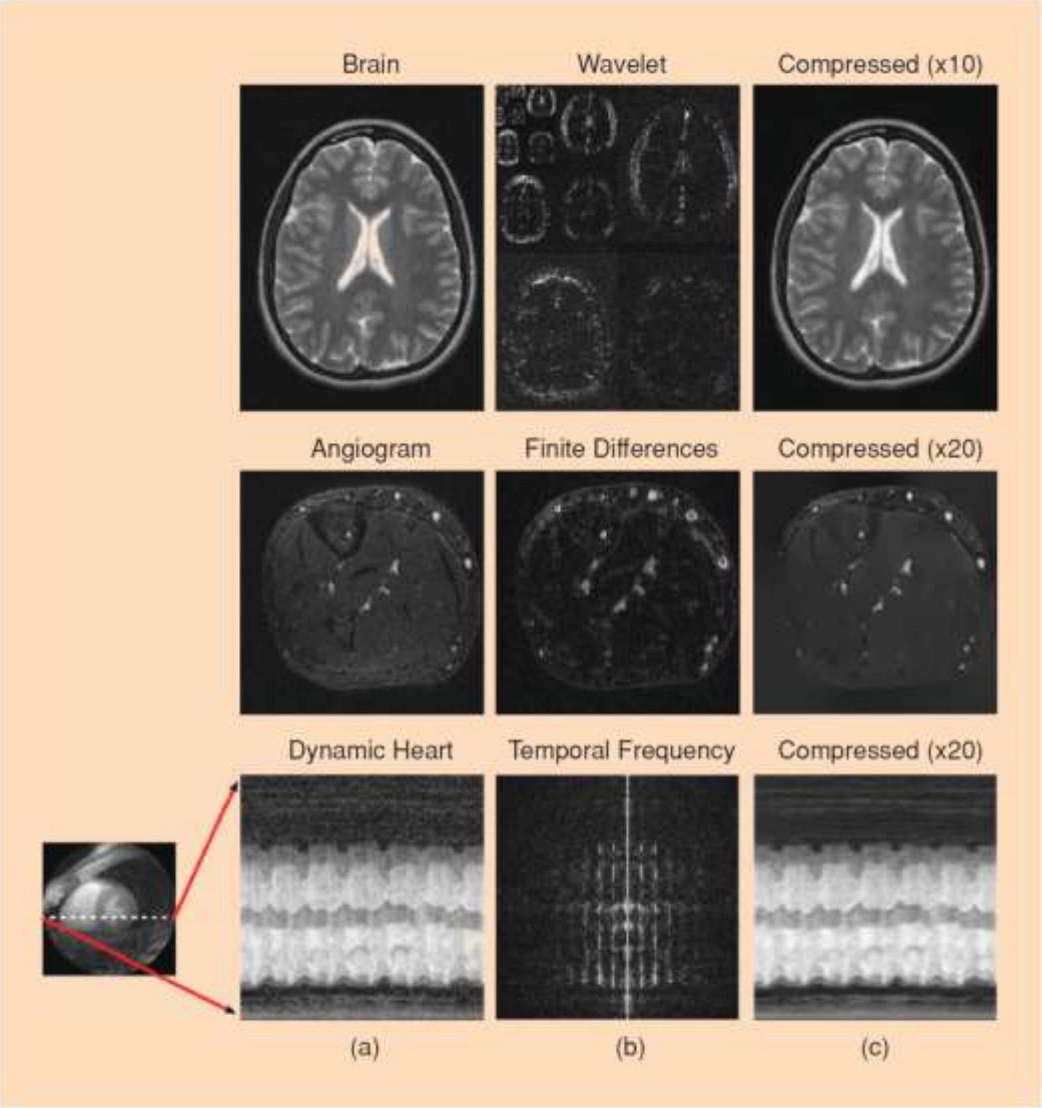


After Lustig et al 2005

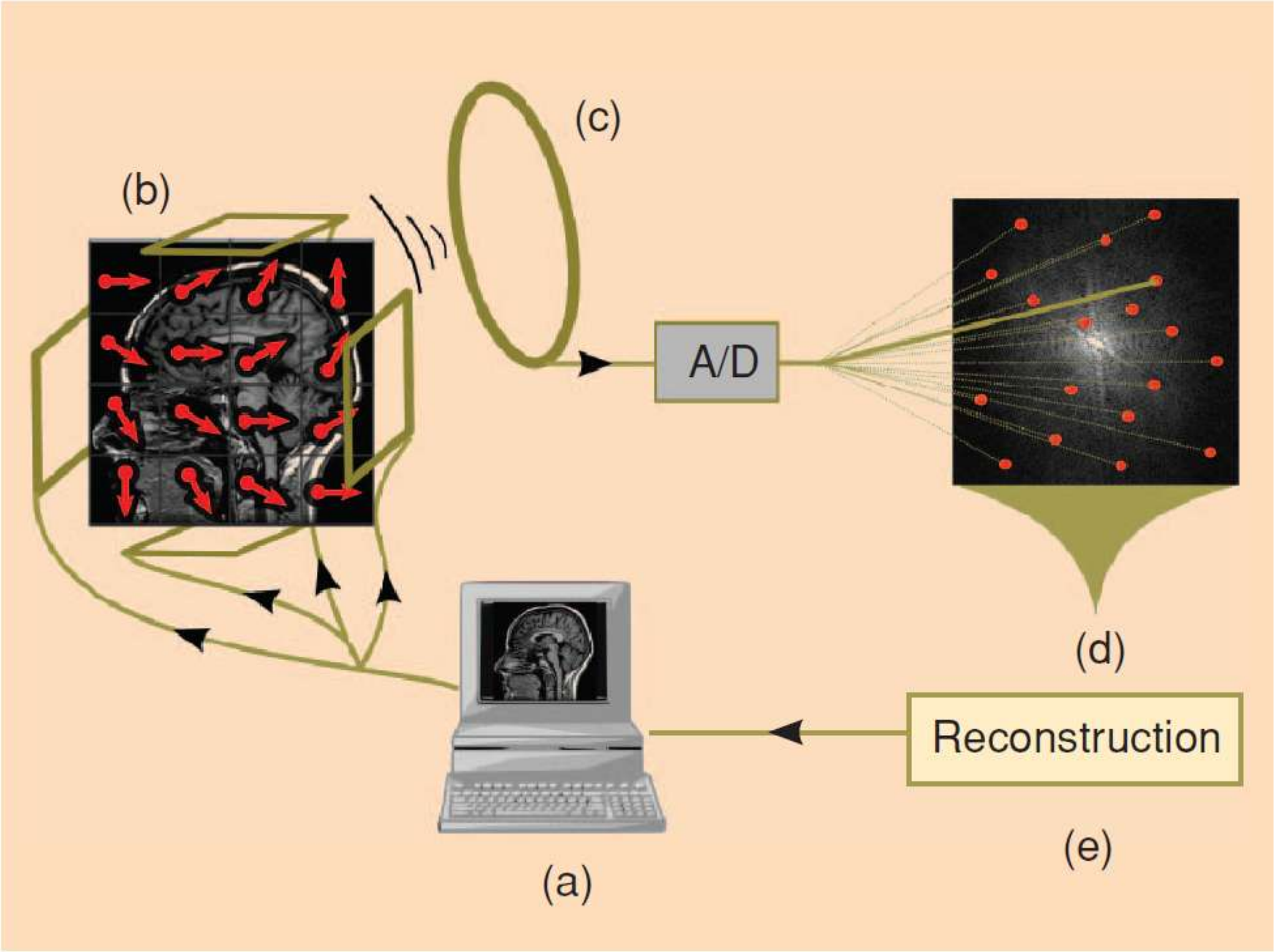
Transform Sparsity of MRI

MANY NATURAL SIGNALS ARE SPARSE OR COMPRESSIBLE IN THE SENSE THAT THEY HAVE CONCISE REPRESENTATIONS WHEN EXPRESSED IN THE PROPER BASIS.

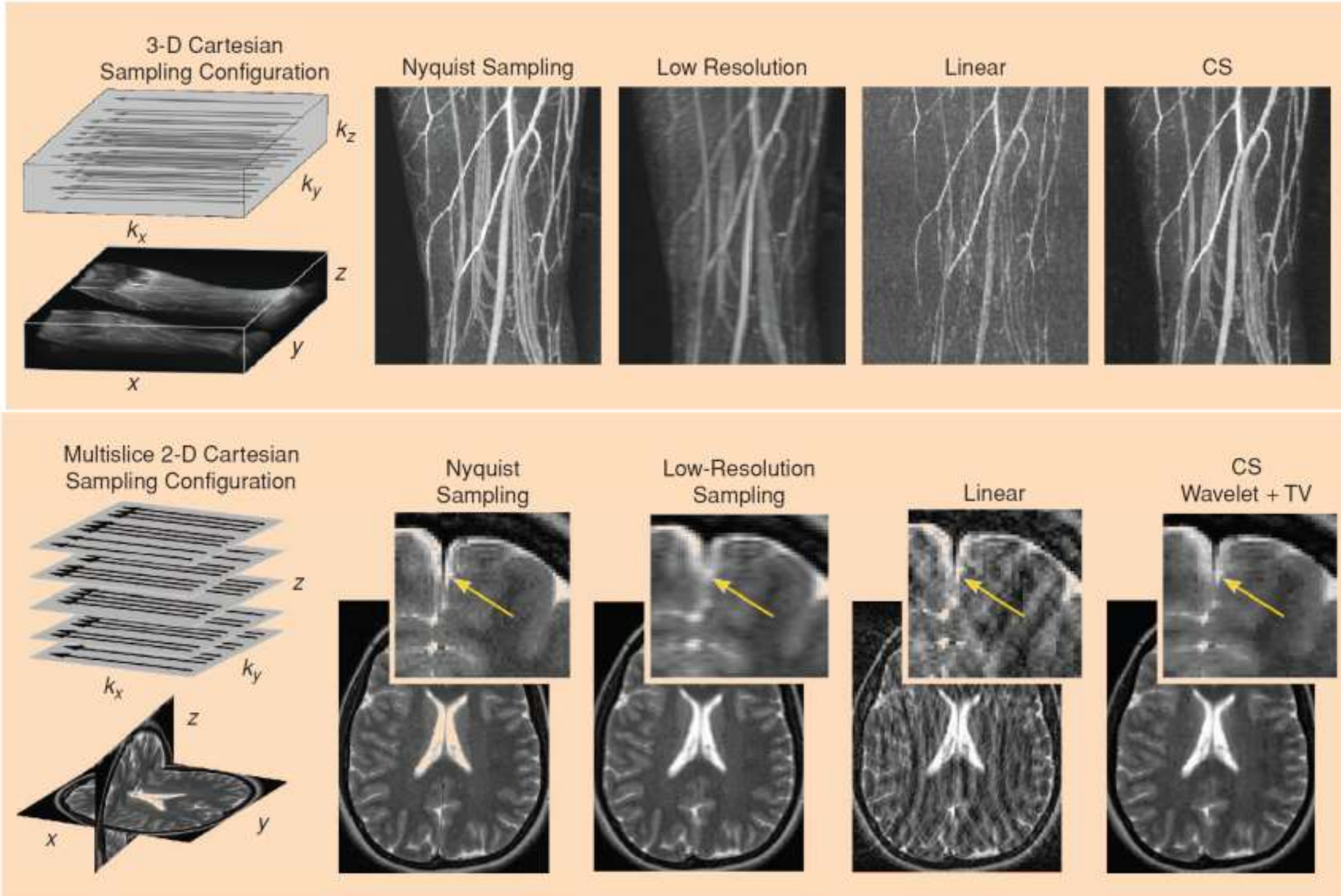
WHAT IS MOST REMARKABLE ABOUT THESE SAMPLING PROTOCOLS IS THAT THEY ALLOW A SENSOR TO VERY EFFICIENTLY CAPTURE THE INFORMATION IN A SPARSE SIGNAL WITHOUT TRYING TO COMPREHEND THAT SIGNAL.



Compressed Sensing MRI



Compressed Sensing Examples



Exercise

- Design an experiment to use compressed sensing to estimate a 1D sparse signal from its random samples. Use different degrees of undersampling and comment on the quality of your results [2 Points]
- Design an experiment to use compressed sensing to compute an image of a randomly sampled k-space of a Shepp-Logan phantom. Use different degrees of undersampling and comment on the quality of your results [4 Point]