Practice Problem Set #2

1. Determine the unilateral Laplace transform of the following signals:

(a) $x(t) = u(t-2)$
(b) $x(t) = u(t+2)$
(c) $x(t) = e^{-2t}u(t+1)$
(d) $x(t) = e^{2t}u(-t+2)$
(e) $x(t) = \sin(\omega_o t)$
(f) $x(t) = u(t) - u(t-2)$
(g) $x(t) = \begin{cases} \sin(\pi t), & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

2. Use the Laplace transform tables and properties to obtain the Laplace transform of the following:

(a) $x(t) = \frac{d}{dt}\{te^{-t}u(t)\}$
(b) $x(t) = tu(t) * \cos(2\pi t)u(t)$
(c) $x(t) = t^3 u(t)$
(d) $x(t) = u(t-1) * e^{-2t}u(t-1)$
(e) $x(t) = \int_0^t e^{-3\tau}\cos(2\tau)d\tau$
(f) $x(t) = t\frac{d}{dt}(e^{-t}\cos(t)u(t))$

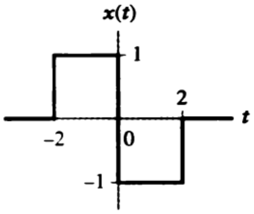
3. Use the tables of transforms and properties to determine the time signals that correspond to the following bilateral Laplace transforms:

(a) $X(s) = e^{5s}\frac{1}{s+2}$ with ROC $\mathrm{Re}(s) < -2$
(b) $X(s) = \frac{d^2}{ds^2}\left(\frac{1}{s-3}\right)$ with ROC $\mathrm{Re}(s) > 3$
(c) $X(s) = s\left(\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s}\right)$ with ROC $\mathrm{Re}(s) < 0$
(d) $X(s) = s^{-2}\frac{d}{ds}\left(\frac{e^{-3s}}{s}\right)$ with ROC $\mathrm{Re}(s) > 0$

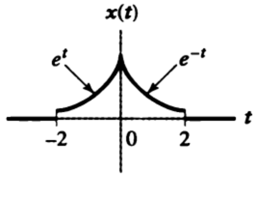
4. Evaluate the frequency-domain representations of the following signals:

(a) $x(t) = e^{-2t}u(t-3)$
(b) $x(t) = e^{-4|t|}$
(c) $x(t) = te^{-t}u(t)$
(d) $x(t) = \sum_{m=0}^{\infty} a^m \delta(t-m)$, $|a| < 1$

5. Evaluate the frequency-domain representations of the shown signals:



(a)



(b)

6. Use the Fourier transform tables and properties to obtain the Fourier transform of the following signals:

(a) $x(t) = \sin(2\pi t)e^{-t}u(t)$

(b) $x(t) = te^{-3|t-1|}$

(c) $x(t) = \left[\dfrac{2\sin(3\pi t)}{\pi t}\right]\left[\dfrac{\sin(2\pi t)}{\pi t}\right]$

(d) $x(t) = \dfrac{d}{dt}(te^{-2t}\sin(t)u(t))$

(e) $x(t) = \displaystyle\int_{-\infty}^{t} \frac{\sin(2\pi\tau)}{\pi\tau}d\tau$

(f) $x(t) = e^{-t+2}u(t-2)$

(g) $x(t) = \left(\dfrac{\sin(t)}{\pi t}\right) * \dfrac{d}{dt}\left[\left(\dfrac{\sin(2t)}{\pi t}\right)\right]$

7. Replace the time variable "t" with the frequency variable "Ω" in all signals in problems 4, 5 and 6 and repeat to obtain the inverse Fourier transform of these signals.