Simulation Systems – Practice Problem Set #1

1. Consider a Markov Chain having state-space p =[0 1 2] and the transition matrix:

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/4 & 1/2 & 1/4 \\ 1/6 & 1/3 & 1/2 \end{bmatrix}.$$

    Determine whether or not this system has a stationary distribution and find out what the stationary distribution will be.

2. Write the steps to implement the following random number generators:
    a. A1/(C1) ^ D1
    b. (F3r(B1)+ E1) mod 2^{64}
    c. B3 + C2 ^ E2(A3)

3. Mark the following questions as either True or False and give reasons:
    a. Random numbers always use linear congruential generators in their formula.
    b. Combined generators are more computationally expensive than simple generator.
    c. Acceptable random generator must combine at least two related random number generators.
    d. Generators that take more than two dozen arithmetic or logical operations are preferred.
    e. Comparison function in rejection method must have an indefinite integral that is known analytically.
    f. It is possible to use rejection method with discrete distributions.
    g. In simulated annealing, cooling schedule determines the performance of the method.
    h. In metropolis method, it is difficult to judge whether the method is getting a good sample.
    i. Importance sampling is useful for sampling from joint distributions on many variables.
    j. Umbrella sampling emphasizes states that are likely to be the stationary distribution.

4. Describe two methods to convert uniform deviates into samples from a normal distribution. Write the steps in detail in each case.

5. Is it always possible to use the transformation method to obtain samples from any distribution? Explain your answer.

6. In random sampling using the rejection method, the first random deviate was obtained from a uniform distribution ~[0, 10] and came out to be 6.3. This corresponds to a value of x=3.1 with desired p(x)=0.3 and upper bound function f(x)=0.4. If the second random deviate was found to be 0.1246, determine whether x will be accepted or rejected and why.

7. In the rejection method, if the area under p(x) is 1 and that of the comparison function f(x) is 1.7, determine the number of samples of x that can be obtained when 1000 uniform deviates are used from each of the two independent uniform random number generators.