Multidimensional Signal Processing Elective Course

Mid-Term Exam Solution - May 2008

Open-Book/ Open-Notes – Time Allowed: 1 Hour – Maximum Grade:100 points

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Q1. Determine whether each of the following signals is periodic. If the signal is periodic, state its fundamental period: (3 points each)

(a) $x(n)= \exp(j \pi n/6) + \exp(j 2 \pi n/31)$ (sol: periodic N=31*12)
(b) $x(n)= \sin(3 \pi n/4) \cos(\sin(\pi n/16)$ (sol: periodic N= 32)
(c) $x(n)= \cos(n)$ (sol: not periodic)
(d) $x(n)= \sin(\pi n/5)/ (\pi n)$ (sol: not periodic)

Q2. For each of the following systems, prove whether the system is (1) linear, (2) time-invariant, (3) causal and (4) stable: (8 points each)

(a) $y(n)= \cos(n)\, x(n)$ (linear, time varying, causal, stable)
(b) $y(n)= \sum_{k=0}^{n} x(k)$ (linear, time varying, causal, stable)
(c) $y(n)= \sum_{k=n-3}^{n+3} x(k)$ (linear, time-invariant, not causal, stable)
(d) $y(n)= x(-n)$ (linear, time invariant, not causal, stable)
(e) $y(n)= x(n) + 3 \sin(\pi n/4)$ (nonlinear, time varying, causal, stable)

Q3. Derive the Fourier transformation of the following functions: (10 points each)

(a) $x(t)= \text{Sinc}(2t)$ $-\infty < t < \infty$
From tables using duality property (looks like a gate function centered at 0)

(b) $x(t)= \text{rect}(t/T) \exp(j \pi t/8)$ $-\infty < t < \infty$
Same as (a) but with a frequency shift of $\pi/8$ (i.e., a gate centered at $\pi/8$)

(c) $x(t)= 1/((1+j t)^2+25)$ $-\infty < t < \infty$
From tables using the duality property

(d) $x(n)= \text{Sinc}(2 n)$ $-\infty < n < \infty$
Same as (a) with periodicity in frequency domain due to the sampling. (i.e., gate centered at 0 with periodicity 2π)

(e) $x(n)= (-1)^n \text{Sinc}(2 n)$ $-\infty < n < \infty$
Recall that $(-1)^n$ is just $\exp(j \pi n)$ – so it is going to be just like (d) with a frequency shift of π

Best of Luck!