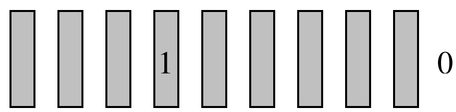
Multidimensional Signal Processing Elective Course
Problem Assignment #2

1. Design a digital band-pass filter that can be used to filter ECG signals given the following specifications: sampling rate 250 Hz, low cutoff frequency 1 Hz, high cutoff frequency 45 Hz, passband ripple < 1dB, stopband attenuation 60 dB. Compare the designs from different FIR/IIR digital methods and state your preferred method and why.

This is just a Matlab exercise for you to do what you saw in the lecture. Just use fdatool and examine the different responses for different filter types.

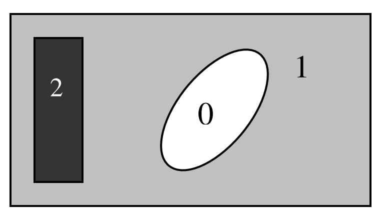
2. Compute the Fourier transformation for the following shapes:

a. (binary)



b. (binary)

c. (ellipse at 45°)



(a) This problem was done in the lecture when we gave an example about the field of an ultrasound linear array and talked about side lobes and grating lobes. The shape is a periodic rect function in the x direction (looks like the above but from -∞ to ∞) multiplied by a gate function in the x direction (to remove all but these 9 periods). So, the Fourier transform looks like :
[Sinc(kx) Sinc (fy) (kx is discrete as a result of periodicity in space)] convolved with Sinc(fx)

(b) The same as (a) but with a Jinc function instead of the Sincx Sincy

(c) Just decompose the shape into basic shapes added together and user the linearity of the Fourier transform to compute the result. Here, it is a large rectangle with intensity 1 plus a small rectangle with intensity 1 shifted with negative x shift plus a 45 deg rotated ellipse with intensity –1 shifted with positive x. You have the Fourier transforms of all these shapes in the 2D FT table and you can use the shift and rotation properties to derive the answer.

3. Verify the projection slice theorem using projections of a rectangular function of width a and height b at angles 0°and 90°.

This part is already given in the lecture and you can also find it in the CT presentation I posted on the web for you.

4. In an embedded DSP system, a DSP processor that allows real-time processing of data. The DSP system computes the spectrogram for a biomedical signal under the following conditions: sampling rate: 10 kHz, window size= 128, number of windows to compute per second=100, a hamming window is used in each case and averaging is not used. Design a suitable digital signal processing method to do that and estimate the processing power required (in terms of an order of computations/second).

Just a block diagram for the spectrogram computation steps (window size, window shift selected to give the desired specs). From the number of DFTs per second, you can estimate the number of computations needed assuming N^2 computations per DFT.

5. In infrared spectroscopy system, it is desired to calculate an accurate power spectrum for a biomedical signal. Assume that the sampling rate is 1 MHz and the number of samples acquired is 100000 samples. Design an algorithm to calculate the power spectrum of the data given that the desired frequency domain resolution is 1 kHz. Make sure that the SNR of the resultant power spectrum is optimal.

Here, you should provide the block diagram or the steps of power spectrum estimation using averaged periodogram with non-overlapping or overlapping windows (it is up to you to choose which one to use). The “optimal SNR” mentioned in the problem indicates that you have to do averaging (since one can just take one periodogram of the whole data as an estimate). The frequency domain resolution will determine the size of the window and you can choose the window shift based on your strategy.

5. Answer the following question with either True (T) or False (F) and give your reasons:

a. The theory of infinite impulse response digital filters has roots in analog filter design methods. (T)
b. Windowing is mainly used to reduce the length of the data to be processed by FFT and consequently reduces the required computation time. (F)
c. The discrete Fourier transform is an approximation for discrete time Fourier transform. (F)
d. It is possible to completely recover an analog signal from its digital samples if the sampling rate was higher than the bandwidth of the signal. (T)

e. Sampling rate required for a given signal is usually much higher than the sampling rate required for the quadrature version of the same signal. (T)
f. Spectral leakage has the same source and meaning as Gibbs ringing. (F)
g. Power spectrum estimation is best used for signals with time-varying frequency content. (F)
h. Stability in digital filters means correct computations without overflow or underflow. (T)
i. It is possible to optimize both the main lobe width and side lobe magnitude simultaneously using different windows. (F)
j. Spectrograms are two dimensional images with a horizontal axis of time and vertical axis of frequency. (T)
k. The only Fourier transform property used in DTFT is the time-domain sampling resulting in a periodic Fourier transform. (T)
l. Linear convolution cannot be computed using the DFT. (F)
m. Real signals always have a symmetric Fourier transformation magnitude. (T)
n. Fourier transformation properties change in its multidimensional extensions. (F)