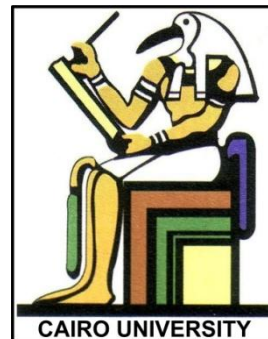


Medical Equipment I - 2009

Chapter 2

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Web: <http://ymk.k-space.org/courses.htm>



[Exponential Growth]

- An exponential growth process is one in which the rate of increase of a quantity is proportional to that quantity

- Example:

Savings account

$$y_t = y_0(1 + b)^t$$

TABLE 2.1. Growth of a savings account earning 5% interest compounded annually, when the initial investment is \$100.

Year	Amount	Year	Amount	Year	Amount
1	\$105.00	10	\$162.88	100	\$13,150.13
2	110.25	20	265.33	200	1,729,258.09
3	115.76	30	432.19	300	2.27×10^8
4	121.55	40	704.00	400	2.99×10^{10}
5	127.63	50	1146.74	500	3.93×10^{12}
6	134.01	60	1867.92	600	5.17×10^{14}
7	140.71	70	3042.64	700	6.80×10^{16}
8	147.75	80	4956.14	800	8.94×10^{18}
9	155.13	90	8073.04	900	1.18×10^{21}

Exponential Growth

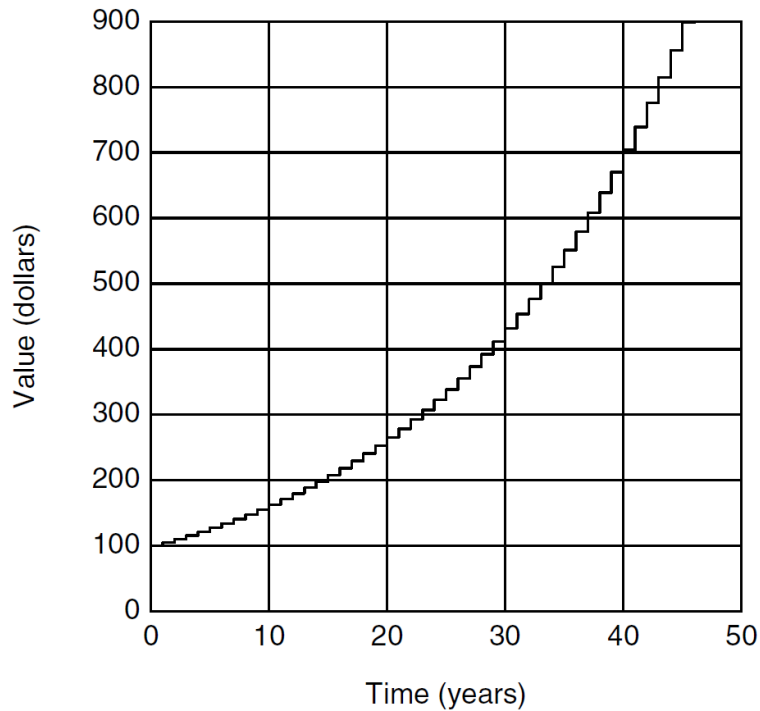


FIGURE 2.1. The amount in a savings account after t years, when the amount is compounded annually at 5% interest.

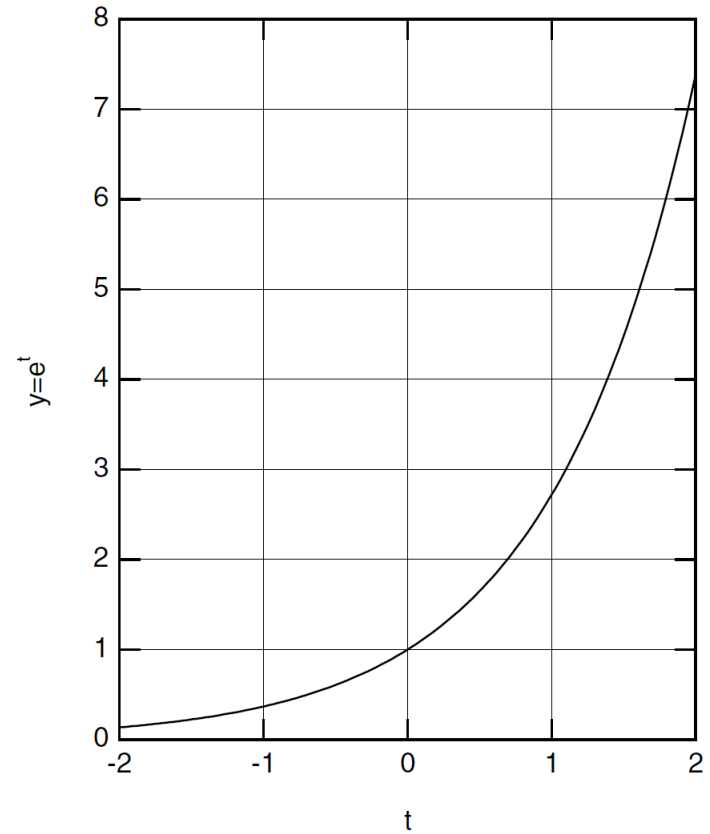


FIGURE 2.2. A graph of the exponential function $y = e^t$.

Exponential Growth: Compounding

- N times/year

TABLE 2.2. Amount of an initial investment of \$100 at 5% annual interest, with different methods of compounding.

Month	Annual	Semiannual	Quarterly	Monthly	Instant
0	\$100.00	\$100.00	\$100.00	\$100.000	\$100.000
1	100.00	100.00	100.00	100.417	100.418
2	100.00	100.00	100.00	100.835	100.837
3	100.00	100.00	101.25	101.255	101.258
4	100.00	100.00	101.25	101.677	101.681
5	100.00	100.00	101.25	102.101	102.105
6	100.00	102.50	102.52	102.526	102.532
7	100.00	102.50	102.52	102.953	102.960
8	100.00	102.50	102.52	103.382	103.390
9	100.00	102.50	103.80	103.813	103.821
10	100.00	102.50	103.80	104.246	104.255
11	100.00	102.50	103.80	104.680	104.690
12	105.00	105.06	105.09	105.116	105.127

$$y_t = y_0 \left(1 + \frac{b}{N} \right)^{Nt}$$

[Exponential Growth]

- As compounding $N \rightarrow \infty$,

$$\begin{aligned} y_t &= \lim_{N \rightarrow \infty} \left\{ y_0 \left(1 + \frac{b}{N} \right)^{Nt} \right\} \\ &= \lim_{N \rightarrow \infty} \left\{ y_0 \left[\left(1 + \frac{b}{N} \right)^N \right]^t \right\} \\ &= y_0 e^{bt} \end{aligned}$$

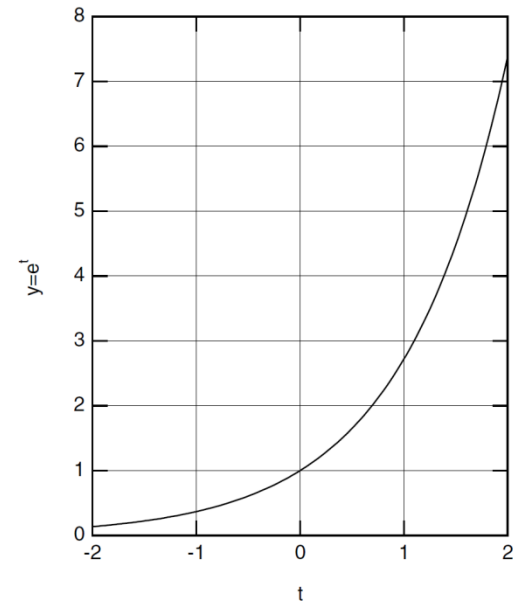


FIGURE 2.2. A graph of the exponential function $y = e^t$.

[Exponential Growth]

- Differential Equation

$$\frac{dy_t}{dt} = \frac{d}{dt} \{y_0 e^{bt}\} = by_0 e^{bt} = by_t$$

[Exponential Decay]

- Example: assume $b > 0$

$$y_t = y_0 e^{-bt}$$

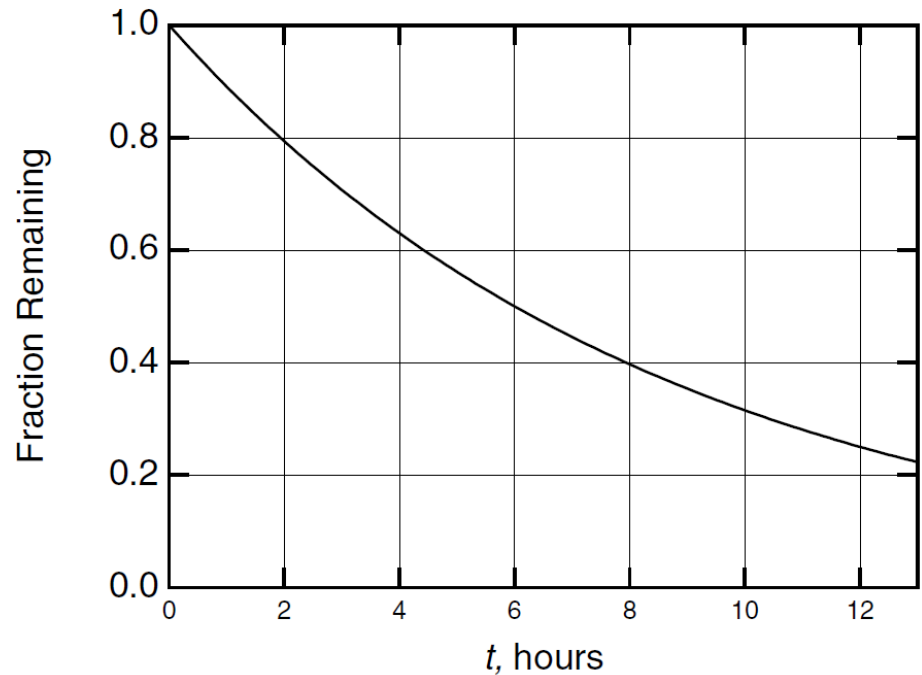


FIGURE 2.3. A plot of the fraction of nuclei of ^{99m}Tc surviving at time t .

Exponential Decay

- Half-Life $T_{1/2}$: Length of time required for y_t to decrease to $1/2$ its original value

$$y_{T_{1/2}} = 0.5 y_0 \Leftrightarrow e^{-bT_{1/2}} = 0.5$$

$$T_{1/2} \approx \frac{0.693}{b}$$

- Note: Doubling time T_2 is same value

Exponential Decay

- Example: Radioactive decay of ^{99m}Tc
 - Decay rate: $b = 0.1155 \text{ h}^{-1}$
 - $T_{1/2} = 0.693/0.1155 = 6 \text{ h}$

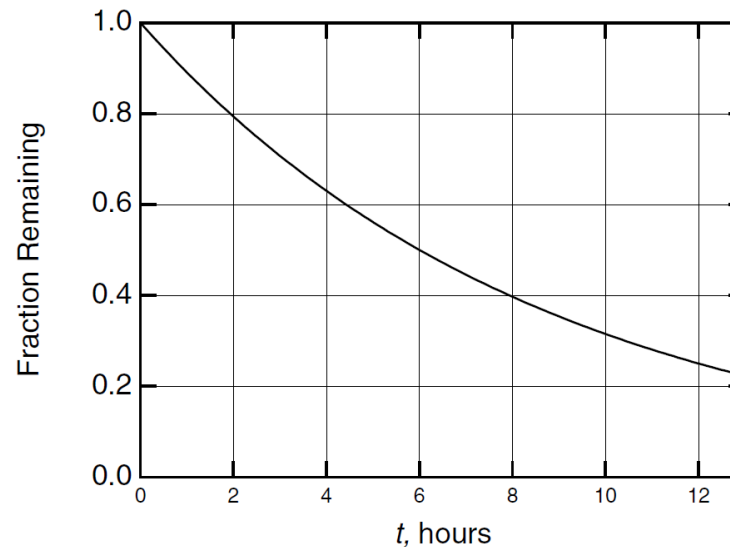


FIGURE 2.3. A plot of the fraction of nuclei of ^{99m}Tc surviving at time t .

[Semilog Paper]

$$\log y_t = \log y_0 e^{bt} = \log y_0 + bt$$

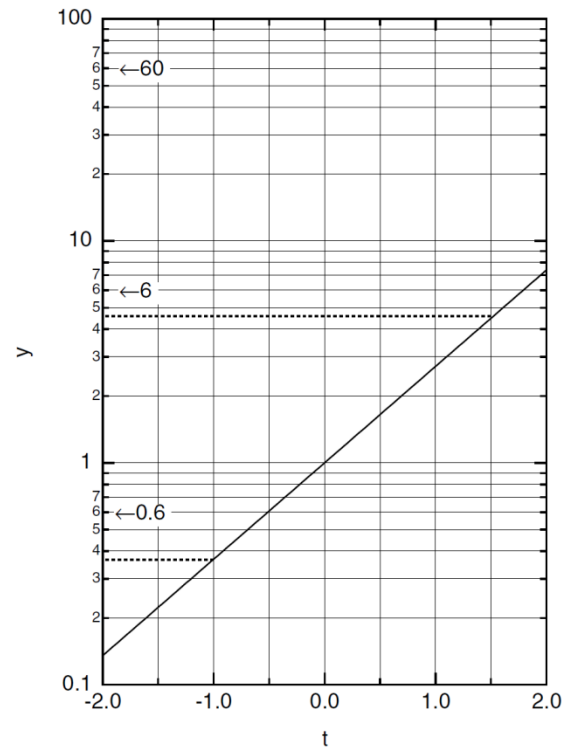
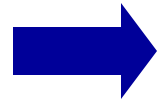
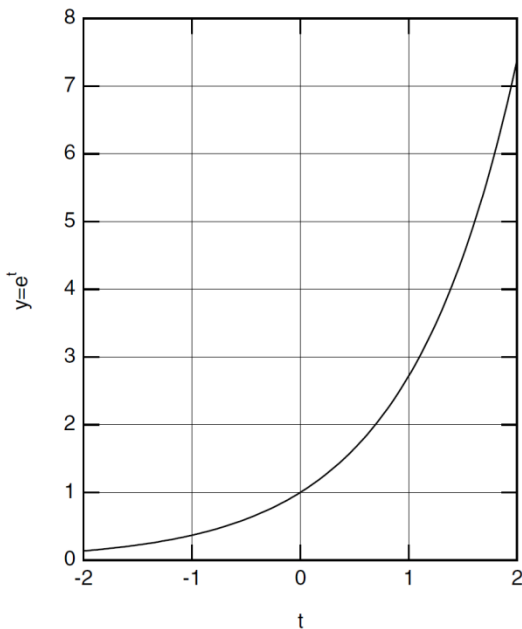


FIGURE 2.2. A graph of the exponential function $y = e^t$.

FIGURE 2.4. A plot of the exponential function on semilog paper.

[Semilog Paper: Example]

$$\begin{aligned}\log y_t &= \log y_0 e^{-bt} \\ &= \log y_0 - bt\end{aligned}$$

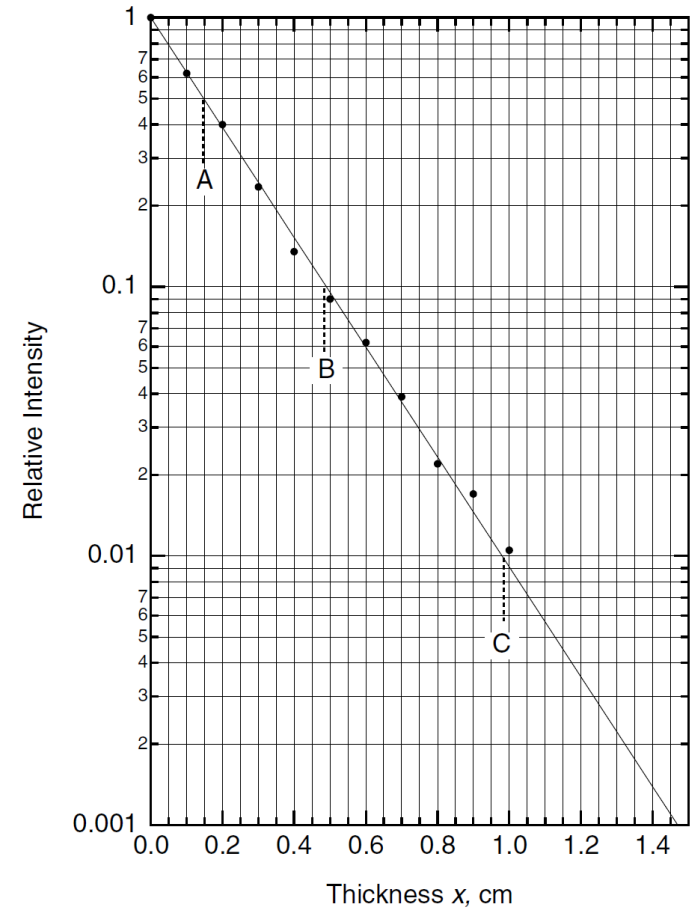


FIGURE 2.5. A semilogarithmic plot of the intensity of light after it has passed through an absorber of thickness x .

[Variable Rates]

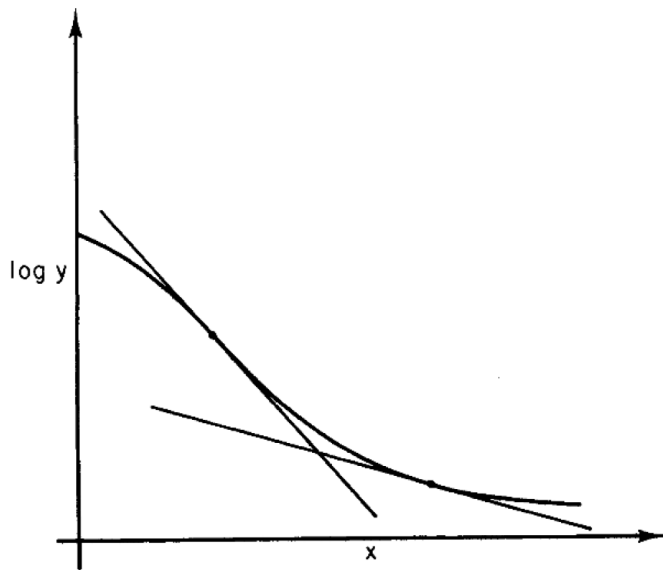


FIGURE 2.6. A semilogarithmic plot of y vs x when the decay rate is not constant. Each tangent line represents the instantaneous decay rate for that value of x .

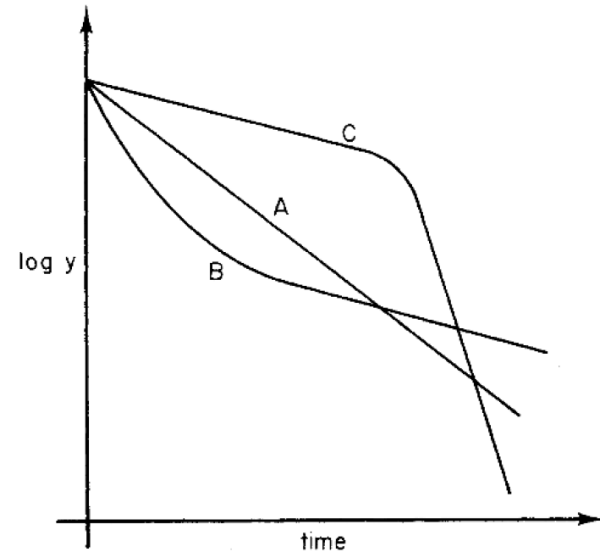


FIGURE 2.7. Semilogarithmic plots of the fraction of a population surviving in three different diseases. The death rates (decay constants) depend on the duration of the disease.

[Variable Rates: Example]

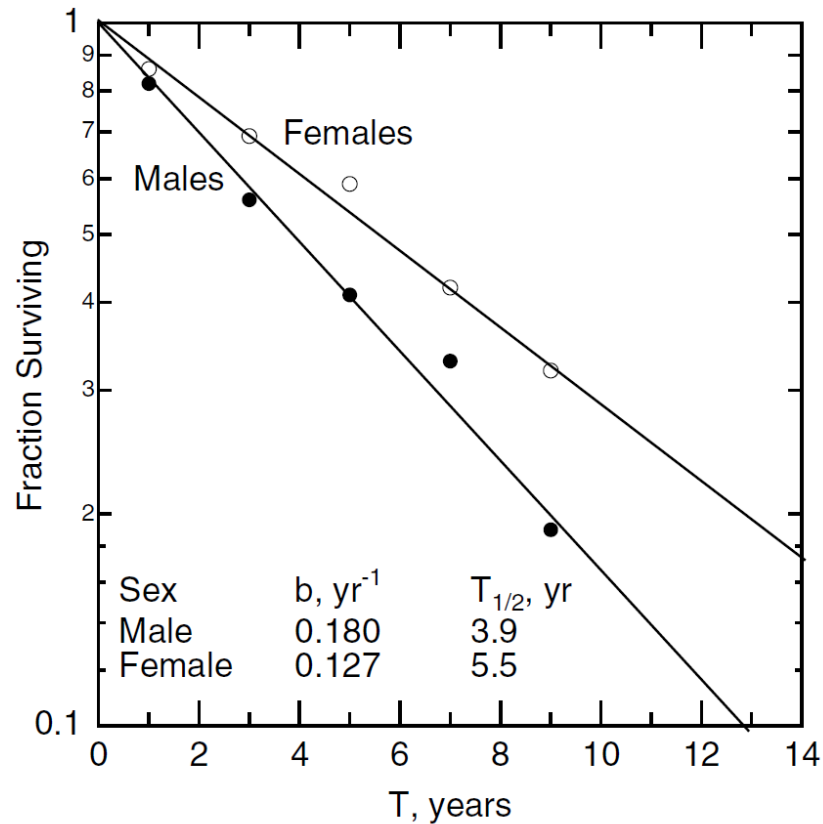
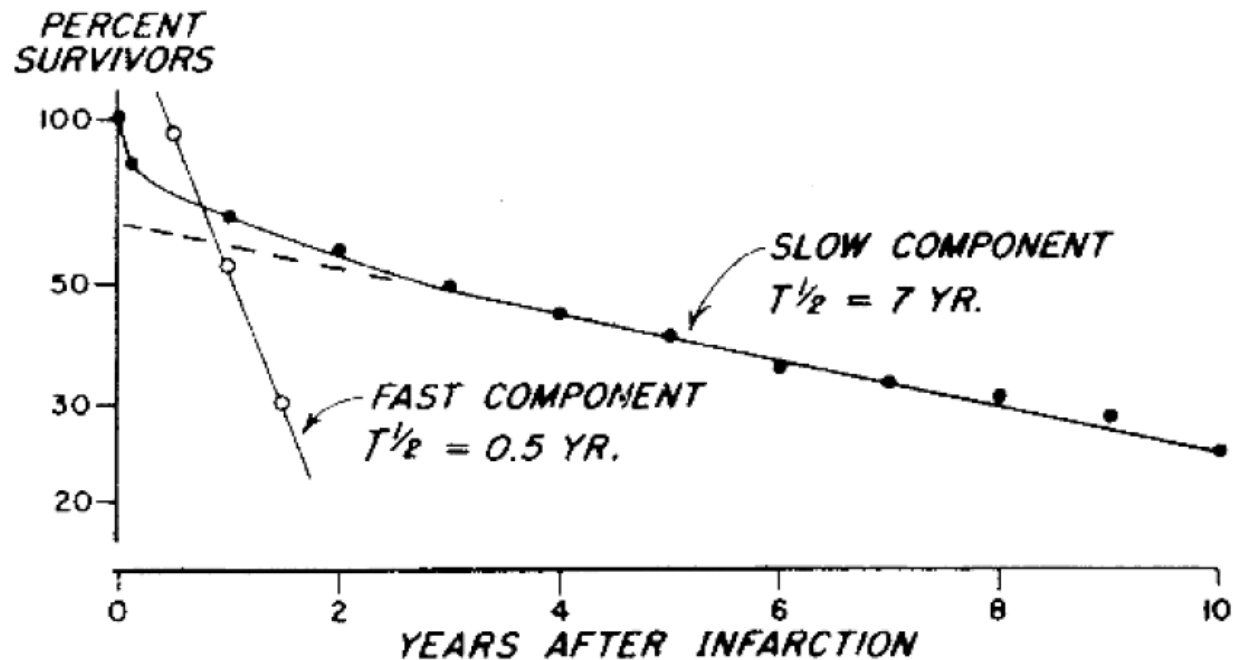


FIGURE 2.8. Survival of patients with congestive heart failure. Data are from McKee *et al.* (1971).

Variable Rates: Example

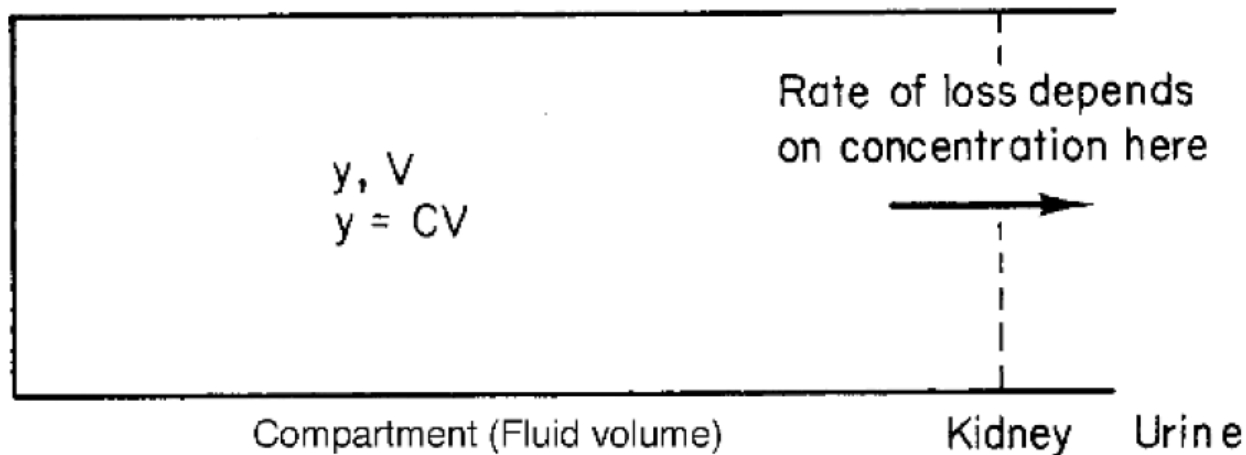
*SURVIVAL AFTER INITIAL MYOCARDIAL INFARCTION
10 YEAR FOLLOW-UP (BLAND and WHITE-1941)*



[Clearance]

- Clearance K is defined by,

$$\frac{dy}{dt} = -KC = -K\left(\frac{y}{V}\right) = -\left(\frac{K}{V}\right)y$$



[Multiple Decay Paths]

- Multiple decay processes

$$\frac{dy}{dt} = -b_1 y - b_2 y - b_3 y - \dots = -(b_1 + b_2 + b_3 + \dots) y = -b y$$

- Half-life

$$\frac{0.693}{T} = \frac{0.693}{T_1} + \frac{0.693}{T_2} + \frac{0.693}{T_3} + \dots \Rightarrow T = (T_1 + T_2 + T_3 + \dots)$$

Decay Plus Input at a Constant Rate

$$\frac{dy}{dt} = a - by \Rightarrow y = \frac{a}{b} (1 - e^{-bt})$$

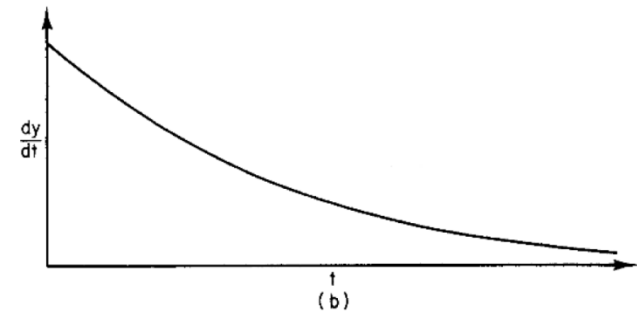
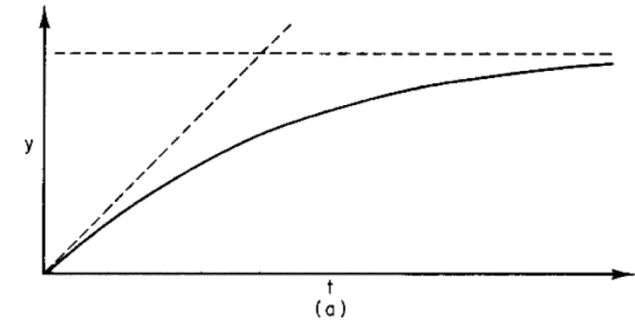
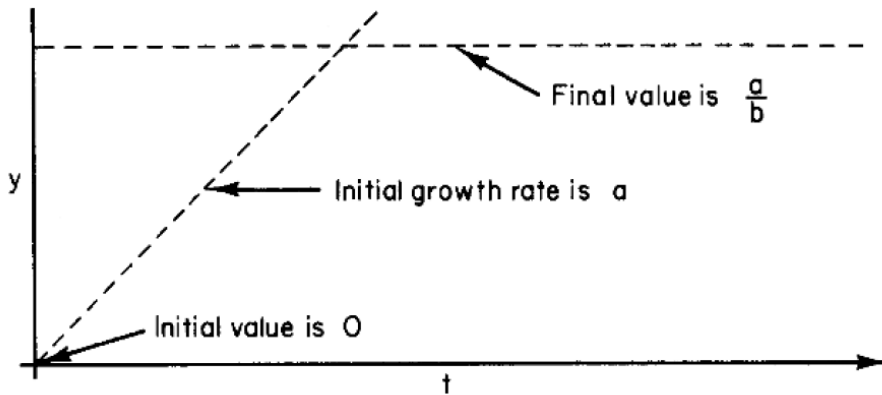


FIGURE 2.11. Sketch of the initial slope a and final value a/b of y when $y(0) = 0$.

FIGURE 2.12. (a) Plot of $y(t)$. (b) Plot of dy/dt .

Decay with Multiple Half-Lives: Fitting Exponentials

$$\begin{aligned}
 y &= y_1 + y_2 \\
 &= A_1 e^{-b_1 t} + A_2 e^{-b_2 t}
 \end{aligned}$$

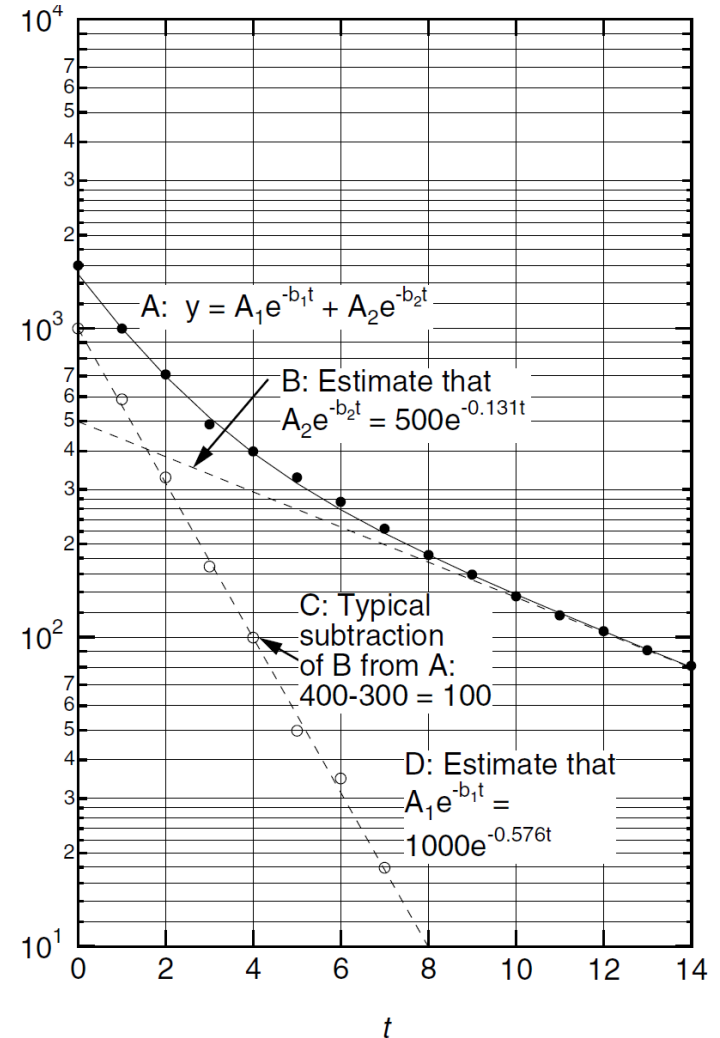


FIGURE 2.13. Fitting a curve with two exponentials.

[Log-Log Plots]

$$y = Bx^n$$

$$\log y = \log B + n \log x$$

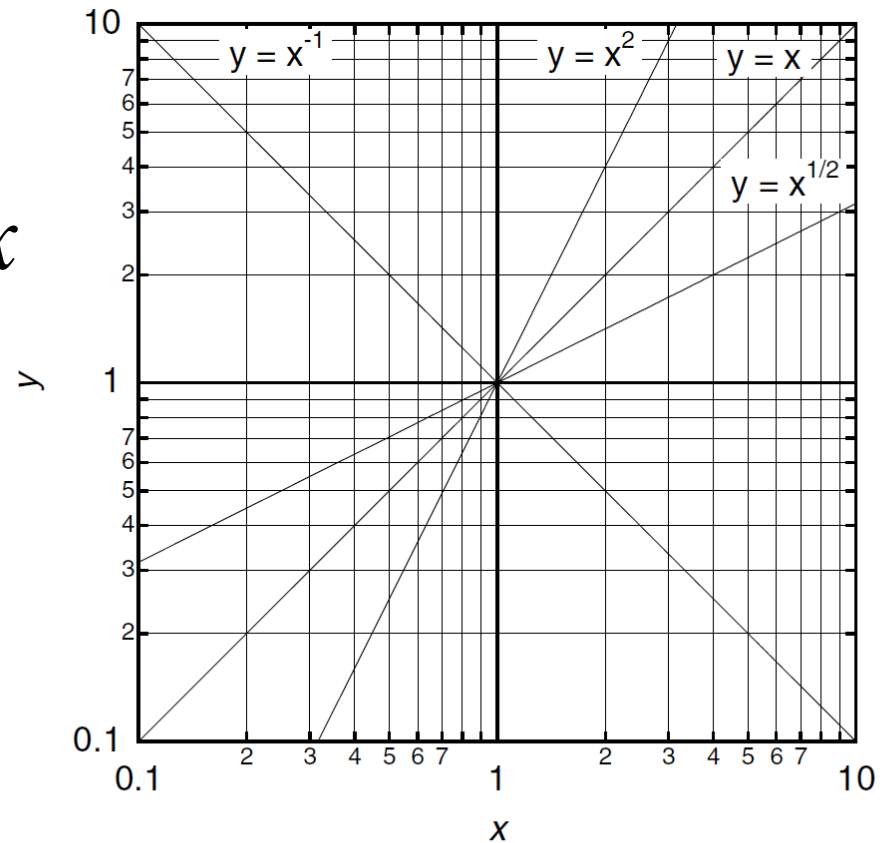
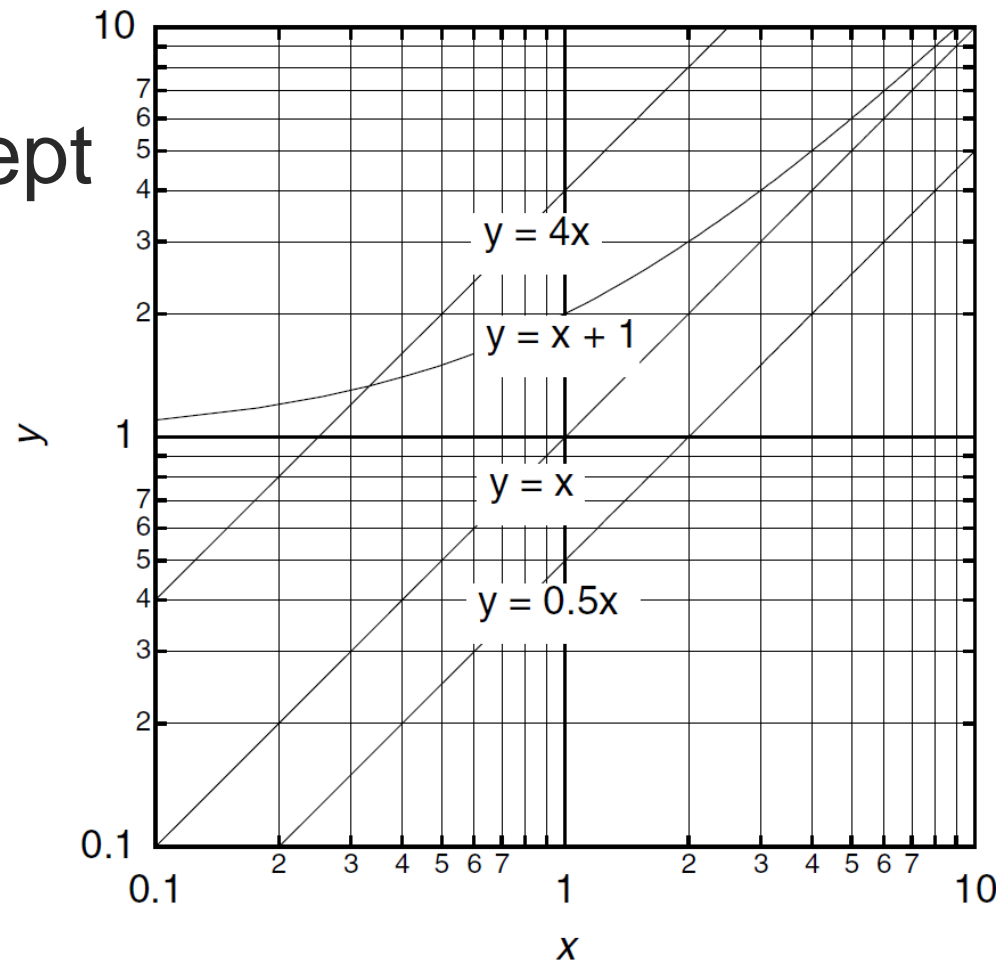


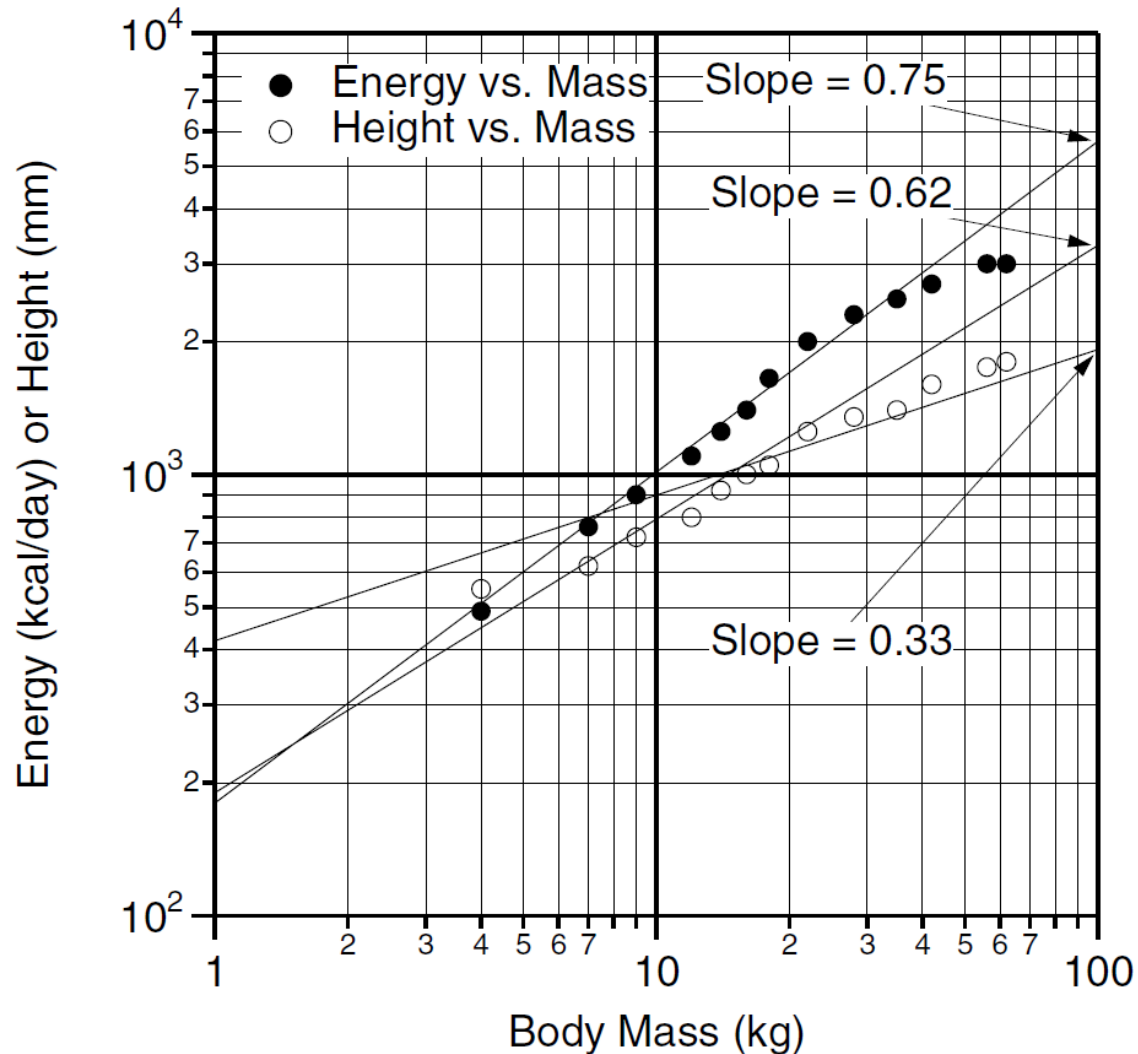
FIGURE 2.15. Log-log plots of $y = x^n$ for different values of n . When $x = 1$, $y = 1$ in every case.

[Log-Log Plots]

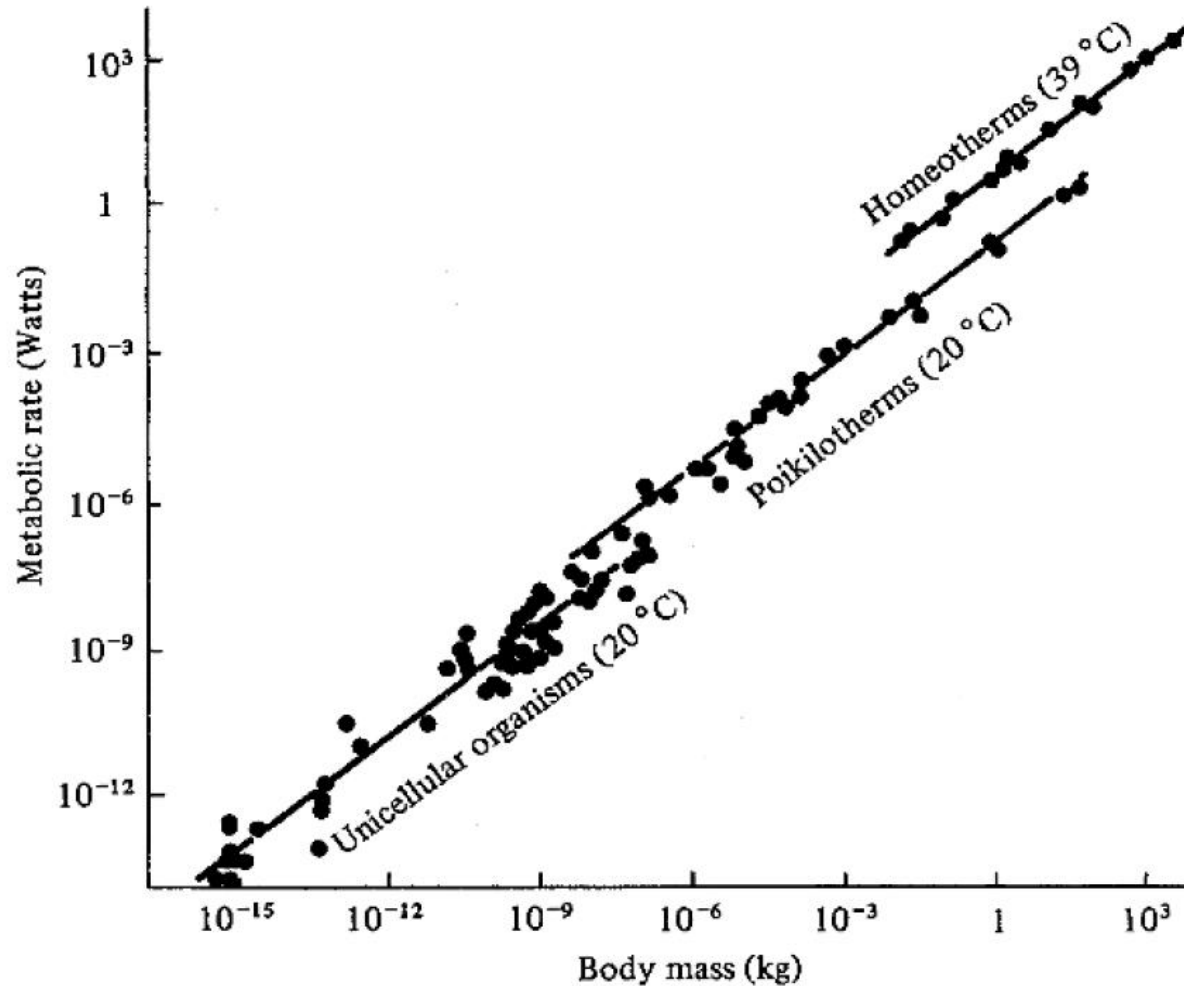
- Scaling
- Nonzero intercept



Example: Food Consumption



Example: Basal Metabolic Rate



[Problem Assignment]

- Posted on class web site

Web: <http://ymk.k-space.org/courses.htm>