Medical Equipment II - 2010
Chapter 14: Atoms and Light

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Web: http://ymk.k-space.org/courses.htm

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Atoms and Light

- This chapter describes some of the biologically important properties of infrared, visible, and ultraviolet light.
- To continue with X-rays in following chapters.

Nature of Light

- Light travels in a vacuum with a velocity $c = 3 \times 10^8$ *m/s*
- When light travels through matter, its speed is less than this and is given by

$$c_n = \frac{c}{n},$$

  - *n is index of refraction of substance*
  - depends on both the composition of substance and color of light.

Nature of Light

- Controversy over the nature of light existed for centuries
  - Isaac Newton: particle model
  - Thomas Young: Interference experiments
  - Late 19th century: waves
  - Early 20th century: dual nature
    - Matter has also wave properties!

Nature of Light

- A traveling wave of light can be described by $f(x - c_n t)$
  - represents a disturbance traveling along the x axis in the positive direction
- If wave is sinusoidal, then the period, T, frequency, ν, and wavelength, λ, are
- related by

$$\nu = \frac{1}{T}, \qquad c_n = \lambda \nu.$$

Nature of Light

- As light moves from one medium into another where it travels with a different speed, frequency remains the same.
  - Wavelength changes as the speed changes.
- Each particle of light or photon has energy E given by:

$$E = h\nu = \frac{hc_n}{\lambda}.$$

TABLE 14.1. The regions of the electromagnetic spectrum and their boundaries

Name	Wavelength	Frequency (Hz)	Energy (eV)
Radio waves			
	1 m	300×10^{6}	1.24×10^{-6}
Microwaves			
	1 mm	300×10^{9}	1.24×10^{-3}
Extreme infrared			
	15 μm	20×10^{12}	0.083
Far infrared			
	6 μm	50×10^{12}	0.207
Middle infrared			
	3 μm	100×10^{12}	0.414
Near infrared			
	750 nm	400×10^{12}	1.65
Visible			
	400 nm	750×10^{12}	3.1
Ultraviolet			
	12 nm	24×10^{15}	100
X rays, γ rays			

TABLE 14.2. The visible electromagnetic spectrum

Color	Wavelength (nm)	Frequency (10^{12} Hz)	Energy (eV)
	750	400	1.65
Red			
	610	490	2.03
Orange			
	590	510	2.10
Yellow			
	570	530	2.17
Green			
	500	600	2.48
Blue			
	450	670	2.76
Violet			
	400	750	3.11

Atomic Energy Levels

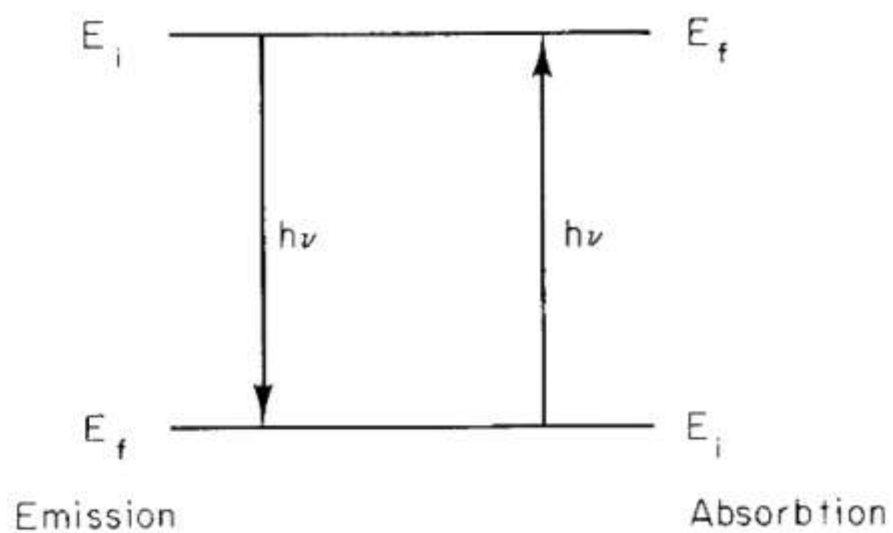
- The simplest system that can emit or absorb light is an isolated atom.
  - An atom is isolated if it is in a monatomic gas.
- In addition to translational kinetic energy, isolated atoms have specific discrete internal energies, called *energy levels.*
- An atom can change from one energy level to another by emitting or absorbing a photon with an energy equal to the energy difference between the levels.

Atomic Energy Levels

- Let the energy levels be labeled by i = 1, 2, 3, ..., with the energy of the ith state being E_i.
- There is a lowest possible internal energy for each atom
  - when the atom is in this state, no further energy loss can take place.

Atomic Energy Levels

- If E_i is greater than the lowest energy, then the atom can lose energy by emitting a photon of energy $E_i - E_f$ and exist in a lower-energy state Ef

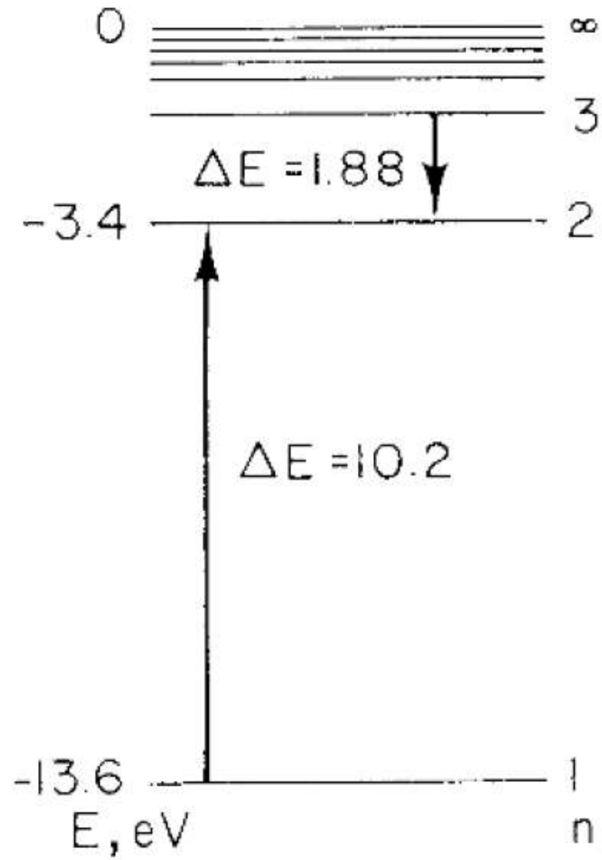


Energy Levels for Hydrogen

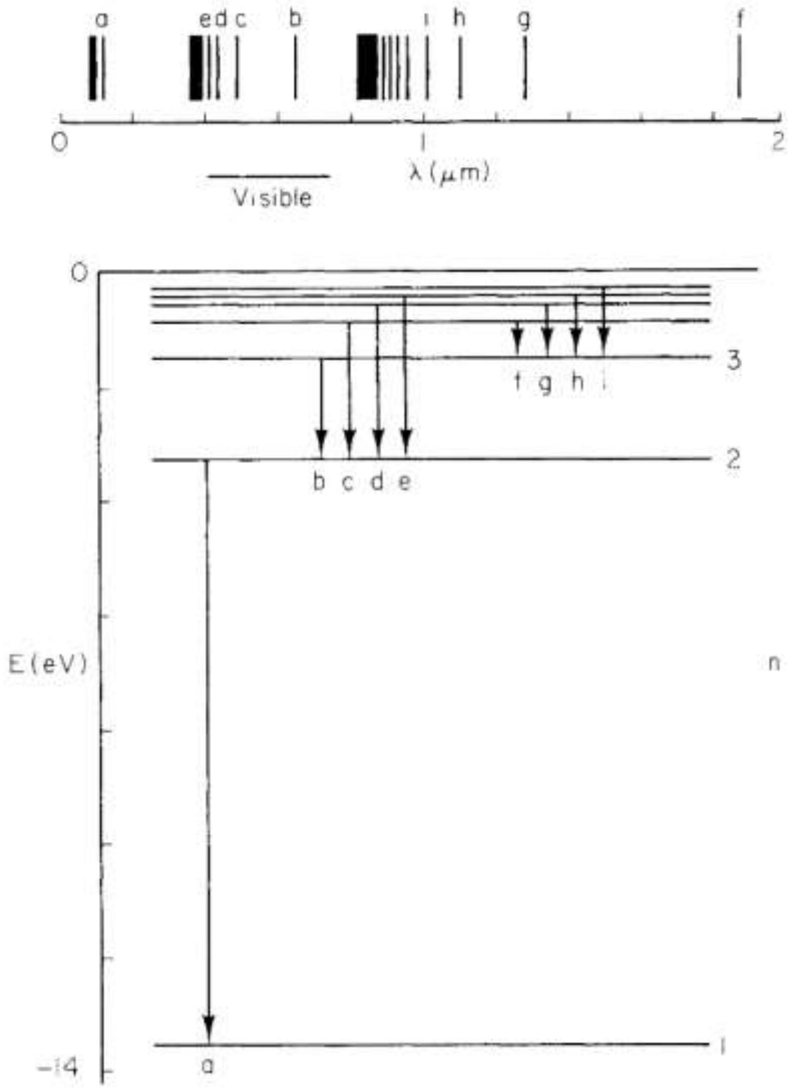
- From quantum mechanics, energy levels given as

$$E_n = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4}{2\hbar^2 n^2}, \quad n = 1, 2, 3, \ldots.$$

$$E_n = -\frac{13.6}{n^2} \quad \text{(in eV)}.$$



Hydrogen Atom Spectra



Atomic Energy Levels

- Internal energy of atom depends on values of five quantum numbers for each electron in atom.
  - Principal n
  - orbital angular momemtum l
  - Spin s
  - "*z component" of the* orbital angular momentum m_l
  - "*z component" of the* spin m_s

Atomic Energy Levels

- Pauli exclusion principle: No two electrons in an atom can have the same values for all their quantum numbers
- Ionization energy *is the smallest amount of energy* required to remove an electron from the atom when the atom is in its ground state.
  - Hydrogen:13.6 eV, Sodium: 5.1 eV

Atomic Energy Levels

- An atom can receive energy from an external source, such as a collision with another atom or some other particle.
  - It can also absorb a photon of the proper energy.
  - Allows one of its electrons to move to a higher energy level, as long as that level is not already occupied.
  - Can get rid of excess energy by emitting a photon
  - Selection rules

$$\Delta l = 1, \quad \Delta j = 0, \pm 1.$$

Scattering and Absorption of Radiation

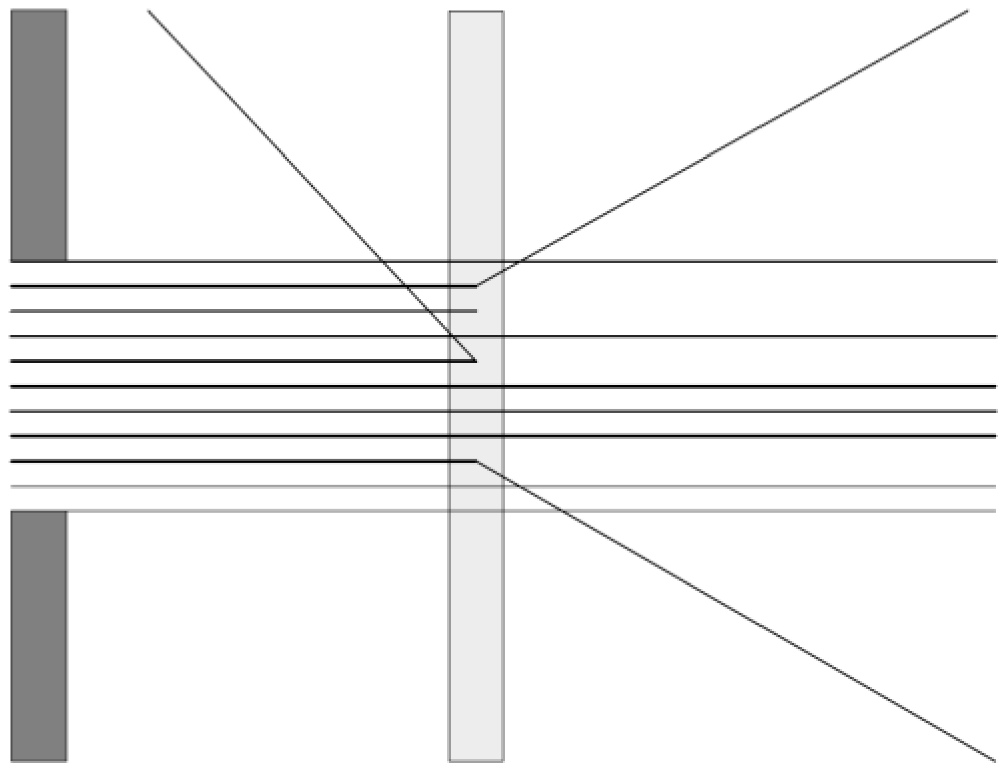
- Photons in a vacuum travel in a straight line.
- When they travel through matter they are apparently slowed down ($n>1$)
- May also be scattered or absorbed
  - Visible light does not go through walls
  - Blue color of sky or white color of clouds

Scattering and Absorption of Radiation

- Imagine that we have a distant source of photons that travel in straight lines, and that we collimate the beam
  - a nearly parallel beam of photons
  - Imagine also that we can see the tracks of the N photons in the beam

Scattering and Absorption of Radiation

- Passing through
- Scattering
- Absorption



Scattering and Absorption of Radiation

- Assume N photons passing through a thin layer of material *dz*

$$dN_s = \mu_s N dz, \qquad dN_a = \mu_a N dz.$$

$$dN = -(dN_s + dN_a) = -N(\mu_s + \mu_a)dz$$

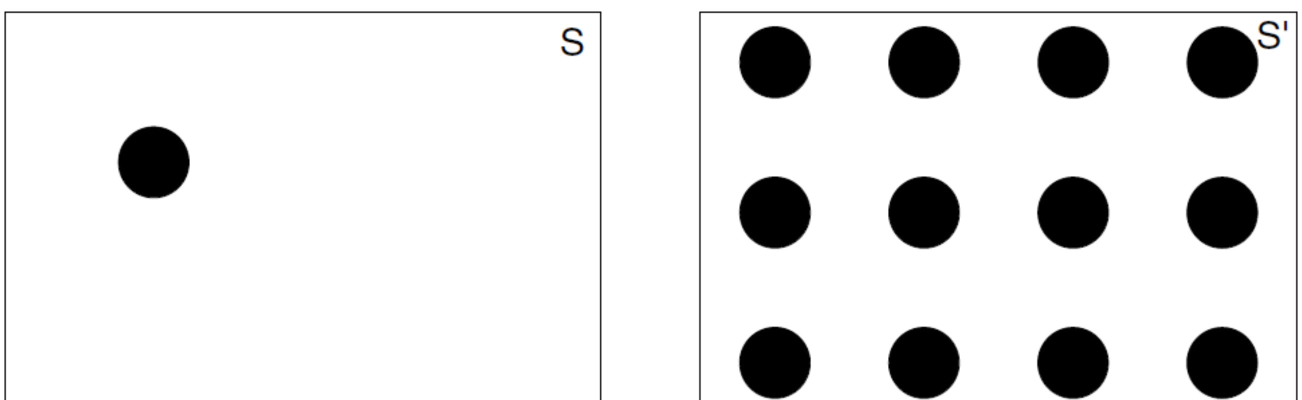
$$N(z) = N_0 e^{-\mu z} = N_0 e^{-(\mu_s + \mu_a)z}.$$

(Beer's law)

  - *μ is the total linear attenuation coefficient.*
  - *μ_s and μ_a are linear scattering and absorption* coefficients

Cross Section

- Interaction of photons with matter is statistical.
- The *cross section σ* is an effective area proportional to the probability that an interaction takes place.



Cross Section

- $\overline{n}$: number of interactions
- N_T: number of target entities
- Φ: average number of photons/unit area
- *p*: Probability of interaction

$$\frac{\overline{n}}{N} = \frac{\sigma}{S}.$$

$$\overline{n} = \sigma\Phi$$

$$\frac{\overline{n}}{N} = \frac{\sigma S' N_T}{S'} = \sigma N_T.$$

$$p = \sigma\Phi.$$

- Mutually exclusive interactions: $\sigma_{\text{tot}} = \sum_i \sigma_i.$

Cross Section Relation to Attenuation

- Number of target entities per unit area is equal to the number per unit volume times the thickness of the target along the beam

$$N_T = \frac{N_A \rho}{A} dz$$

$$\mu_s = \frac{N_A \rho}{A} \sigma_s,$$

$$\mu_a = \frac{N_A \rho}{A} \sigma_a,$$

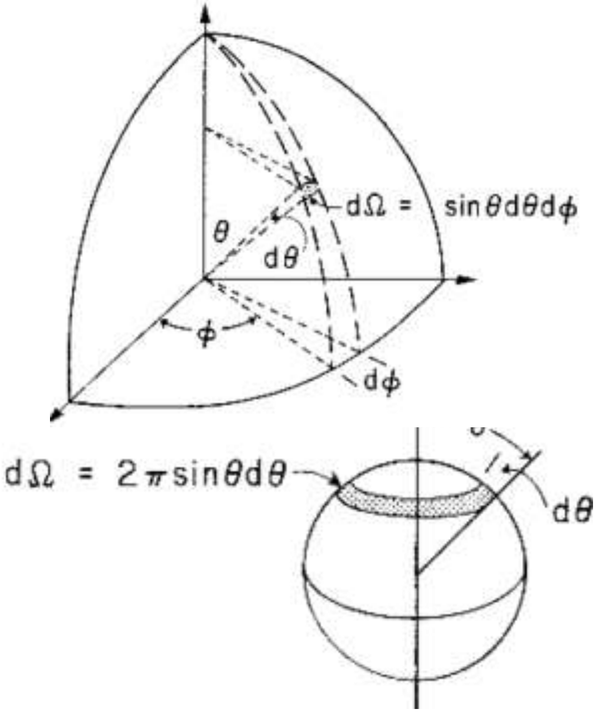
$$\mu = \frac{N_A \rho}{A} (\sigma_s + \sigma_a) = \frac{N_A \rho}{A} \sigma_{\mathrm{tot}}.$$

Differential Scattering Cross Section

- probability that particles are scattered in a certain direction.
  - probability that they are scattered into a small solid angle *dΩ*

$$\frac{d\sigma}{d\Omega}d\Omega \qquad \text{or} \qquad \sigma(\theta)d\Omega.$$

$$d\Omega = \sin\theta\, d\theta\ d\phi$$



Interpretation of Exponential Decay

- First interpretation:
  - The number of particles remaining in the beam that have undergone no interaction decreases as the target becomes thicker, so that the number of particles available to interact in the deeper layers is less

Interpretation of Exponential Decay

- Second interpretation:
  - the exponential can be regarded as taking into account the fact that in a thicker sample some of the target atoms are hidden behind others and are therefore less effective in causing new interactions.

Interpretation of Exponential Decay

- Third interpretation:
  - Poisson probability distribution
  - Homework to show

Cross Section: Final Words

- Large cross section
  - Complicated – multiple interactions likely
  - Approximations exist
- Reduced scattering coefficient
  - Let g= average value of scattering angle
  - g=0: isotropic , g=-1:backscattering

$$g = \frac{\int_0^\pi \sigma(\theta) \cos\theta \, 2\pi \sin\theta \, d\theta}{\int_0^\pi \sigma(\theta) \, 2\pi \sin\theta \, d\theta}.$$

$$\mu_s' = (1 - g)\mu_s$$

Problem Assignments

- Information posted on web site
- Chapter 14 Problems 1, 2, 3, 4, 10, 11, 13, 14