Medical Equipment I - 2010
Chapter 14

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Atoms and Light

- This chapter describes some of the biologically important properties of infrared, visible, and ultraviolet light.
- To continue with X-rays in following chapters.

Nature of Light

- Light travels in a vacuum with a velocity $c = 3 \times 10^8$ *m/s*
- When light travels through matter, its speed is less than this and is given by

$$c_n = \frac{c}{n},$$

  - *n is index of refraction of substance*
  - depends on both the composition of substance and color of light.

Nature of Light

- Controversy over the nature of light existed for centuries
  - Isaac Newton: particle model
  - Thomas Young: Interference experiments
  - Late 19th century: waves
  - Early 20th century: dual nature
    - Matter has also wave properties!

Nature of Light

- A traveling wave of light can be described by $f(x - c_n t)$
  - represents a disturbance traveling along the x axis in the positive direction
- If wave is sinusoidal, then the period, T, frequency, ν, and wavelength, λ, are
- related by

$$\nu = \frac{1}{T}, \qquad c_n = \lambda \nu.$$

Nature of Light

- As light moves from one medium into another where it travels with a different speed, frequency remains the same.
  - Wavelength changes as the speed changes.
- Each particle of light or photon has energy E given by:

$$E = h\nu = \frac{hc_n}{\lambda}.$$

TABLE 14.1. The regions of the electromagnetic spectrum and their boundaries

Name	Wavelength	Frequency (Hz)	Energy (eV)
Radio waves			
	1 m	300×10^6	1.24×10^{-6}
Microwaves			
	1 mm	300×10^9	1.24×10^{-3}
Extreme infrared			
	15 μm	20×10^{12}	0.083
Far infrared			
	6 μm	50×10^{12}	0.207
Middle infrared			
	3 μm	100×10^{12}	0.414
Near infrared			
	750 nm	400×10^{12}	1.65
Visible			
	400 nm	750×10^{12}	3.1
Ultraviolet			
	12 nm	24×10^{15}	100
X rays, γ rays			

TABLE 14.2. The visible electromagnetic spectrum

Color	Wavelength (nm)	Frequency (10^{12} Hz)	Energy (eV)
	750	400	1.65
Red			
	610	490	2.03
Orange			
	590	510	2.10
Yellow			
	570	530	2.17
Green			
	500	600	2.48
Blue			
	450	670	2.76
Violet			
	400	750	3.11

Atomic Energy Levels

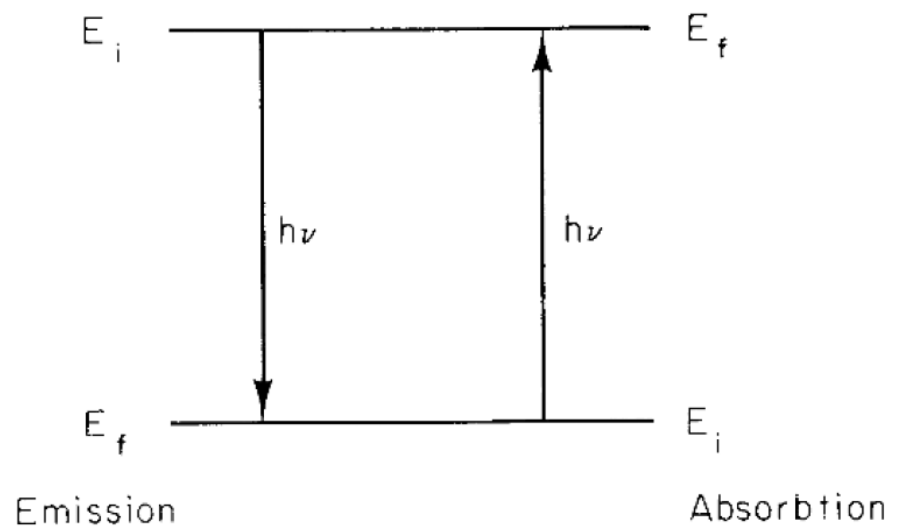
- The simplest system that can emit or absorb light is an isolated atom.
  - An atom is isolated if it is in a monatomic gas.
- In addition to translational kinetic energy, isolated atoms have specific discrete internal energies, called *energy levels.*
- An atom can change from one energy level to another by emitting or absorbing a photon with an energy equal to the energy difference between the levels.

Atomic Energy Levels

- Let the energy levels be labeled by i = 1, 2, 3, ..., with the energy of the ith state being E_i.
- There is a lowest possible internal energy for each atom
  - when the atom is in this state, no further energy loss can take place.

Atomic Energy Levels

- If E_i is greater than the lowest energy, then the atom can lose energy by emitting a photon of energy $E_i - E_f$ and exist in a lower-energy state Ef

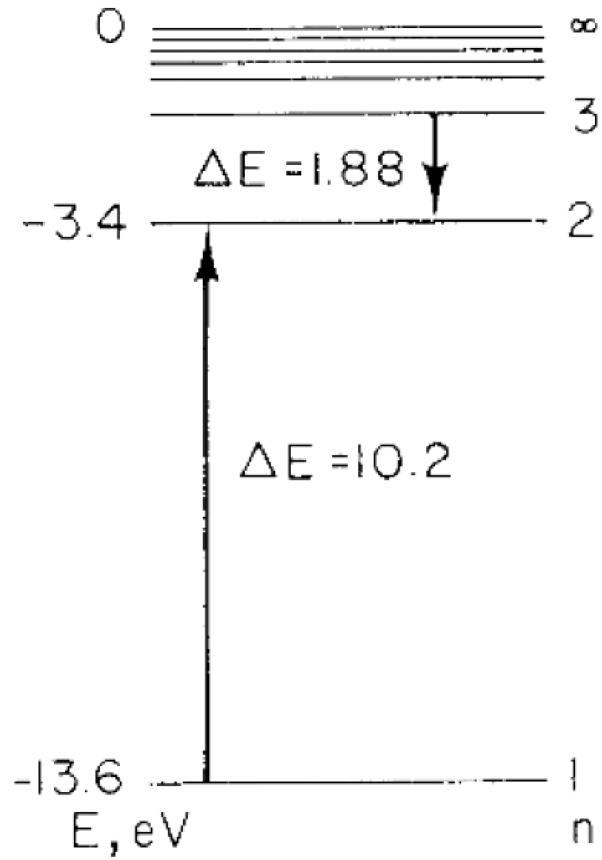


Energy Levels for Hydrogen

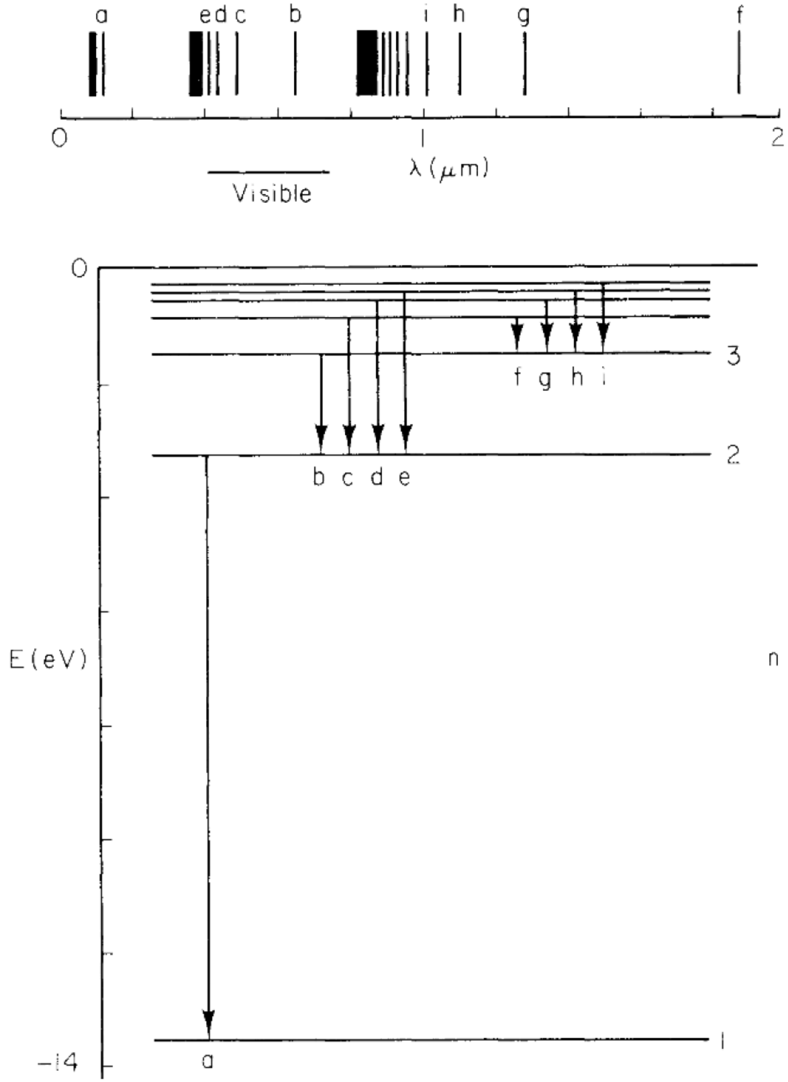
- From quantum mechanics, energy levels given as

$$E_n = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4}{2\hbar^2 n^2}, \quad n = 1, 2, 3, \ldots.$$

$$E_n = -\frac{13.6}{n^2} \quad (\text{in eV}).$$



Hydrogen Atom Spectra



Atomic Energy Levels

- Internal energy of atom depends on values of five quantum numbers for each electron in atom.
  - Principal n
  - orbital angular momemtum l
  - Spin s
  - "*z component" of the* orbital angular momentum m_l
  - "*z component" of the* spin m_s

Atomic Energy Levels

- Pauli exclusion principle: No two electrons in an atom can have the same values for all their quantum numbers
- Ionization energy *is the smallest amount of energy* required to remove an electron from the atom when the atom is in its ground state.
  - Hydrogen:13.6 eV, Sodium: 5.1 eV

Atomic Energy Levels

- An atom can receive energy from an external source, such as a collision with another atom or some other particle.
  - It can also absorb a photon of the proper energy.
  - Allows one of its electrons to move to a higher energy level, as long as that level is not already occupied.
  - Can get rid of excess energy by emitting a photon
  - Selection rules

$$\Delta l = 1, \quad \Delta j = 0, \pm 1.$$

Scattering and Absorption of Radiation

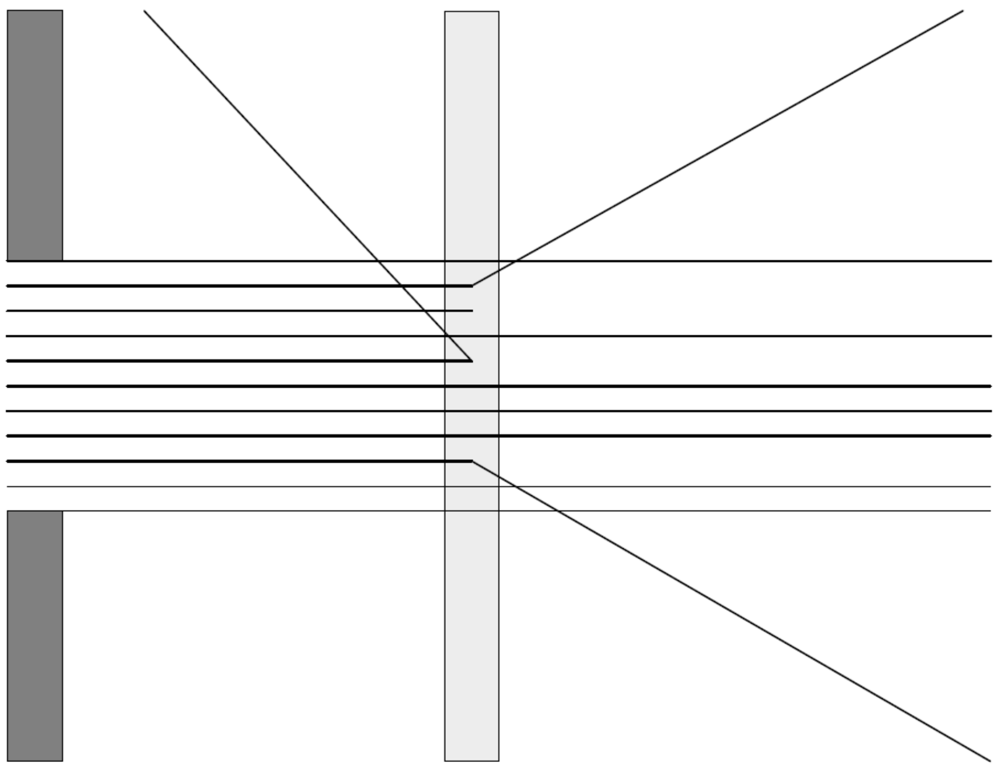
- Photons in a vacuum travel in a straight line.
- When they travel through matter they are apparently slowed down (n>1)
- May also be scattered or absorbed
  - Visible light does not go through walls
  - Blue color of sky or white color of clouds

Scattering and Absorption of Radiation

- Imagine that we have a distant source of photons that travel in straight lines, and that we collimate the beam
  - a nearly parallel beam of photons
  - Imagine also that we can see the tracks of the N photons in the beam

Scattering and Absorption of Radiation

- Passing through
- Scattering
- Absorption



Scattering and Absorption of Radiation

- Assume N photons passing through a thin layer of material *dz*

$$dN_s = \mu_s N dz, \qquad dN_a = \mu_a N dz.$$

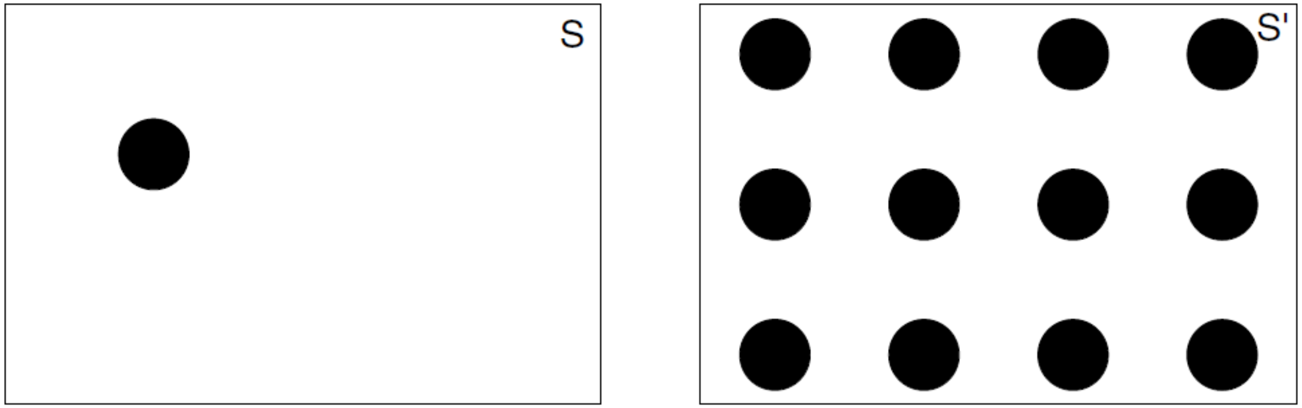
$$dN = -(dN_s + dN_a) = -N(\mu_s + \mu_a)dz$$

$$N(z) = N_0 e^{-\mu z} = N_0 e^{-(\mu_s + \mu_a)z}.$$ *(Beer's law)*

  - *μ is the total linear attenuation coefficient.*
  - *μ_s and μ_a are linear scattering and absorption* coefficients

Cross Section

- Interaction of photons with matter is statistical.
- The *cross section* σ is an effective area proportional to the probability that an interaction takes place.



Cross Section

- $\overline{n}$: number of interactions
- N_T: number of target entities
- Φ: average number of photons/unit area
- *p*: Probability of interaction

$$\frac{\overline{n}}{N} = \frac{\sigma}{S}.$$

$$\overline{n} = \sigma\Phi$$

$$\frac{\overline{n}}{N} = \frac{\sigma S' N_T}{S'} = \sigma N_T.$$

$$p = \sigma\Phi.$$

- Mutually exclusive interactions: $\sigma_{\text{tot}} = \sum_i \sigma_i.$

Cross Section Relation to Attenuation

- Number of target entities per unit area is equal to the number per unit volume times the thickness of the target along the beam

$$N_T = \frac{N_A \rho}{A} dz$$

$$\mu_s = \frac{N_A \rho}{A} \sigma_s ,$$

$$\mu_a = \frac{N_A \rho}{A} \sigma_a ,$$

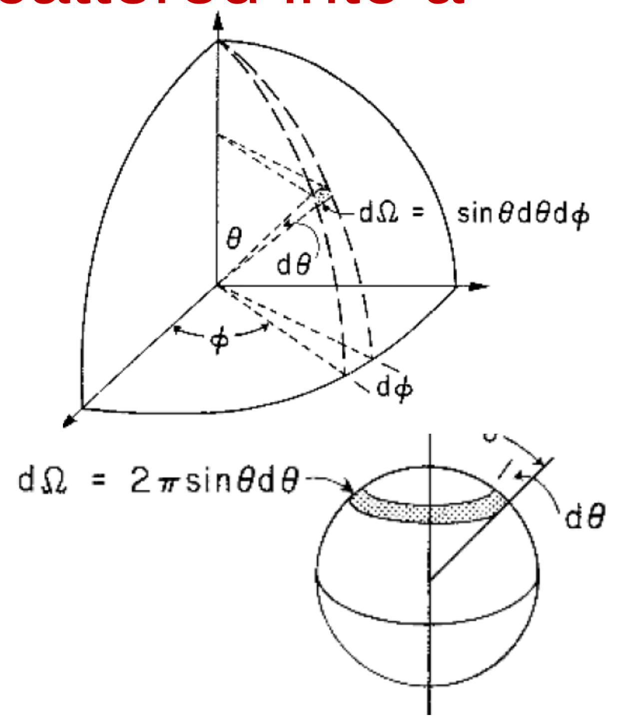
$$\mu = \frac{N_A \rho}{A} (\sigma_s + \sigma_a) = \frac{N_A \rho}{A} \sigma_{\mathrm{tot}} .$$

Differential Scattering Cross Section

- probability that particles are scattered in a certain direction.
  - probability that they are scattered into a small solid angle $d\Omega$

$$\frac{d\sigma}{d\Omega}d\Omega \qquad \text{or} \qquad \sigma(\theta)d\Omega.$$

$$d\Omega = \sin\theta \, d\theta \;\; d\phi$$



Interpretation of Exponential Decay

- First interpretation:
  - The number of particles remaining in the beam that have undergone no interaction decreases as the target becomes thicker, so that the number of particles available to interact in the deeper layers is less

Interpretation of Exponential Decay

- Second interpretation:
  - the exponential can be regarded as taking into account the fact that in a thicker sample some of the target atoms are hidden behind others and are therefore less effective in causing new interactions.

Interpretation of Exponential Decay

- Third interpretation:
  - Poisson probability distribution
  - Homework to show

Cross Section: Final Words

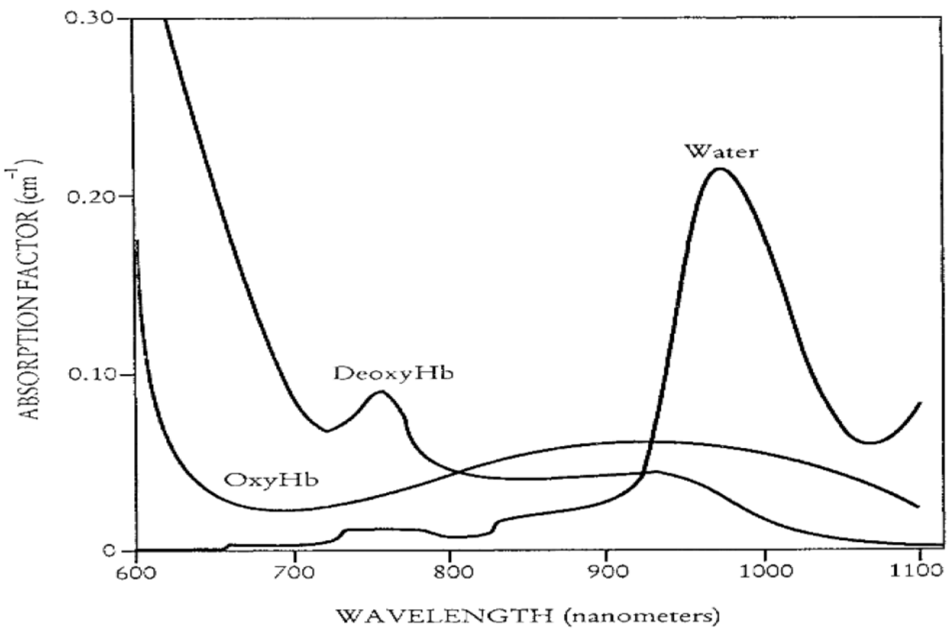
- Large cross section
  - Complicated – multiple interactions likely
  - Approximations exist
- Reduced scattering coefficient
  - Let g= average value of scattering angle
  - g=0: isotropic , g=-1:backscattering

$$g = \frac{\int_0^\pi \sigma(\theta) \cos\theta \, 2\pi \sin\theta \, d\theta}{\int_0^\pi \sigma(\theta) \, 2\pi \sin\theta \, d\theta}.$$

$$\mu_s' = (1 - g)\mu_s$$

Applications of NIR: Blood Oximetry

- NIR (600–1000 nm) is used to measure the oxygenation of the blood
  - Measure absorption at two different λ's
  - Isosbestic point
  - Calibration

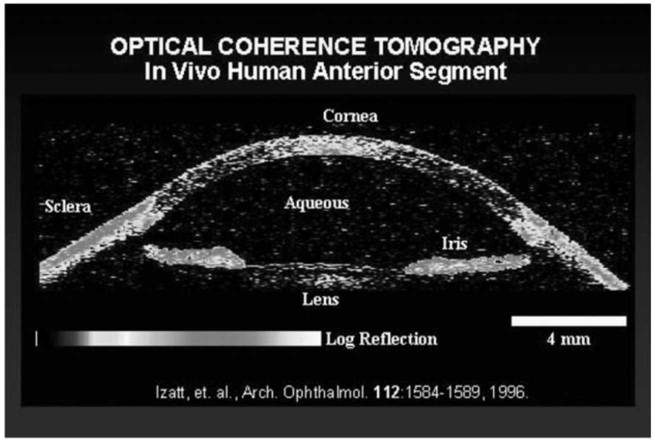


Optical Coherence Tomography (OCT)

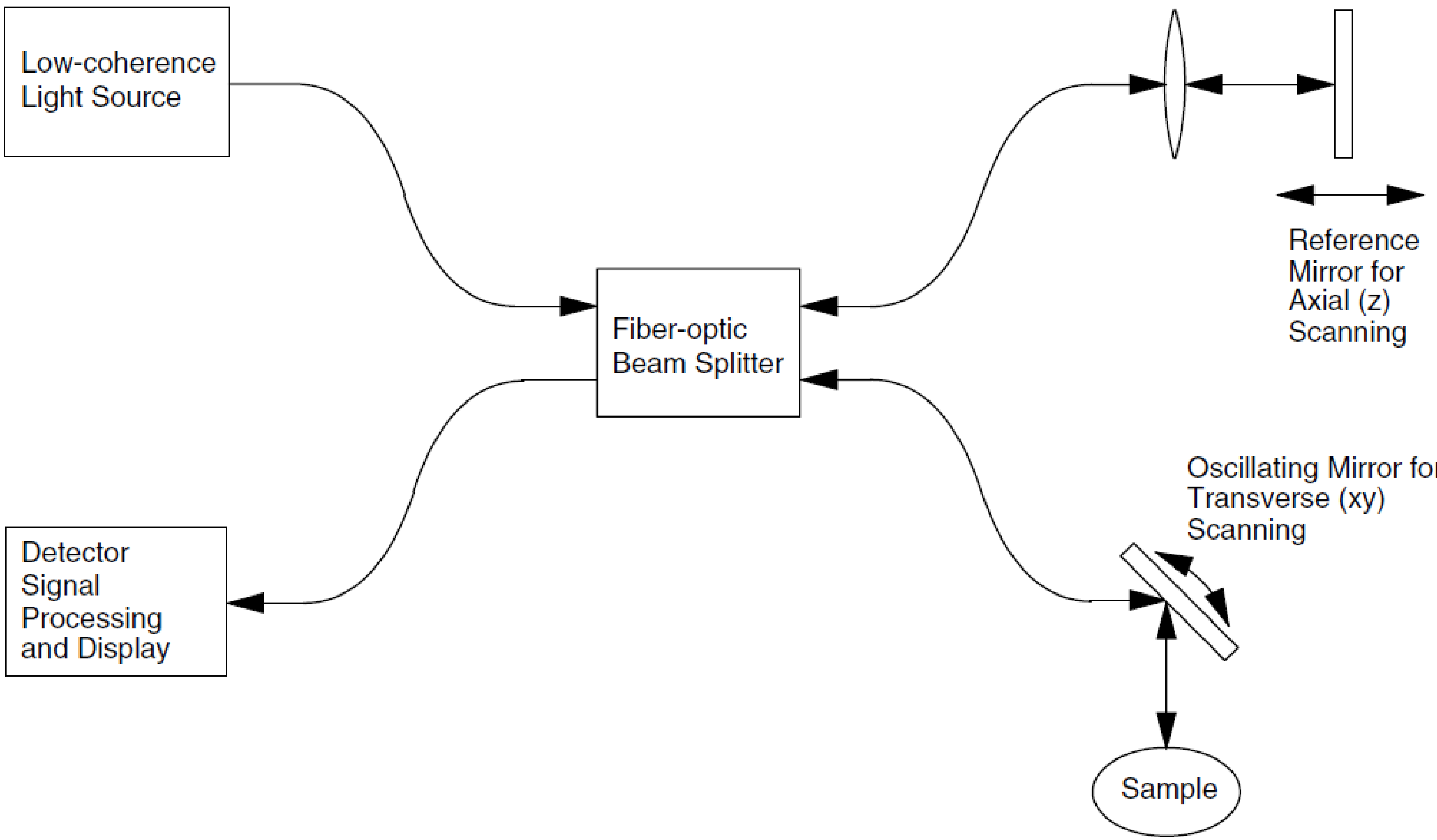
- Optical range measurements using the time delay of reflected or backscattered light from pulses of a few femtosecond (10^{-15} s) duration can be used to produce images similar to those of ultrasound A- and B-mode scans.
  - Spatial extent of a 30 fs pulse in water ~7 μm.
  - Difficult to measure delays that short

- OCT system design:

  Use interference principles to solve problem with simple method



OCT System Diagram



OCT Theory

- Wave received: $A \sin \frac{2\pi}{\lambda}(x - c_n t)$
- Detector: intensity averaged over time

$$y \propto A^2 \overline{\sin^2 \omega(x/c_n - t)} = A^2/2.$$

- Split wave and add two waves before detection : Interferometry

$$y \propto (A/2)^2 \overline{[\sin \omega(x_1/c_n - t) + \sin \omega(x_2/c_n - t)]^2}$$
$$= \frac{A^2}{4}\left(1 + \cos \frac{\omega}{c_n}(x_2 - x_1)\right).$$

  - Change position of reference beam to get position of the reflector and use to form image

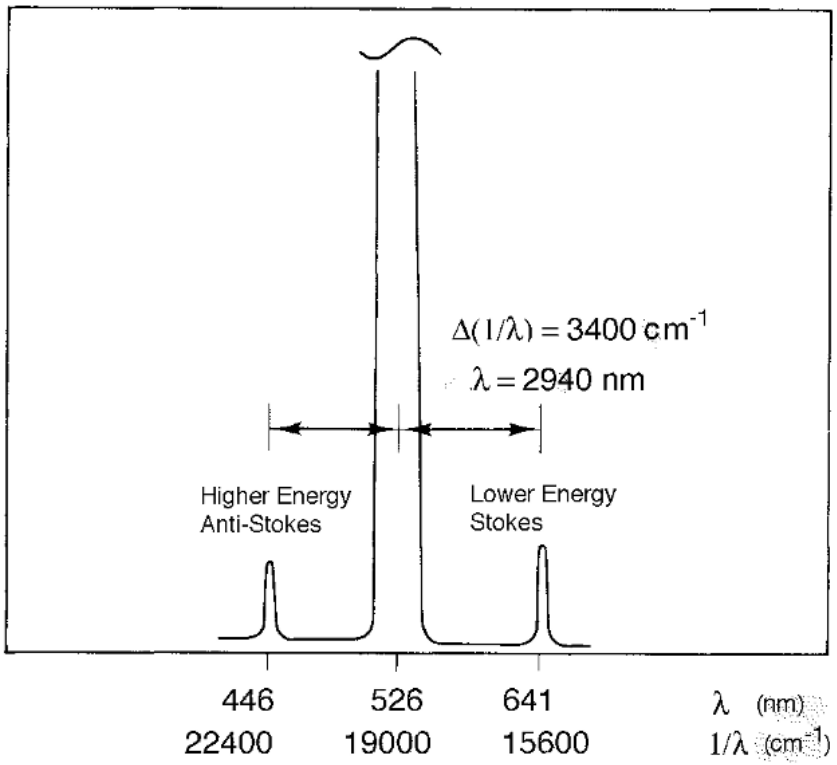
OCT: Coherence of Source

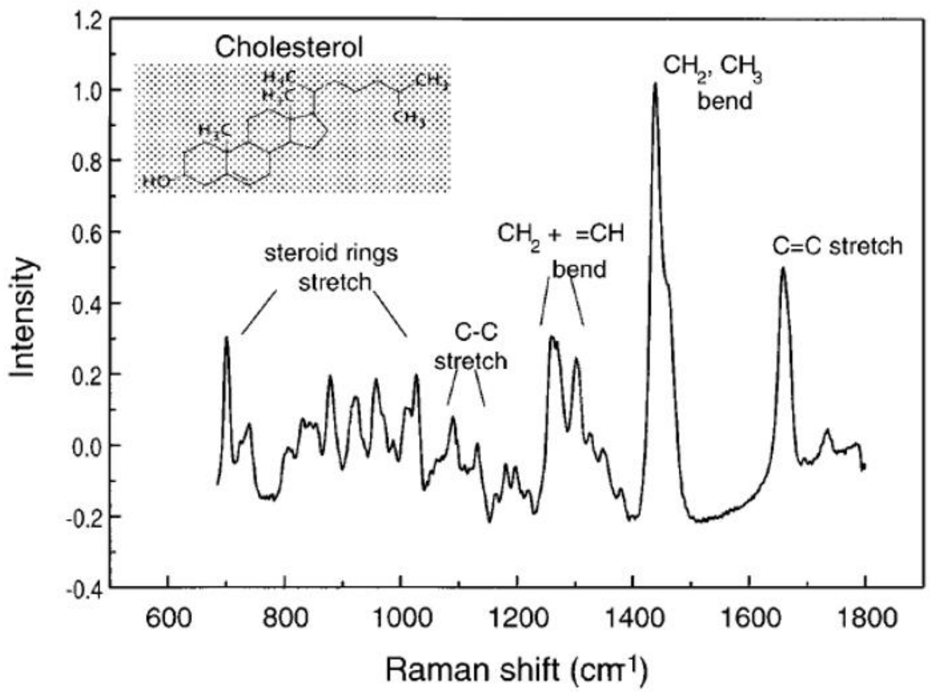
- Coherence of the light beam
  - number of cycles over which the phase of the wave does not change.
- When an atom emits light, wave lasts for a finite time, τ_{coh} (~10^{-8} s).
  - When another atom emits light, the phase is unrelated to the phase of light already emitted.
- If $(x_2-x_1)/c_n > \tau_{coh}$, time average will go to 0
  - Distance measured when $x_2 - x_1 < c_n \tau_{\text{coh}}$.
  - Hi Res: use light source with short τ_{coh}

Raman Spectroscopy

- Scattering of light in which the scattered photon does not have its original energy, but has lost or gained energy corresponding to a rotational or vibrational transition.
- Raman scattering can be done with light of any wavelength, from infrared to ultraviolet

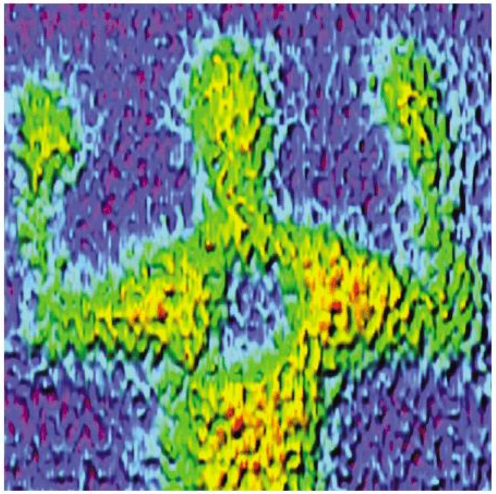
Raman Scattering





Far Infrared or Terahertz Radiation

- Medical use of THz radiation ("T rays")
- Polarization is important
- High attenuation
  - Restricted to superficial applications
- Applications
  - Spectroscopy
  - Imaging



Thermal Radiation

- Any atomic gas emits light if it is heated to a few thousand kelvin.
  - Light consists of a line spectrum.
  - Example: famous yellow line of sodium

$$\lambda = 589.2 \text{ nm},$$

$$\nu = c/\lambda = 5.092 \times 10^{14} \text{ Hz},$$

$$E = h\nu = hc/\lambda = 3.38 \times 10^{-19} \text{ J} = 2.11 \text{ eV}.$$

Thermal Radiation

- At any temperature, ratio of excited sodium atoms to those in ground state is given by Boltzmann ratio:,

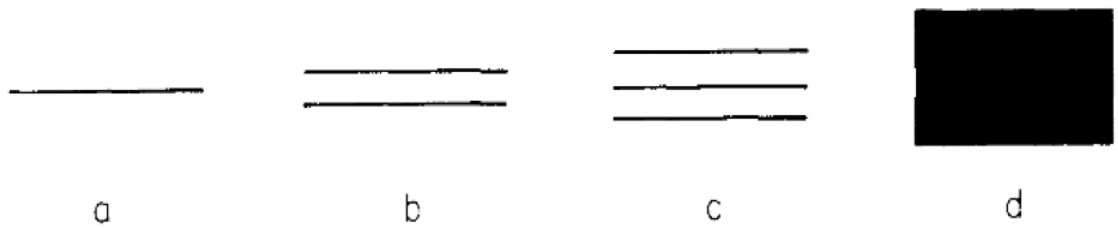
$$\frac{P_{\text{excited}}}{P_{\text{ground}}} = e^{-E/k_B T}$$

- At room temperature 3.8×10^{-36} (negligible)
- At 1500 K 8×10^{-8} (enough atoms give off light)

Thermal Radiation

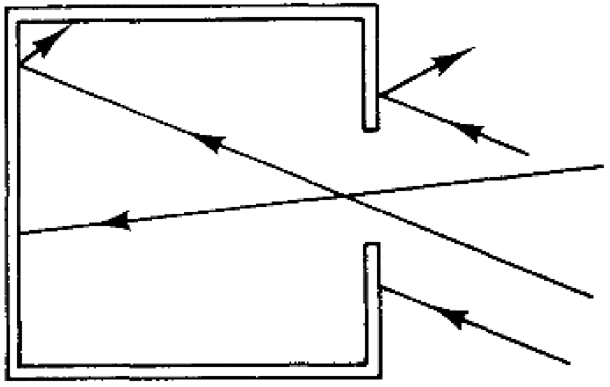
- Heating a piece of iron
  - Splitting
  - Shifting
  - Bands

Color	T (K)
Red, just visible in daylight	750–800
"Cherry" red	975–1175
Yellow	1200–1505
White	1425-1800
Dazzling (bluish) white	1900



Blackbody

- A substance that has so many closely spaced levels that it can absorb every photon that strikes it appears black.
- Ideal not practical
  - Hole in cavity wall approximation



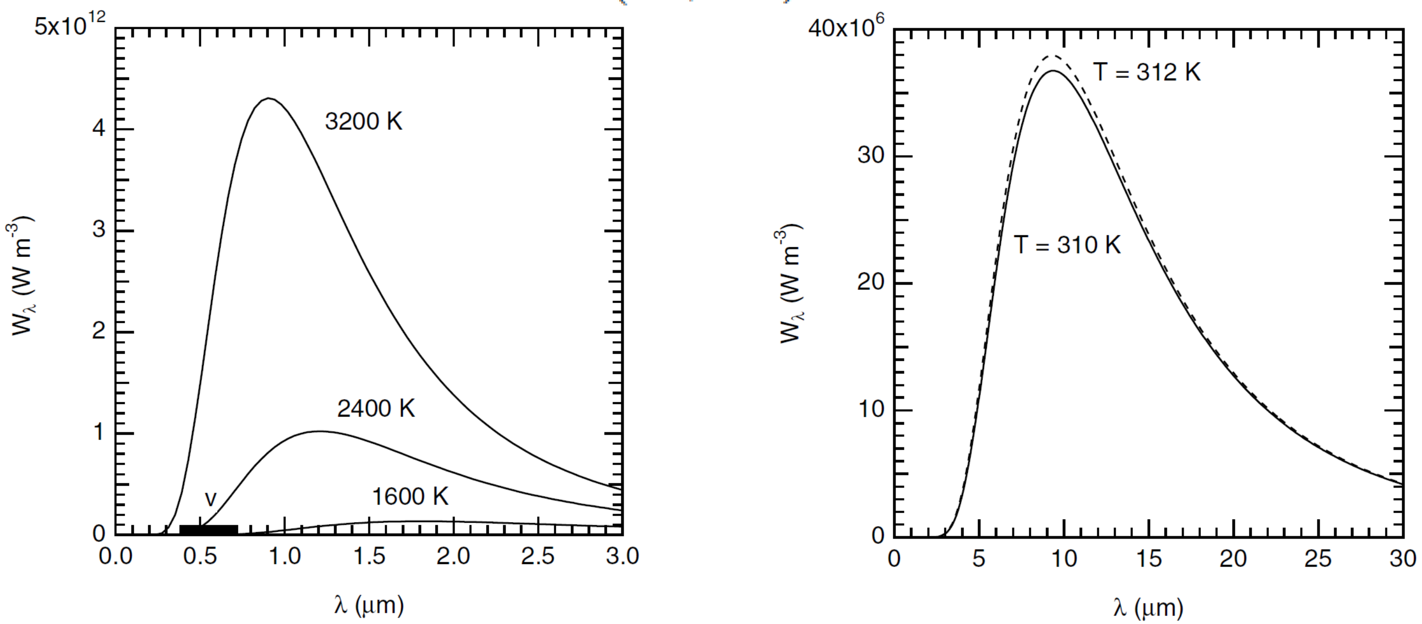
Emissivity

- If the surface is not completely absorbing, we define the emissivity $\varepsilon(\lambda)$, which is the fraction of light absorbed at wavelength λ
  - $\varepsilon = 0$ for transparent or reflective material
  - $\varepsilon = 1$ for complete absorption
  - $\varepsilon(\lambda) = 1$ for a blackbody
  - $\varepsilon(\lambda) =$ constant <1 for a "gray body"
- When a blackbody is heated, the light given off has a continuous spectrum because the energy levels are so closely spaced.

Blackbody Radiation Function

- The spectrum of power per unit area emitted by a completely black surface in wavelength interval between λ and λ+dλ (units W/m³)

$$W_\lambda(\lambda, T)d\lambda$$



Blackbody Radiation Function

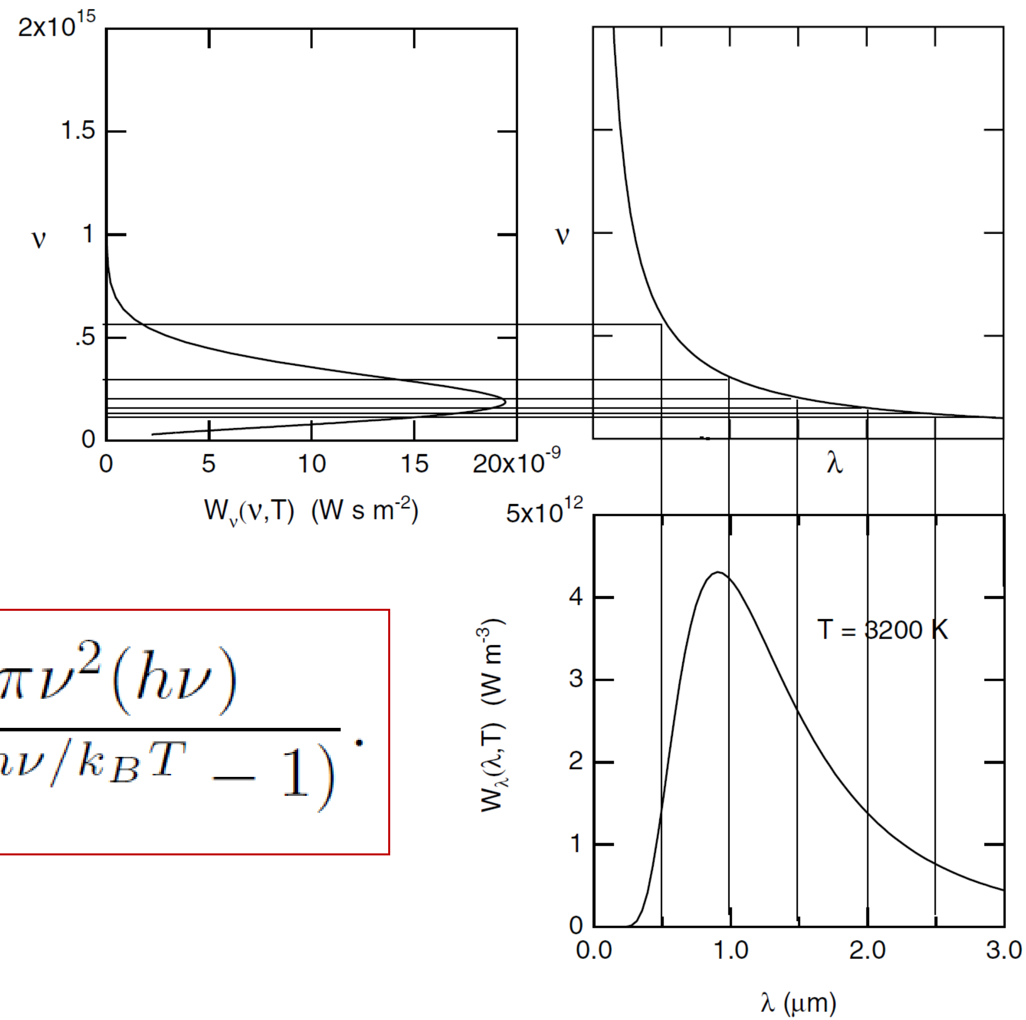
- Total power from Planck's theoretical formula

$$W_\lambda(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5 \left(e^{hc/\lambda k_B T} - 1\right)}.$$

$$W_{\text{tot}}(T) = \int_0^\infty W_\lambda(\lambda, T)\, d\lambda$$
$$= \frac{2\pi^5 k_B^4}{15 c^2 h^3} T^4 = \sigma_{SB} T^4.$$

Stefan-Boltzmann law with σ_{SB} defined as Stefan-Boltzman constant $\sigma_{SB} = 5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}.$

Blackbody Radiation Function



$$W_\nu(\nu, T) = \frac{2\pi\nu^2(h\nu)}{c^2(e^{h\nu/k_BT} - 1)}.$$

The transformation from $W_\lambda(\lambda, T)$ to $W_\nu(\nu, T)$

Blackbody Radiation Function: Peak Location

- Differentiate and equate by 0

$$\lambda_{\max} T = \frac{hc}{4.9651 k_B} = 2.90 \times 10^{-3} \text{ m K}.$$

$$\frac{\nu_{\max}}{T} = \frac{2.82144 k_B}{h} = 5.88 \times 10^{10} \text{ K}^{-1} \text{ s}^{-1}.$$

$$\lambda_{\max} \nu_{\max} = 1.705 \times 10^{8} \text{ m s}^{-1} = 0.57c.$$

- General case (not blackbody)

$$\boxed{\epsilon(\lambda) W_\lambda(\lambda, T).}$$

Infrared Radiation from The Human Body

- Emissivity of human skin:

$$\text{for } 1\ \mu\text{m} < \lambda \leq 14\ \mu\text{m},\ \epsilon(\lambda) = 0.98 \pm 0.01$$

- Assuming surface area of 1.73m2, temperature of 33 °C, total power radiated is

$$w_{\text{tot}} = SW_{\text{tot}} = S\sigma_{SB}T^4 = 860\ \text{W}$$

  - 9 times basal metabolic rate of 100W
  - Error: assuming surroundings at 0 °K

Infrared Radiation from The Human Body

- Assume surrounding temperature T_s=20°C

$$w_{\text{tot}} = S\sigma_{SB}(T^4 - T_s^4).$$

$$w_{\text{tot}} = (1.73)(5.67 \times 10^{-8})(306^4 - 293^4) = 137 \text{ W}.$$

  - Subject has to exercise to keep body temperature
- Radiation to a cold window
  - Glass has high emissivity in infrared region
  - Heat from body to glass window more than from
  - Application to infant incubator near window

Infrared Imaging

- Illumination and imaging of shallow structures
- Thermal imaging of the body
- Body temperature measurement
- Atherosclerotic coronary heart disease (ACHD) diagnosis
  - Temperature sensor (thermistor)
  - Raman spectroscopy and near infrared spectroscopy

Blue and Ultraviolet Radiation

- The energy of individual photons of blue and ultraviolet light is high enough to trigger chemical reactions in the body.
  - Can be both harmful and beneficial.
  - Beneficial: treatment of neonatal jaundice
  - Harmful: sunburn, skin cancer, and premature aging of the skin

Treatment of Neonatal Jaundice

- Neonatal jaundice occurs when bilirubin builds up in the blood
  - Bilirubin is a toxic waste product of decomposition of hemoglobin from dead RBCs.
  - Insoluble in water and cannot be excreted through either the kidney or the gut.
  - It is excreted only after being conjugated with glucuronic acid in liver (become water soluble). through bile and leave via the gut.
  - Occurs when newborns have immature livers or with increased rate of hemolysis

Treatment of Neonatal Jaundice

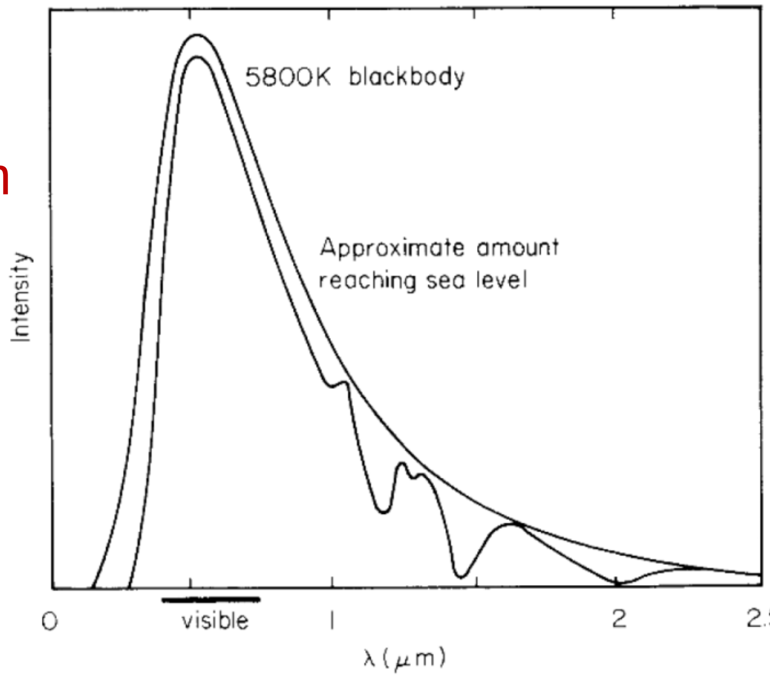
- When skin of a newborn with jaundice was exposed to bright light, jaundice went away.
  - Photons of blue light have sufficient energy to convert the bilirubin molecule into more soluble and apparently less harmful forms.
  - Photons of longer wavelength have less energy and are completely ineffective.

Treatment of Neonatal Jaundice

- Standard form of phototherapy used to be to place the baby "under the lights."
  - Place eye patches to protect the baby's eyes.
  - Baby placed in an incubator to keep warm
- Fiberoptic blanket
  - Wrapped around the baby's torso under clothing
  - Still controversial

UV Spectrum

- UV can come from the sun or from lamps.
- Maximum intensity of solar radiation is in the green, at about 500 nm.
  - Sun emits approximately like a thermal radiator at a temperature of 5,800 °K.
  - O_3 absorbs 200-320 nm
  - Molecular O_2 absorbs <180 nm
- Solar constant
  - Theoretical 1,390 W m^{-2}
  - Satellite 1,372 W m^{-2}
  - Surface: 1,000 W m^{-2}



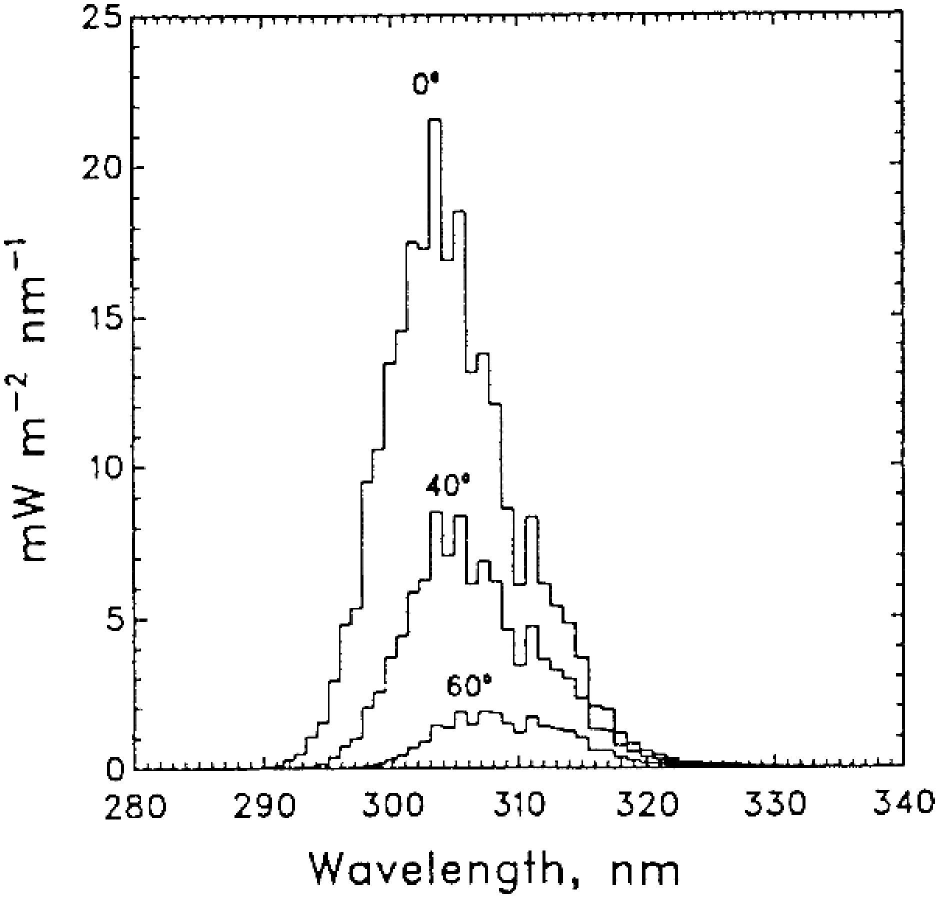
UV Spectrum

UVA	315–400 nm
UVB	280–315 nm[12]
UVC or middle UV	200–280 nm
Vacuum UV	<240 nm
Far UV	120–200 nm
Extreme UV	10–120 nm

- Only the first three are of biological significance, because the others are strongly absorbed in the atmosphere

UV Spectrum

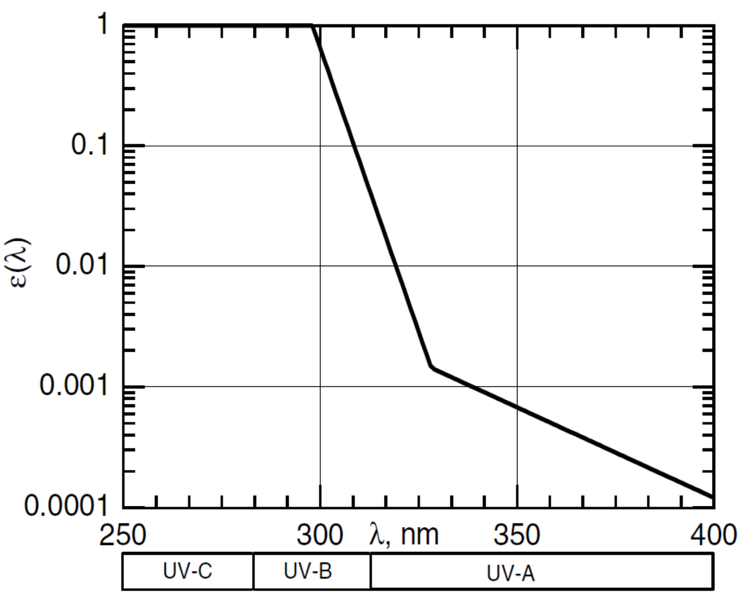
- Solar radiation reaching ground when sun is at different angles from the zenith weighted for DNA sensitivity.



Response of Skin to UV

- Acute (inflammatory) response
  - Erythema : reddening of the skin
  - Minimum erythemal dose
  - Erythema action spectrum: relative sensitivity of skin vs. wavelength

$$\epsilon(\lambda) = \begin{cases} 1.0, & 250 \le \lambda \le 298 \text{ nm} \\ 10^{0.094(298-\lambda)}, & 298 \le \lambda \le 328 \text{ nm} \\ 10^{0.015(139-\lambda)}, & 328 \le \lambda \le 400 \text{ nm} \end{cases}$$



Response of Skin to UV

- Chronic (prolonged) response
  - Abnormal cell growth
  - Hypertrophy or hyperplasia

TABLE 14.4. Abnormal changes in tissue

Metaplasia	A reversible change in which one cell type is replaced by another.
Dysplasia	Variation in size, shape, and organization of the cells. Literally, "deranged development."
Anaplasia	A marked, irreversible, and regressive change from adult cells that are differentiated in form to more primitive, less differentiated cells.

Ultraviolet Light Causes Skin Cancer

- Chronic exposure to UV causes premature aging of the skin.
  - Photoaged different from skin with normal aging
- UVA and UVB suppress the body's immune system
  - This plays a major role in cancer caused by UV

TABLE 14.5. Estimates of skin cancer incidence rates per 100,000.[a]

Cancer type	Population	Males	Females
Melanoma	White, New Orleans, 1983–87	6.9	5.3
	White, Hawaii, 1983–1987	22.2	14.9
SCC	White, U.S., 1994 (rough est.)	100	45
BCC	White, U.S., 1994 (rough est.)	400	200

Protection of Skin from UV

- Sunscreen (SPF: sun protection factor)
  - SPF is based on erythema (mainly UVB effect)
  - Some sunscreens do not protect against UVA
- Important for children
  - 3x exposure to sum + skin more susceptible

UV Damages the Eye

- Acute effects
  - Keratitis and conjunctivitis
  - Thickening of the cornea
  - Disruption of corneal metabolism.
  - Retina susceptible to trauma from blue light.
  - Low doses cause photochemical changes in tissues
  - High doses also cause thermal damage.
- Chronic exposure to UV causes permanent damage to the cornea

Ultraviolet Light Synthesizes Vitamin D

- Ultraviolet light has one beneficial effect: it allows the body to synthesize vitamin D.
- Brief exposures are sufficient to do that.
  - Remember exposure hazards
- Many foods are fortified with vitamin D, which has caused occasional overdoses

Heating Tissue with Light

- Hyperthermia
  - Heating of tissue as part of cancer therapy
- Tissue ablation
  - Sufficient energy is deposited to vaporize tissue.
- Heating may be a side effect of phototherapy.
- Modeling
  - Source term for deposition of photon energy and
  - Term for flow of energy away in warmed blood

Bioheat Equation

- Linear equation for heat conduction

$$j_H = -K\frac{dT}{dx}$$

- Heat-conduction equation (same as Fick's)

$$\rho_t C_t \frac{\partial T}{\partial t} = K\nabla^2 T$$

- Perfusion P (analogous to clearance)
  - volume flow of blood per unit mass of tissue

Bioheat Equation

- Perfusion extra term

$$C_b \frac{\mathrm{J}}{\mathrm{K\ kg\ (blood)}} \times \rho_b \frac{\mathrm{kg\ (blood)}}{\mathrm{m^3\ (blood)}} \times \rho_t P \frac{\mathrm{m^3\ (blood)}}{\mathrm{m^3\ (tissue)\ s}} \times [(T - T_0)\ \mathrm{K}]$$

or

$$C_b \rho_b \rho_t P (T - T_0) \frac{\mathrm{J}}{\mathrm{m^3\ (tissue)\ s}},$$

$$\rho_t C_t \frac{\partial T}{\partial t} = K \nabla^2 T - C_b \rho_b \rho_t P (T - T_0)$$

Bioheat Equation

- Heat deposition by the beam
  - Beam energy fluence rate
  - Rate of absorption proportional to μ_a
- Final form of Bioheat Equation

$$\psi = \frac{d\Psi}{dt}.$$

$$\rho_t C_t \frac{\partial T}{\partial t} = K\nabla^2 T - C_b \rho_b \rho_t P(T - T_0) + \mu_a \psi.$$

Radiometry and Photometry

- Radiometry
  - Measurement of radiant energy
  - Independent of any detector
- Photometry
  - Measurement of ability of EM radiation to produce human visual sensation
- Actinometry
  - Measurement of photon flux or photon dose (total number of photons) independent of any subsequent photoactivated process

Radiometric Definitions

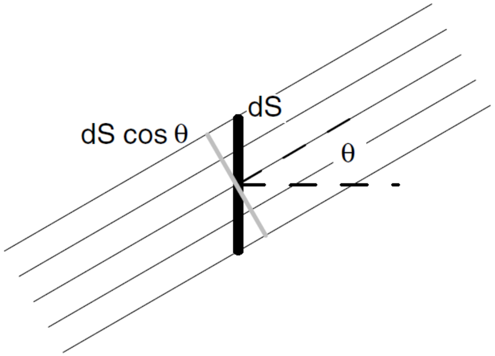
- Radiant Energy
  - The total amount of energy being considered
  - Measured in joules.
  - Emitted by a source, transferred from one region to another or received by a detector.
- Radiant Power
  - Rate at which the energy is radiated, transferred, or received
  - Measured in Watts

Radiometric Definitions

- Point Source: Radiant Intensity
  - Radiant power per unit solid angle (no $1/r^2$)

$$\frac{dP}{d\Omega} = \frac{P}{4\pi} \quad (\mathrm{W\ sr^{-1}})$$

- Extended Source: Radiance L
  - Amount of radiant power per unit solid angle per unit surface area projected perpendicular to the direction of the radiant energy



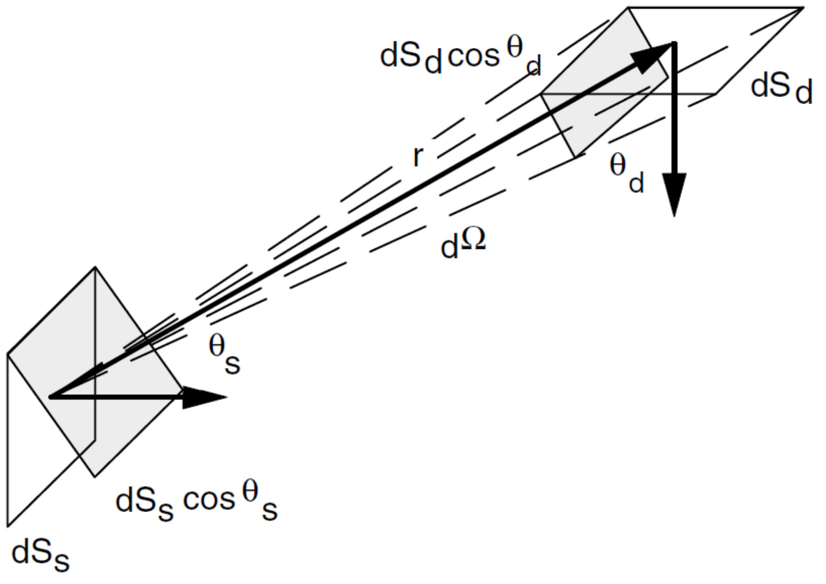
Radiometric Definitions

- Energy Striking a Surface: Irradiance E
  - Power per unit area incident on a surface
  - Point source

$$E = \frac{dP}{\cos\theta_d \, dS_d}.$$

  - Extended source

$$L dS_s d\Omega = \frac{d^2 P}{\cos\theta_s \, dS_s \, d\Omega} dS_s \, d\Omega,$$



Radiometric Definitions

- For perfectly diffuse surfaces, the radiation is isotropic ($L=L_o$)
  - Lambert's law of illumination
  - Isotropic or Lambertian surface

$$E = \frac{dS_d 2\pi L_0 \int_0^{\pi/2} \cos\theta_d \sin\theta_d d\theta_d}{dS_d} = \pi L_0 .$$

$$\psi = 4\pi L_0 = 4E \quad \text{(isotropic radiation)}.$$

Radiometric Definitions

- Spectrum
  - When the energy is not monochromatic, we define amount of energy per unit wavelength interval as R_λ, with units J m^{-1} or J nm^{-1}.
  - Total energy between wavelengths *λ_1 and λ_2 is*

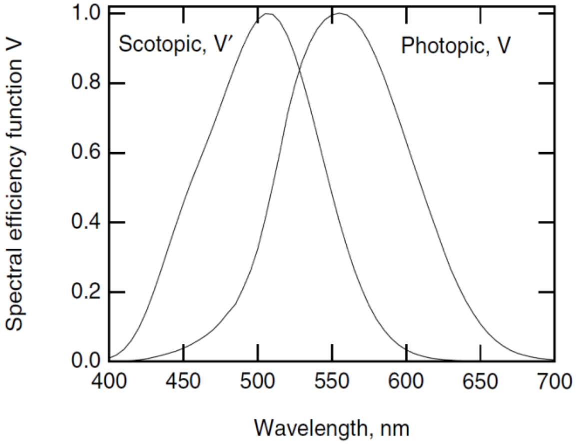
$$\int_{\lambda_1}^{\lambda_2} R_\lambda(\lambda)\, d\lambda$$

and between frequencies ν_1 and ν_2 it is,

$$\int_{\nu_1}^{\nu_2} R_\nu(\nu) d\nu.$$

Photometric Definitions

- Rods (sensitive, no color) and cones (color)
- Photopic vision
  - Normal vision at high levels of illumination in which the eye can distinguish colors.
- Scotopic vision
  - occurs at low light with dark-adapted eye.



Photometric Definitions

- Luminous flux P_v in lumens (lm)
- Peak sensitivity for photopic vision is for green light, *λ = 555 nm.*

$$P = 1\ \mathrm{W} \Longleftrightarrow P_v = 683\ \mathrm{lm},$$
$$P_v = 1\ \mathrm{lm} \Longleftrightarrow P = 1.464 \times 10^{-3}\ \mathrm{W}.$$

- Ratio P_v/P at 555 nm is Luminous efficacy for photopic vision, $K_m = 683\ \mathrm{lm\ W^{-1}}$
- For scotopic vision, $K_m \approx$ *1700 lm W^{-1}*

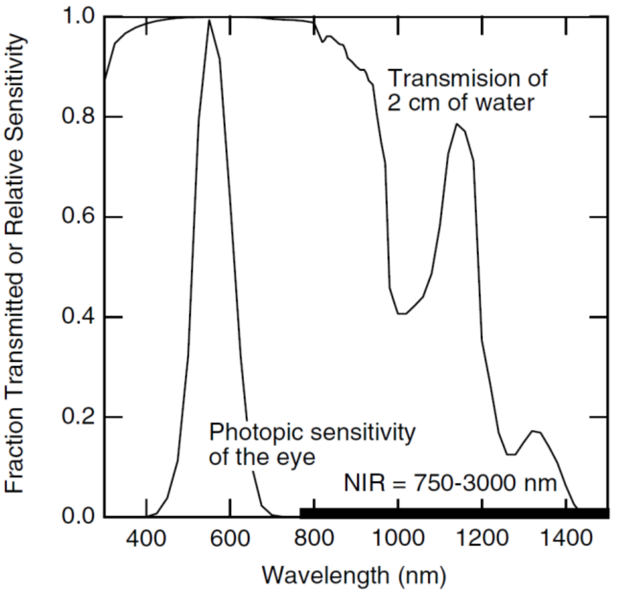
Photometric Definitions

$$P_v = K_m \int_{400\ \mathrm{nm}}^{700\ \mathrm{nm}} V(\lambda) P_\lambda(\lambda)\, d\lambda.$$

- If P were spread uniformly over the visible spectrum, overall conversion efficiency would be about 200 lm W−1
  - Incandescent lamp: 10–20 lm W^{-1}
  - Florescent lamp: 60–80 lm W^{-1}
- Number of lumens per steradian is luminous intensity, in lm sr−1 (candle)

Photometric Definitions

- Peak of the eye's spectral efficiency function is at about the peak of the sun's blackbody spectrum
  - Simple yet incorrect explanation: Evolution
  - Vertebrate eye composition
  - Insects: no water (more UV)



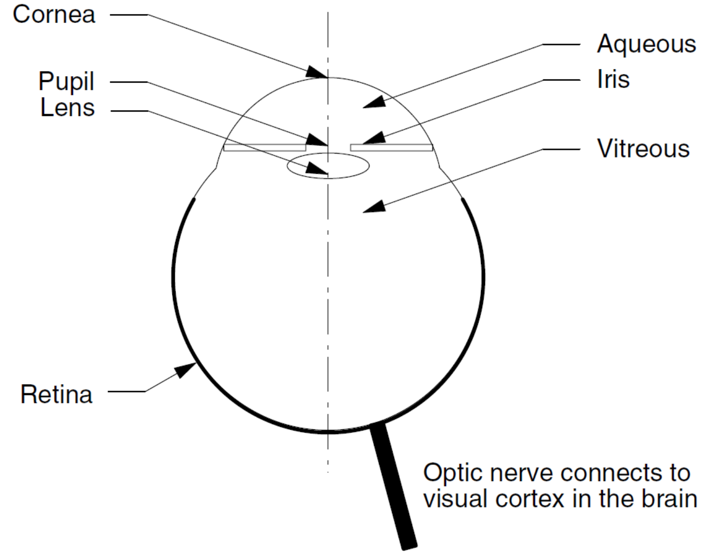
The Eye

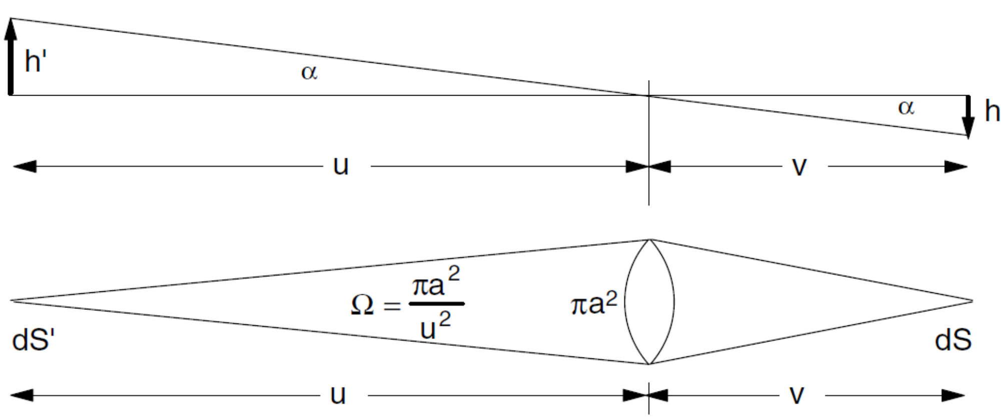
- Snell's law

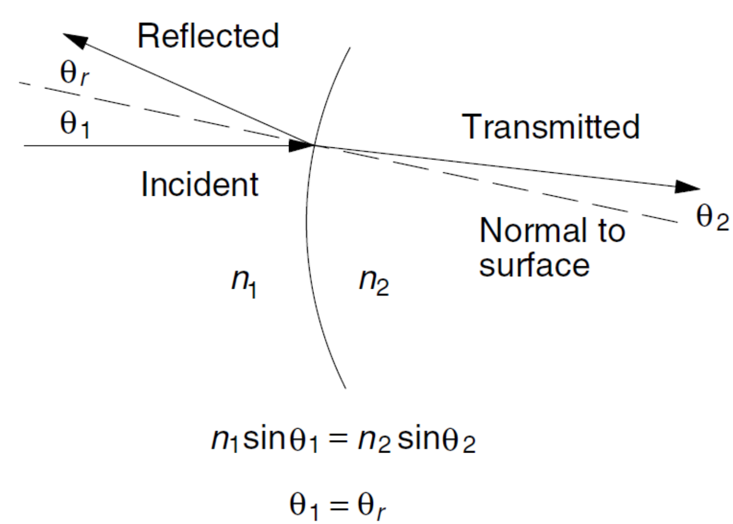
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- Thin lens equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$







The Eye

- Optometry: Vergence (diopters)

$$U = -\frac{1}{u} \quad \text{(diverging from the object)},$$

$$V = \frac{1}{v} \quad \text{(converging to the image)},$$

$$F = \frac{1}{f} \quad \text{(a converging lens)}.$$

$$V = F + U.$$

The Eye

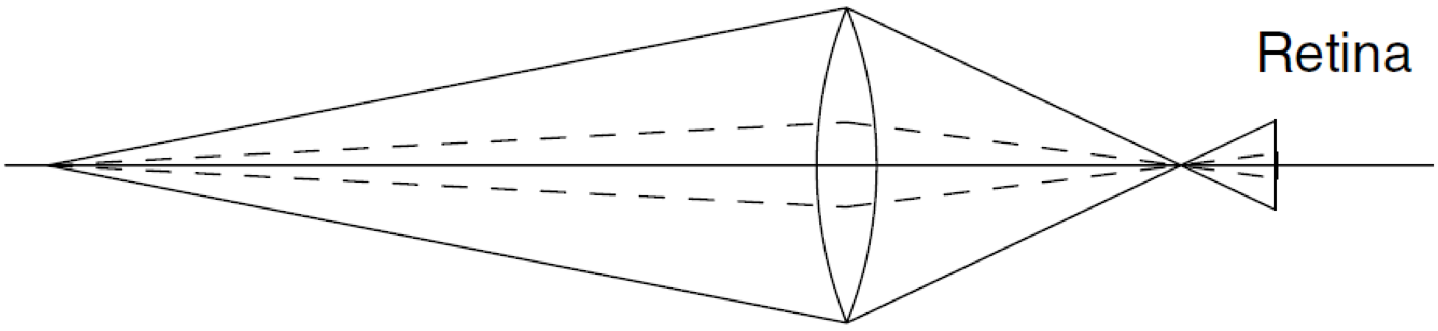
- Accomodation
  - Decreases with age: bifocals
- Emmetropic (normal) eye: V=F (when U=0)
- Nearsightedness or myopia: F>V
- Farsighted or hypermetropic: F<V
- Astigmatism
  - Eye is not symmetric about an axis through center of lens
  - Occurs at surface of cornea: corrected by lenses

The Eye

- Chromatic aberration
  - Index of refraction varies with wavelength.
  - Nearly a 2-diopter change in overall refractive power from the red to the blue.
- Spherical aberration
  - Refractive power changes with distance from the axis of the eye.
  - Different from astigmatism, which is a departure from symmetry at different angles about the axis

The Eye

- Depth of field
  - Brighter light: smaller pupil and sharper image



Problem Assignments

- Information posted on web site

Web: http://ymk.k-space.org/courses.htm