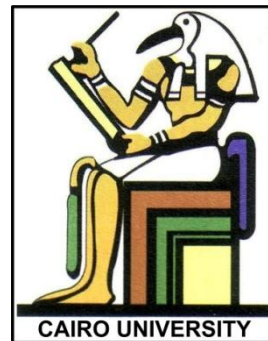


Medical Equipment I - 2010

Chapter 15

Professor Yasser M. Kadah

Web: <http://ymk.k-space.org/courses.htm>



[Photon Interactions]

- A number of different ways in which a photon can interact with an atom
- Notation: (γ, bc)
 - γ : *incident photon*
 - b and c are the results of the interaction
 - Ex1: (γ, γ) initial and final photons of same energy
 - Ex2: (γ, e) *photon absorbed and electron emerges.*

Photoelectric Effect

- Photon is absorbed by the atom and a single electron is ejected (γ , e)
- Initial photon energy $h\nu_0$ is equal to the final energy

$$h\nu_0 = T_{e1} + B.$$

- T_{e1} : Kinetic energy of electron, B : binding energy
- Photoelectric cross section is τ .

Compton and Incoherent Scattering

- Original photon disappears and photon of lower energy and electron emerge. ($\gamma, \gamma' e$)

$$h\nu_0 = h\nu + T_{el} + B.$$

- Compton cross section for scattering from a single electron is σ_C .
- Incoherent scattering is Compton scattering from all the electrons in the atom, with cross section σ_{incoh} .

[Coherent Scattering]

- Photon is elastically scattered from the entire atom.
 - Internal energy of atom does not change
 - Equal energy of incident and scattered photons

$$h\nu_0 = h\nu.$$

- Cross section for coherent scattering is σ_{coh} .

[Inelastic Scattering]

- Final photon with different energy from the initial photon (γ, γ') without emission of electron.
 - Internal energy of target atom increases or decreases by a corresponding amount.
 - Examples: fluorescence and Raman scattering
 - In fluorescence, $(\gamma, \gamma' \gamma'')$, $(\gamma, 2\gamma)$, $(\gamma, 3\gamma)$ possible

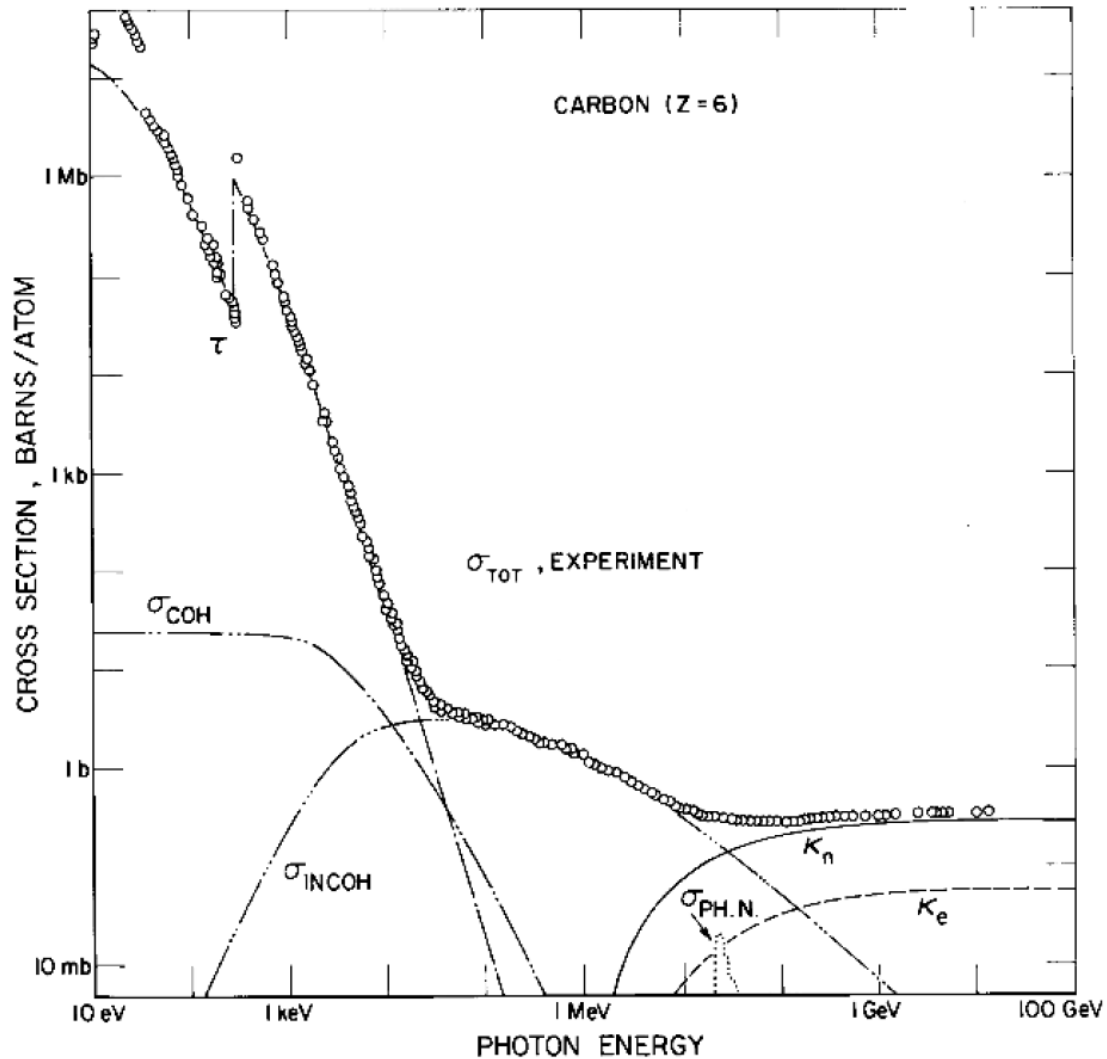
[Pair Production]

- High energy ($\gamma, e^+ e^-$) *interaction*
- Since it takes energy to create negative electron and positive electron or positron, their rest energies must be included in the energy balance

$$h\nu_0 = T_+ + m_e c^2 + T_- + m_e c^2 = T_+ + T_- + 2m_e c^2.$$

- Cross section for pair production is κ .

[Energy Dependence]



Photoelectric Effect

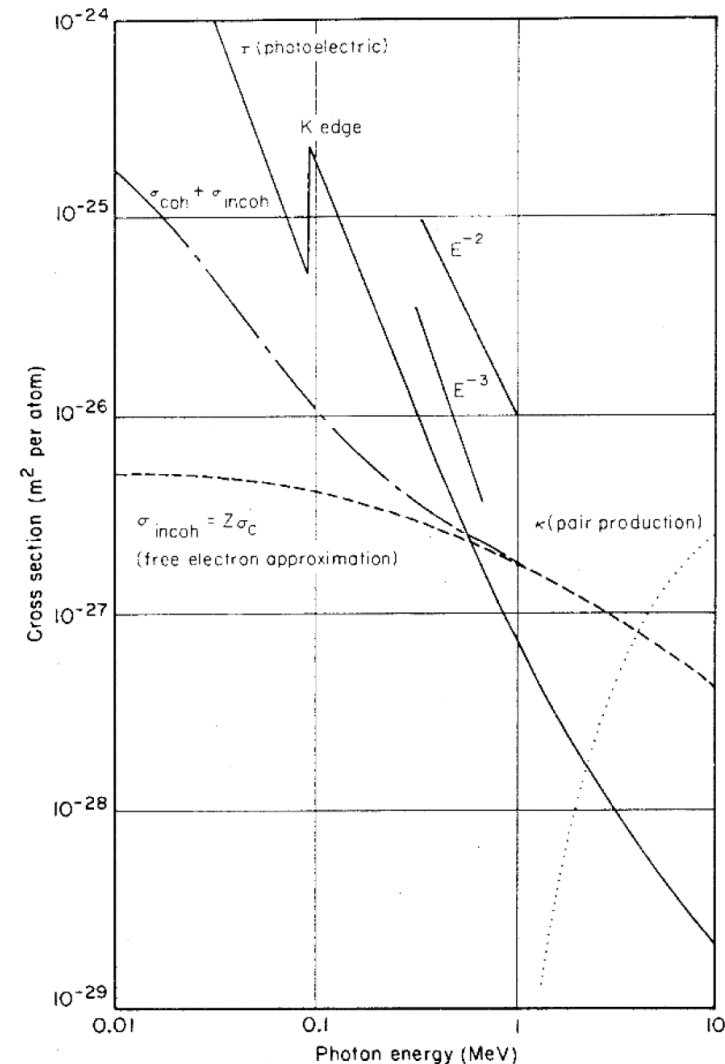
- (γ, e) Photon interaction – $h\nu_0 = T_{e1} + B$
 - T_{e1} : Kinetic energy of electron, B : binding energy
- Binding energy depends on shell
 - B_K, B_L , and so on.
- Photoelectric cross section is τ .

$$\tau = \tau_K + \tau_L + \tau_M + \dots$$

Photoelectric Effect

- For photon energies too small to remove an electron from the K shell, τ_K is zero.
 - **K edge**
 - **Can still remove L electron**
- Model around 100 KeV:

$$\tau \propto Z^4 E^{-3}.$$



Compton Scattering: Kinematics

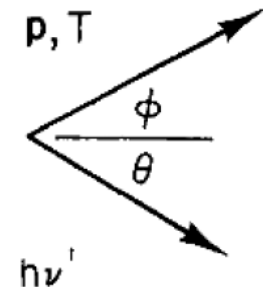
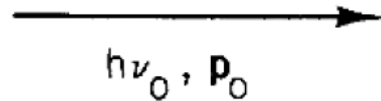
- (γ, γ', e) photon interaction $h\nu_0 = h\nu + T_{el} + B.$
- Photon kinematics: Special relativity

$$E^2 = (pc)^2 + (m_0c^2)^2.$$



$$E = h\nu = pc.$$

- Conservation of energy and momentum can be used to derive angle and energy of scattered photon



Compton Scattering: Kinematics

- Conservation of momentum in direction of the incident photon:

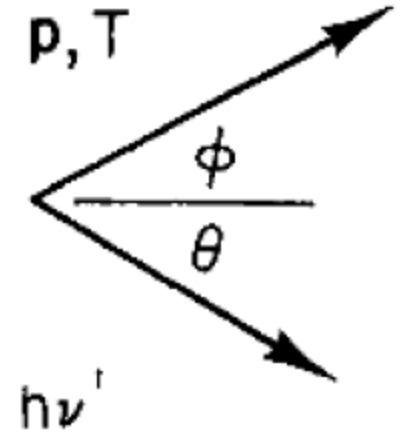
$$\frac{h\nu_0}{c} = \frac{h\nu'}{c} \cos \theta + p \cos \phi.$$

- Conservation of momentum at 90°

$$\frac{h\nu'}{c} \sin \theta = p \sin \phi.$$

- Conservation of energy

$$h\nu_0 = h\nu' + T.$$



Compton Scattering: Kinematics

- Electron energy:

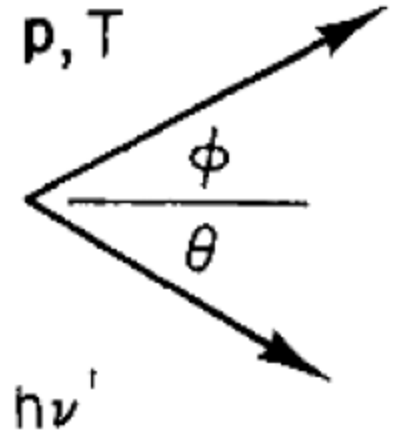
$$E = T + m_e c^2$$

- Combining with special relativity:

$$E^2 = (pc)^2 + (m_0 c^2)^2.$$

$$(pc)^2 = T^2 + 2m_e c^2 T.$$

- Solve 4 equations in 4 unknowns
 - Unknowns: T, ν', θ, ϕ



Compton Scattering: Kinematics

- Wavelength of scattered photon:

$$\lambda' - \lambda_0 = \frac{c}{\nu'} - \frac{c}{\nu_0} = \frac{h}{m_e c} (1 - \cos \theta).$$

- Difference is independent of incident wavelength
- Compton length of electron $h/m_e c$

- Energy of scattered photon

$$h\nu' = \frac{m_e c^2}{1 - \cos \theta + 1/x}$$

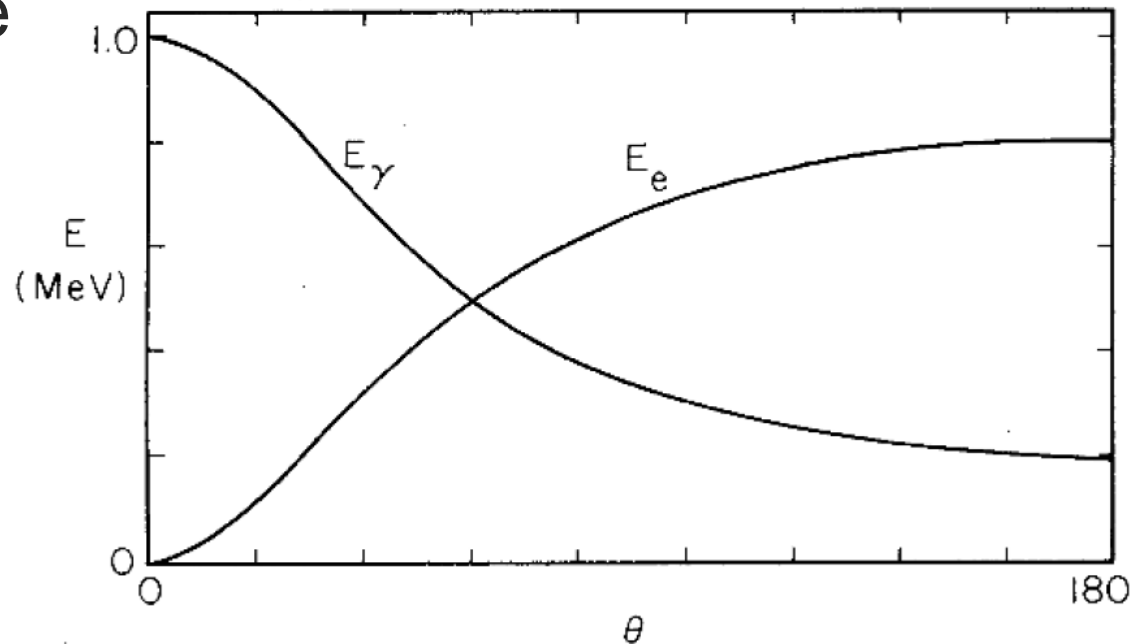
$$x = \frac{h\nu_0}{m_e c^2}.$$

Compton Scattering: Kinematics

- Energy of recoil electron

$$T = \frac{h\nu_0(2x \cos^2 \phi)}{(1+x)^2 - x^2 \cos^2 \phi} = \frac{h\nu_0 x(1 - \cos \theta)}{1 + x(1 - \cos \theta)}$$

- Dependence on angle θ



Compton Scattering: Cross Section

- Compton cross section σ_C .
- Quantum mechanics: Klein–Nishina Formula

$$\frac{d\sigma_C}{d\Omega} = \frac{r_e^2}{2} \left[\frac{1 + \cos^2 \theta + \frac{x^2(1 - \cos \theta)^2}{1 + x(1 - \cos \theta)}}{[1 + x(1 - \cos \theta)]^2} \right]$$

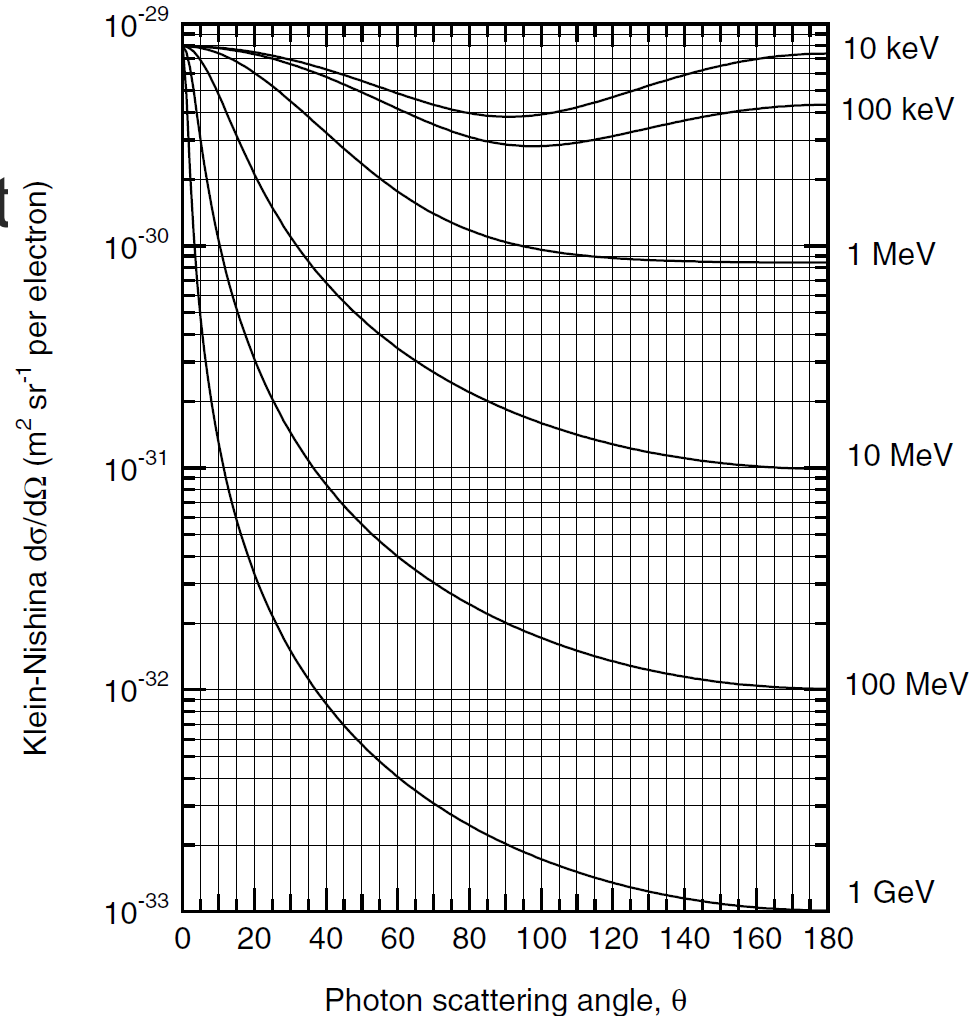
- Classical radius of electron

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} = 2.818 \times 10^{-15} \text{ m.}$$

Compton Scattering: Cross Section

- σ_C peaked in the forward direction at high energies.
- As $x \rightarrow 0$ (low energy):

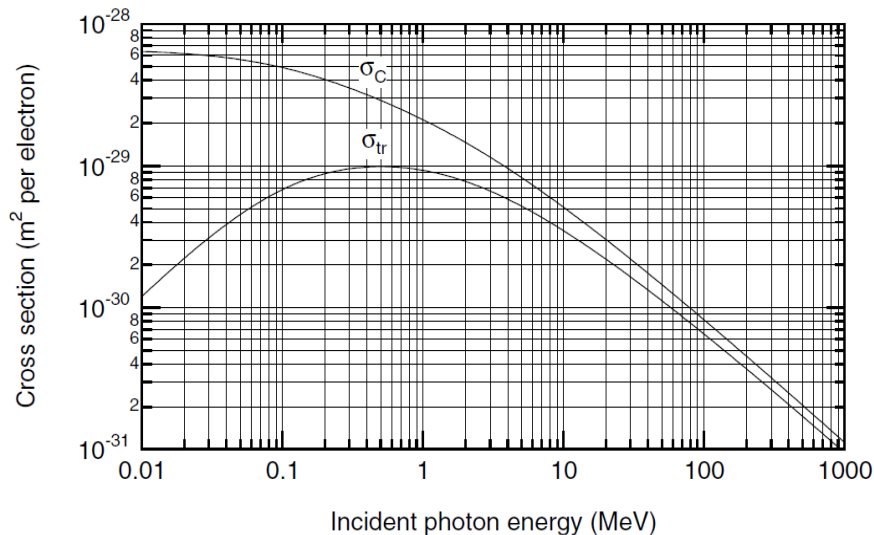
$$\frac{d\sigma_C}{d\Omega} = \frac{r_e^2(1 + \cos^2 \theta)}{2}$$



Compton Scattering: Cross Section

- Integrated over all angles

$$\sigma_C = 2\pi r_e^2 \left[\frac{1+x}{x^2} \left(\frac{2(1+x)}{1+2x} - \frac{\ln(1+2x)}{x} \right) + \frac{\ln(1+2x)}{2x} - \frac{1+3x}{(1+2x)^2} \right]. \quad (15.19)$$

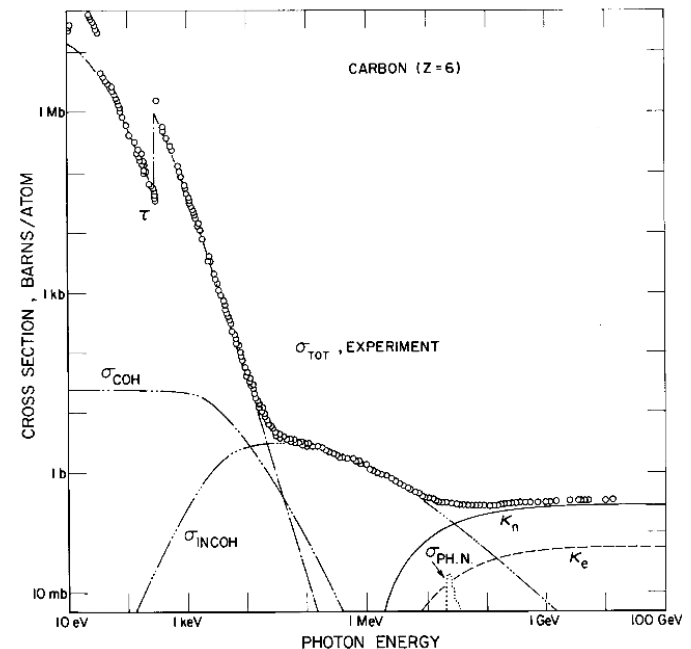


Compton Scattering: Incoherent Scattering

- σ_C is for a single electron.
- For an atom containing Z electrons, maximum value of σ_{incoh} occurs if all Z electrons take part in Compton scattering

$$\sigma_{incoh} \leq Z\sigma_C.$$

- For carbon, equality near 10 keV.

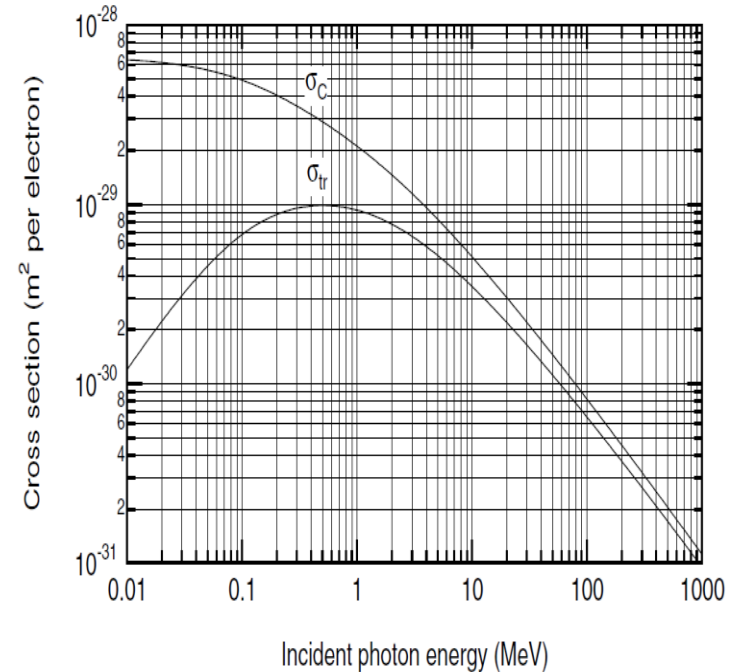


Compton Scattering: Energy Transferred to Electron

- Integrating T equation over all angles

$$\sigma_{\text{tr}} = \int_0^\pi \frac{d\sigma_C}{d\Omega} \frac{T(\theta)}{h\nu_0} 2\pi \sin\theta d\theta = f_C \sigma_C.$$

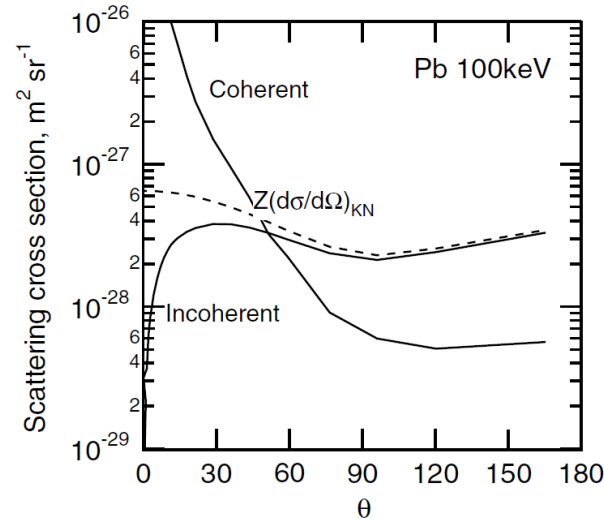
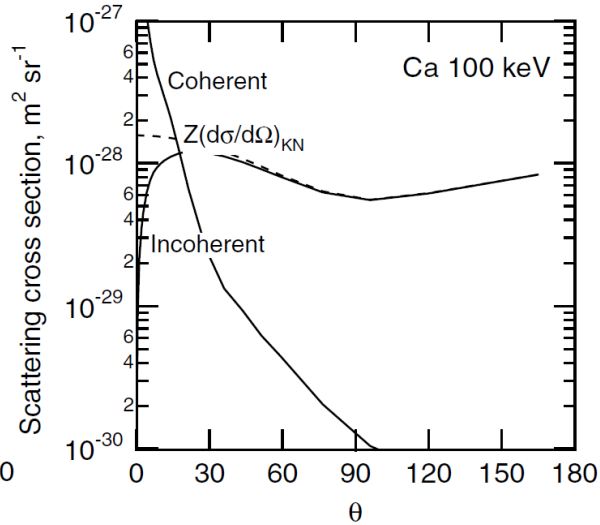
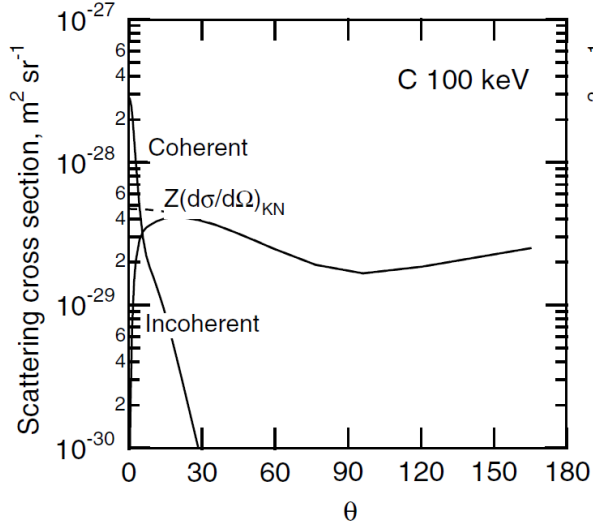
$$\sigma_{\text{tr}} = 2\pi r_e^2 \left[\frac{2(1+x)^2}{x^2(1+2x)} - \frac{1+3x}{(1+2x)^2} - \frac{(1+x)(2x^2-2x-1)}{x^2(1+2x)^2} - \frac{4x^2}{3(1+2x)^3} - \left(\frac{1+x}{x^3} - \frac{1}{2x} + \frac{1}{2x^3} \right) \ln(1+2x) \right].$$



[Coherent Scattering]

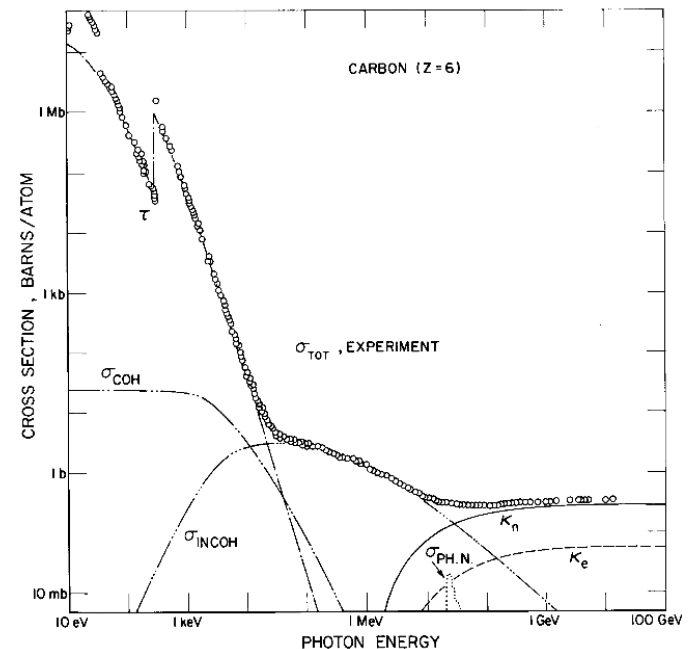
- (γ, γ) photon interaction.
- Primary mechanism is oscillation of electron cloud in the atom in response to the electric field of the incident photons.
- Cross section for coherent scattering is σ_{coh}
 - σ_{coh} peaked in the forward direction because of interference effects between EM waves scattered by various parts of the electron cloud.
 - Peak is narrower for elements of lower atomic number and for higher energies.

Coherent Scattering



Coherent Scattering

- If wavelength of incident photon \gg size of the atom, all Z electrons behave like a single particle with charge $-Ze$ and mass Zm_e .
 - Limit is almost $Z^2\sigma_c$



Pair Production

- High energy ($\gamma, e^+ e^-$) interaction

$$h\nu_0 = T_+ + m_e c^2 + T_- + m_e c^2 = T_+ + T_- + 2m_e c^2.$$

- One can show that momentum is not conserved by the positron and electron if the former equation is satisfied.
 - Interaction takes place in the Coulomb field of another particle (usually a nucleus) that recoils to conserve momentum.
 - Cross section for pair production involving nucleus is κ_n .

[Pair Production]

- Pair production with excitation or ionization of the recoil atom can take place at energies that are only slightly higher than the threshold
 - Cross section does not become appreciable until the incident photon energy exceeds *2.04 MeV*
 - A free electron (rather than a nucleus) recoils to conserve momentum.
 - $(\gamma, e^+ e^- e^-)$ process : Triplet production.
- Total cross section: $\kappa = \kappa_n + \kappa_e$

Linear Attenuation Coefficient

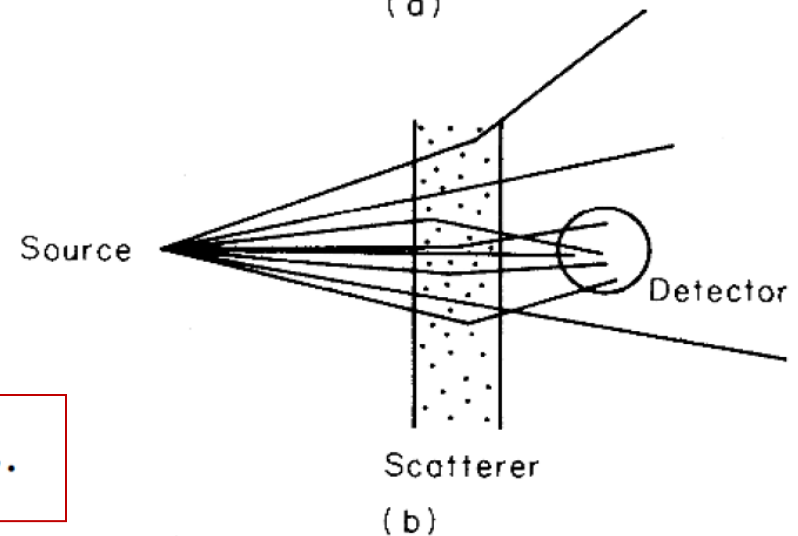
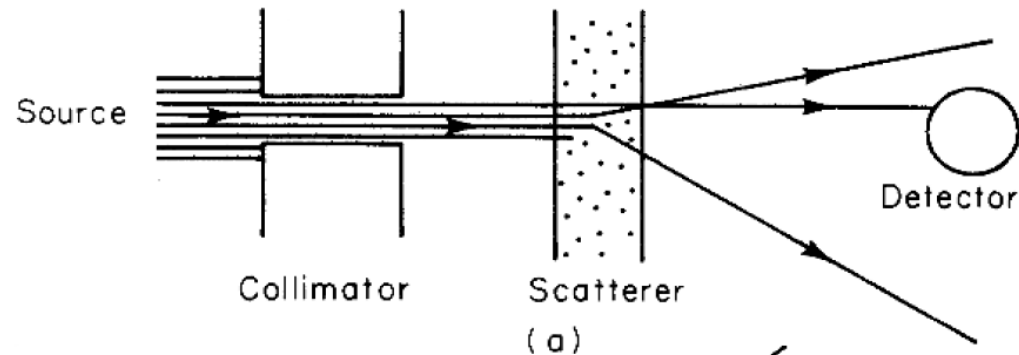
- Narrow- vs. Broad-beam geometries
 - Idealization ?

$$dN = -\frac{\sigma_{\text{tot}} N_A \rho}{A} N dz,$$

$$N(z) = N_0 e^{-\mu_{\text{atten}} z}.$$

$$\mu_{\text{atten}} = \frac{N_A \rho \sigma_{\text{tot}}}{A}.$$

$$\sigma_{\text{tot}} = \sigma_{\text{coh}} + \sigma_{\text{incoh}} + \tau + \kappa.$$



Mass Attenuation Coefficient

- Mass attenuation coefficient
 - Independent of density: very useful in gases

$$\frac{\mu_{\text{atten}}}{\rho} = \frac{N_A \sigma_{\text{tot}}}{A}$$

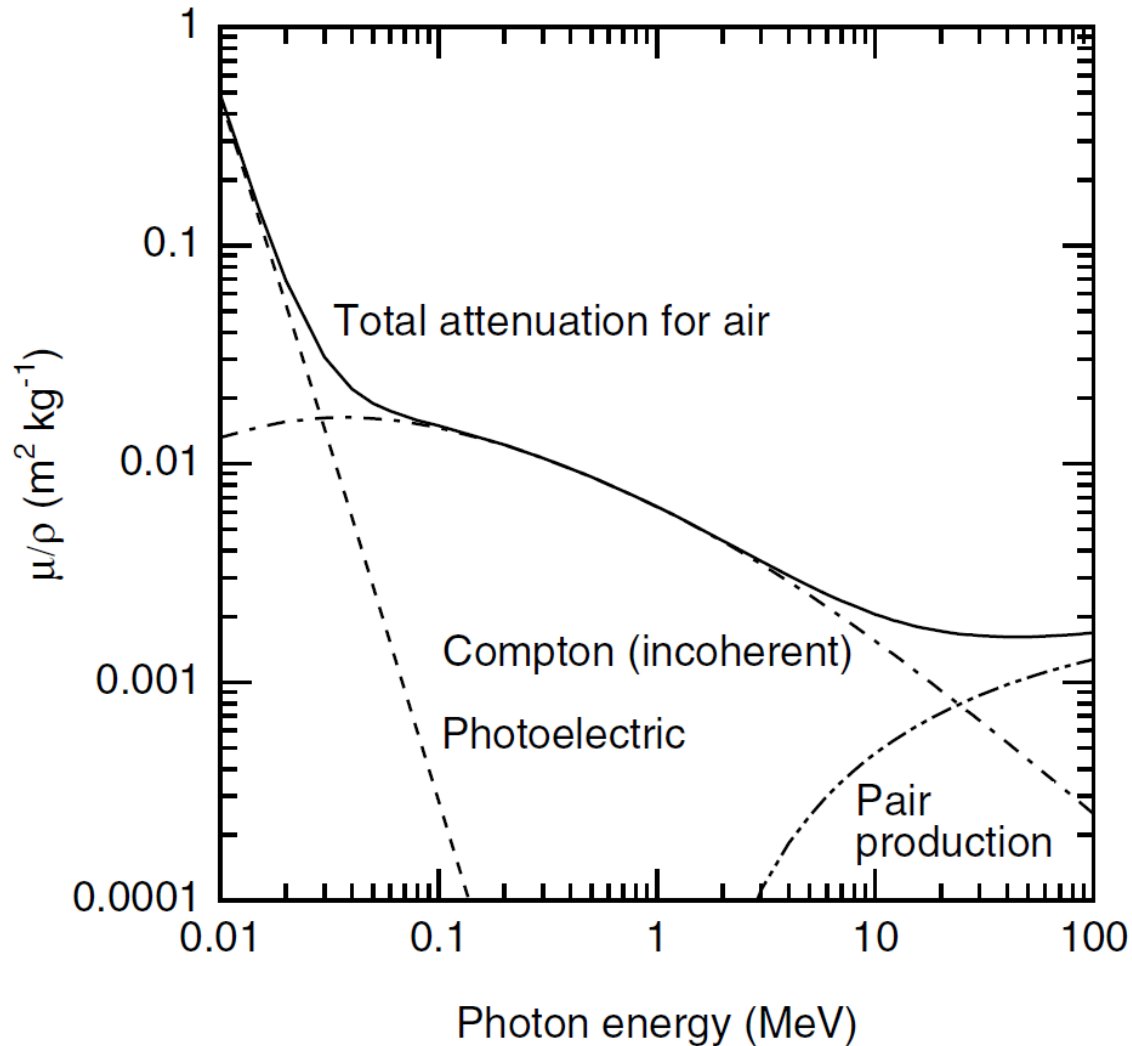


$$N(\rho z) = N_0 e^{-(\mu_{\text{atten}}/\rho)(\rho z)}$$

- Additional advantage in incoherent scattering:
Z/A is nearly 1/2 for all elements except H¹: minor variations over periodic table

$$\frac{\mu_{\text{atten}}}{\rho} = \frac{Z \sigma_C N_A}{A}$$

Mass Attenuation Coefficient



Compounds and Mixtures

- Usual procedure for dealing with mixtures and compounds is to assume that each atom scatters independently.

$$w_i = \frac{a_i A_i}{A_{\text{mol}}}$$

$$\frac{\bar{n}}{N} = \sum_i \sigma_i (N_T)_i = \left(\sum_i \sigma_i (N_{TV})_i \right) dz,$$

$$(N_{TV})_i = \frac{M_i N_A}{A_i V} = \frac{w_i}{A_i} \rho N_A.$$

$$\begin{aligned} \sum_i \sigma_i (N_{TV})_i &= \left(\sum_i \frac{a_i \sigma_i}{A_{\text{mol}}} \right) \rho N_A \\ &= \left(\sum_i a_i \sigma_i \right) \frac{\rho N_A}{A_{\text{mol}}} = \sigma_{\text{mol}} (N_{TV})_{\text{mol}}. \end{aligned}$$

[Compounds and Mixtures]

- When a target entity (molecule) consists of a collection of subentities (atoms), we can say that in this approximation (all subentities interacting independently), the cross section per entity is the sum of the cross sections for each subentity.
 - For example, for CH₄, total molecular cross section is $\sigma_{\text{carbon}} + 4\sigma_{\text{hydrogen}}$ and the molecular weight is $[(4 \times 1) + 12] \times 10^{-3} \text{ kg mol}^{-1}$

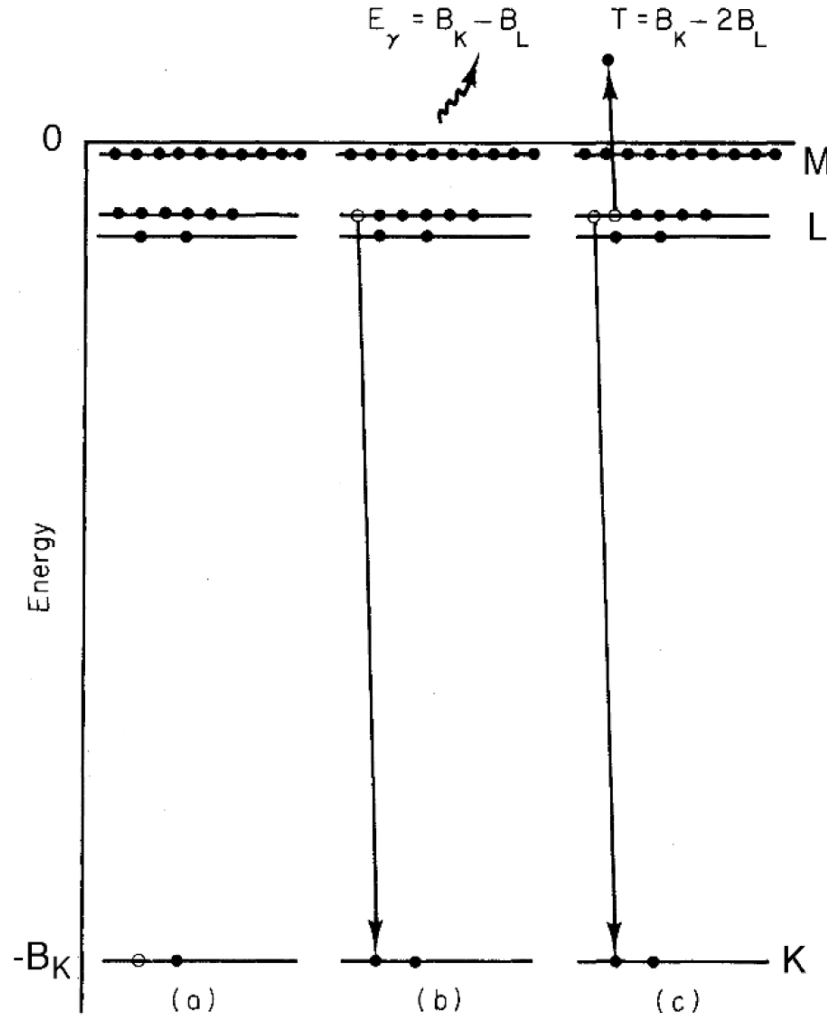
Deexcitation of Atoms

- Excited atom is left with a hole in some electron shell.
 - Similar state when an electron is knocked out by a passing charged particle or by certain transformations in the atomic nucleus
- Two competing processes:
 - Radiative transition: photon is emitted as an electron falls into the hole from a higher level,
 - Nonradiative or radiationless transition: emission of an Auger electron

Deexcitation of Atoms

Process	Total photon energy	Total electron energy	Atom excitation energy	Sum
Before photon strikes atom	$h\nu$	0	0	$h\nu$
After photoelectron is ejected [Fig. 15.12(a)]	0	$h\nu - B_K$	B_K	$h\nu$
Case 1: Deexcitation by the emission of a K and an L photon				
Emission of K fluorescence photon [Fig. 15.12(b)]	$B_K - B_L$	$h\nu - B_K$	B_L	$h\nu$
Emission of L fluorescence photon	$B_K - B_L, B_L$	$h\nu - B_K$	0	$h\nu$
Case 2: Deexcitation by emission of an Auger electron from the L shell				
Emission of Auger electron [Fig. 15.12(c)]	0	$h\nu - B_K, B_K - 2B_L$	$2B_L$	$h\nu$
First L -shell hole filled by fluorescence	B_L	$h\nu - B_K, B_K - 2B_L$	B_L	$h\nu$
Second L -shell hole filled by fluorescence	B_L, B_L	$h\nu - B_K, B_K - 2B_L$	0	$h\nu$

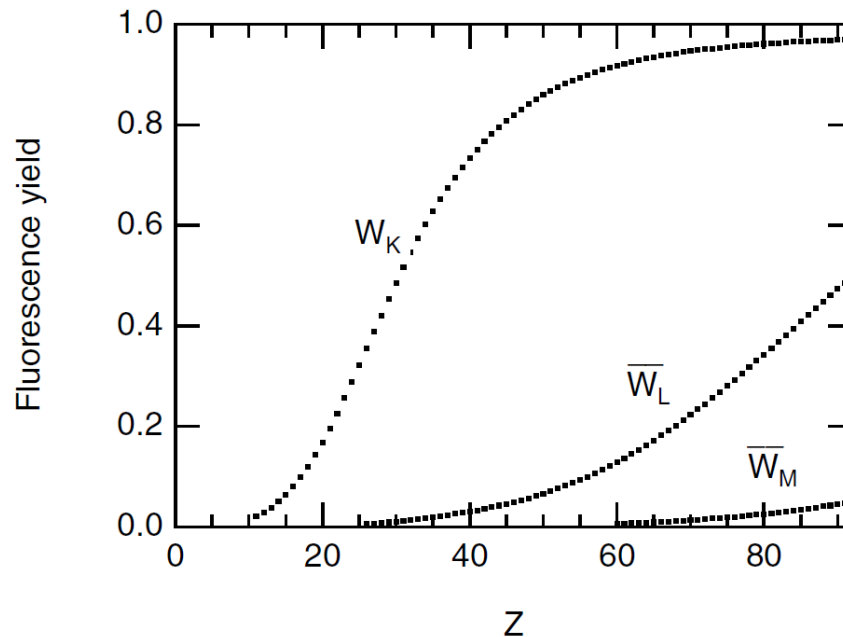
Deexcitation of Atoms



$$\Delta l = \pm 1, \quad \Delta j = 0, \pm 1.$$

Deexcitation of Atoms

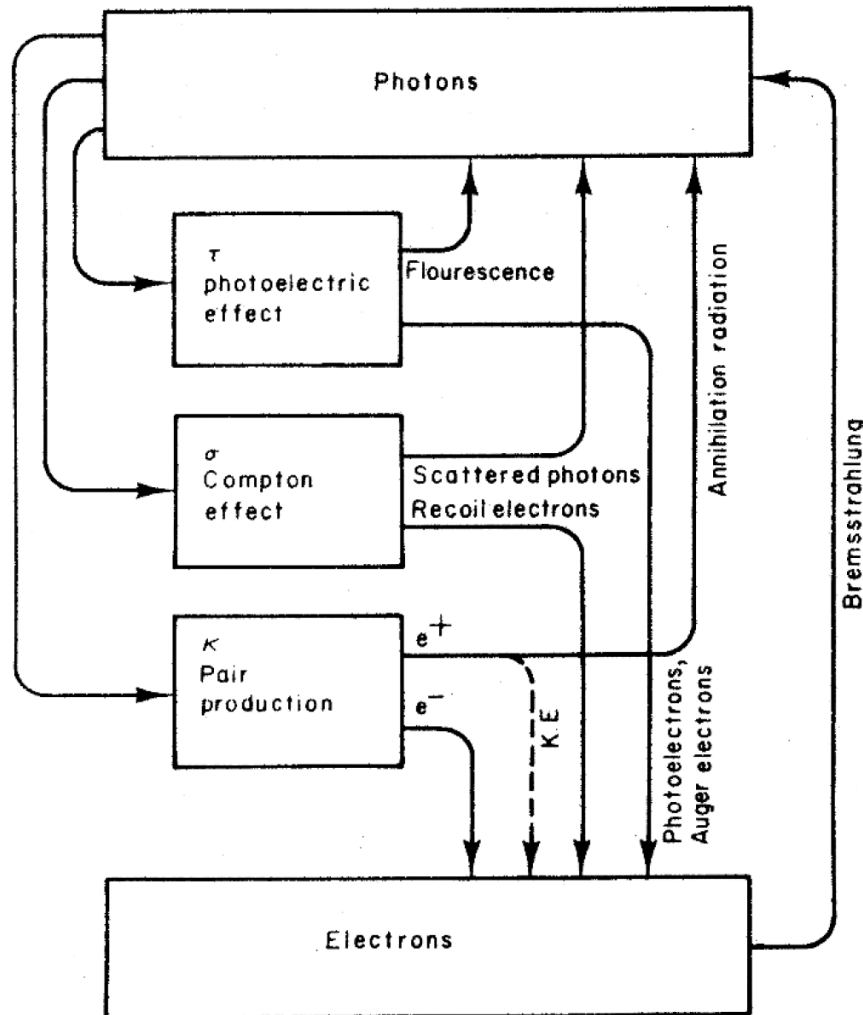
- Probability of photon emission is called the **fluorescence yield**, W_K .
 - *Auger yield is $A_K = 1 - W_K$.*
 - *L or higher shells: consider yield for each subshell*



Deexcitation of Atoms

- Coster–Kronig transitions
 - Radiationless transitions within the subshell
 - Hole in L_I -shell can be filled by an electron from the L_{III} -shell with the ejection of an M-shell electron
- Super-Coster–Kronig transitions
 - Involves electrons all within same shell (e.g., all M)
- Auger cascade
 - Bond breaking – important for radioactive isotopes

Energy Transfer from Photons to Electrons

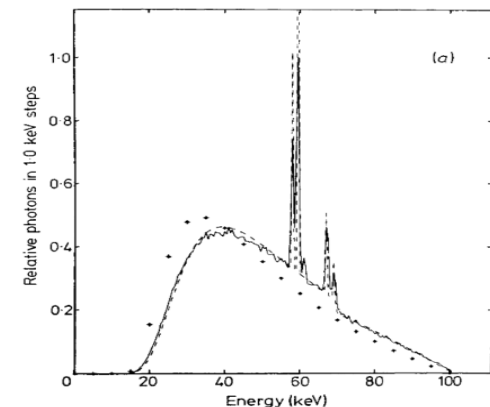
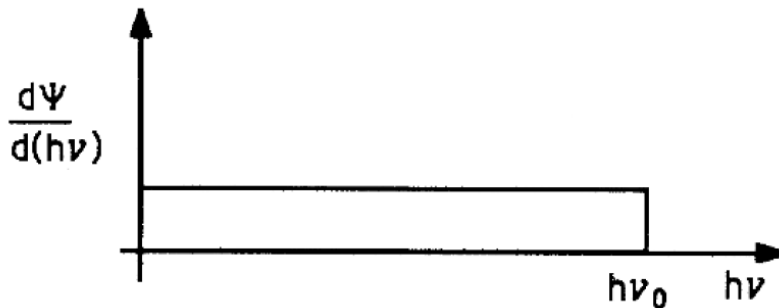


[Bremsstrahlung]

- Classically, a charged particle at rest creates an electric field which is inversely proportional to squared distance from charge.
- When in motion with a constant velocity it creates both electric and magnetic fields.
- When accelerated, additional electric and magnetic fields appear
 - fall off less rapidly—inversely with the first power of distance from charge with continuous distribution.

Bremsstrahlung

- Quantum-mechanically, when a charged particle undergoes acceleration or deceleration, it emits photons.
- Radiation is called deceleration radiation, braking radiation, or *bremsstrahlung*.
 - It has a continuous distribution of frequencies up to some maximum value.



[Problem Assignments]

- Information posted on web site

Web: <http://ymk.k-space.org/courses.htm>