# Ultrasound Bioinstrumentation

Topic 4: Lectures 6,7,8

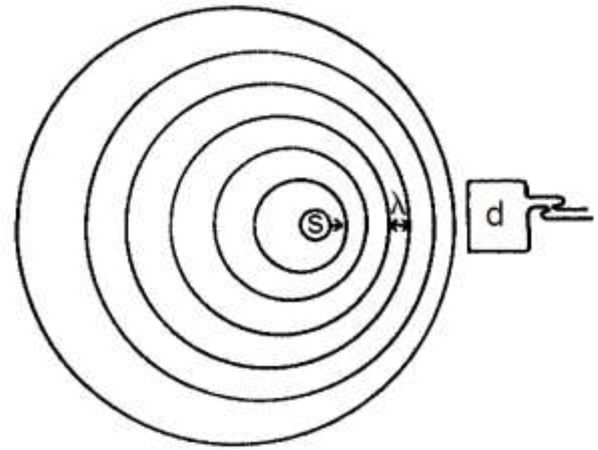
Doppler Ultrasound

CAIRO UNIVERSITY

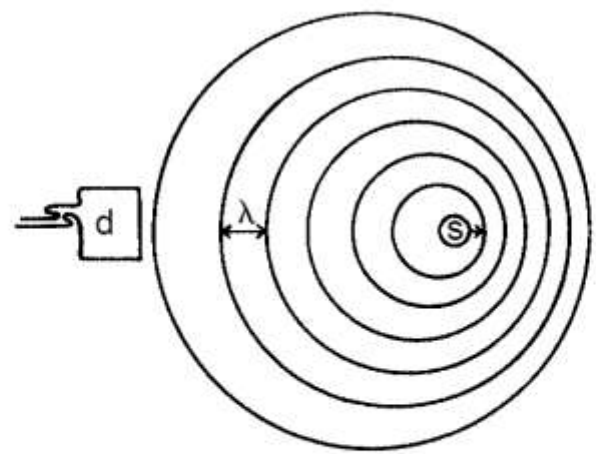
# The Doppler Effect

- Christian Doppler described the change in wave frequency because of motion of source/receiver

$$\Delta\nu = 2\nu_0\left(\frac{v}{c}\right)\cos\theta$$



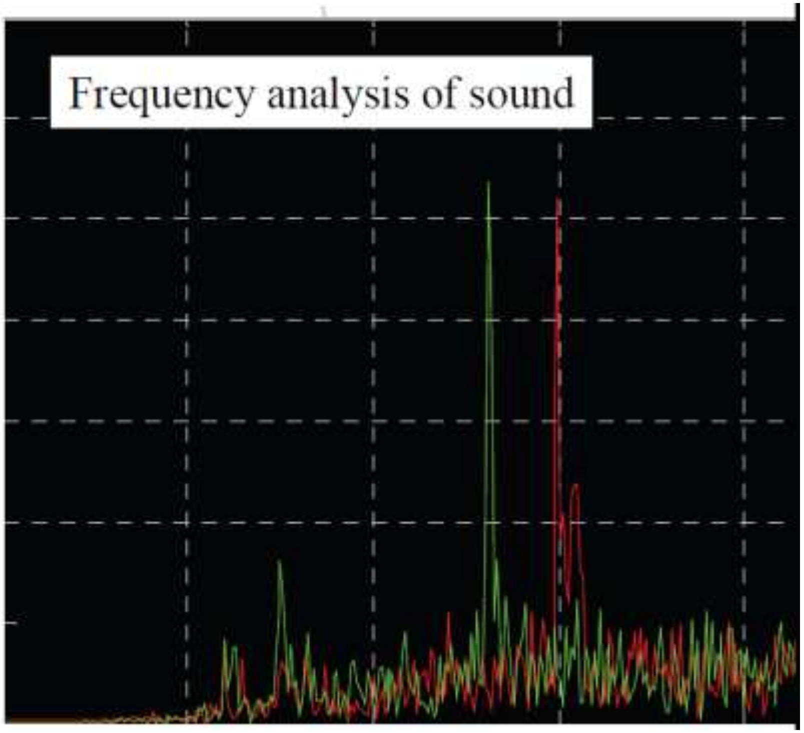

(a)




(b)

# Doppler Application Example

- Speed control on highways



# Exact Doppler Model Derivation

- Fixed target (v=0)
- Transmitted signal is sinusoidal

$$s_t(t) = \sqrt{2P_t}\cos\omega_c t = \sqrt{2}\,\mathrm{Re}\,[\sqrt{P_t}\,e^{j\omega_c t}], \qquad -\infty < t < \infty.$$

$$s_r(t) = \sqrt{2}\,\mathrm{Re}\left\{\sqrt{P_t}\sum_{i=1}^{K} g_i \exp\,[j\omega_c(t-\tau)+\theta_i]\right\}.$$

$$\tau \triangleq \frac{2R}{c}.$$

# Exact Doppler Model Derivation

- Central limit theorem

$$s_r(t) = \sqrt{2}\,\mathrm{Re}\,\{\sqrt{P_t}\,\tilde{b}\exp(j\omega_c(t-\tau)]\},$$

$$E\{|\tilde{b}|\} = \sqrt{\frac{\pi}{2}}\,\sigma_b$$

$$E\{|\tilde{b}|^2\} = 2\sigma_b{}^2.$$

# Exact Doppler Model Derivation

- Assume reflection process is linear

$$s_t(t) = \sqrt{2}\,\mathrm{Re}\,[\sqrt{E_t}\,\tilde{f}(t)e^{j\omega_c t}]$$

$$s_r(t) = \sqrt{2}\,\mathrm{Re}\,[\sqrt{E_t}\,\tilde{b}\tilde{f}(t-\tau)e^{j\omega_c t}].$$

# Exact Doppler Model Derivation

- Slowly moving point target

$$R(t) = R_0 - vt.$$

$$s_t(t) = \sqrt{2}\,\mathrm{Re}\,[\sqrt{E_t}\,\tilde{f}(t)e^{j\omega_c t}]$$

$$s_r(t) = \sqrt{2}\,\mathrm{Re}\,[\sqrt{E_t}\,\tilde{b}\tilde{f}(t - \tau(t))\exp\,[\,j\omega_c(t - \tau(t))]],$$

# Exact Doppler Model Derivation

$$R\left(t - \frac{\tau(t)}{2}\right) = R_0 - v\left(t - \frac{\tau(t)}{2}\right).$$

$$\tau(t) = \frac{2R(t - \tau(t)/2)}{c}.$$

$$\tau(t) = \frac{2R_0/c}{1 + v/c} - \frac{(2v/c)t}{1 + v/c}.$$

# Exact Doppler Model Derivation

- For velocities of interest $\frac{v}{c} \ll 1.$

Then,

$$\tau(t) \simeq \frac{2R_0}{c} - \frac{2v}{c}t \triangleq \tau - \frac{2v}{c}t.$$

Hence,

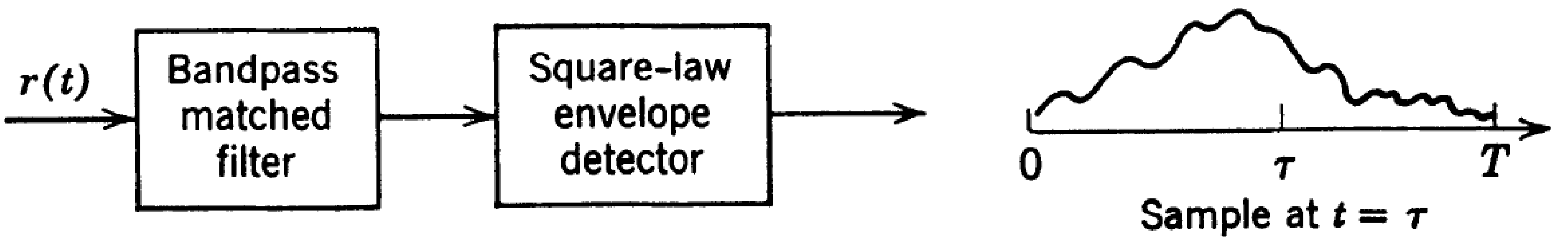
$$s_r(t) = \sqrt{2}\,\mathrm{Re}\left[\sqrt{E_t}\,\tilde{b}\tilde{f}\left(t - \tau + \frac{2v}{c}t\right)\exp\left[j\omega_c\left(t + \frac{2v}{c}t\right)\right]\right].$$

# Exact Doppler Model Derivation

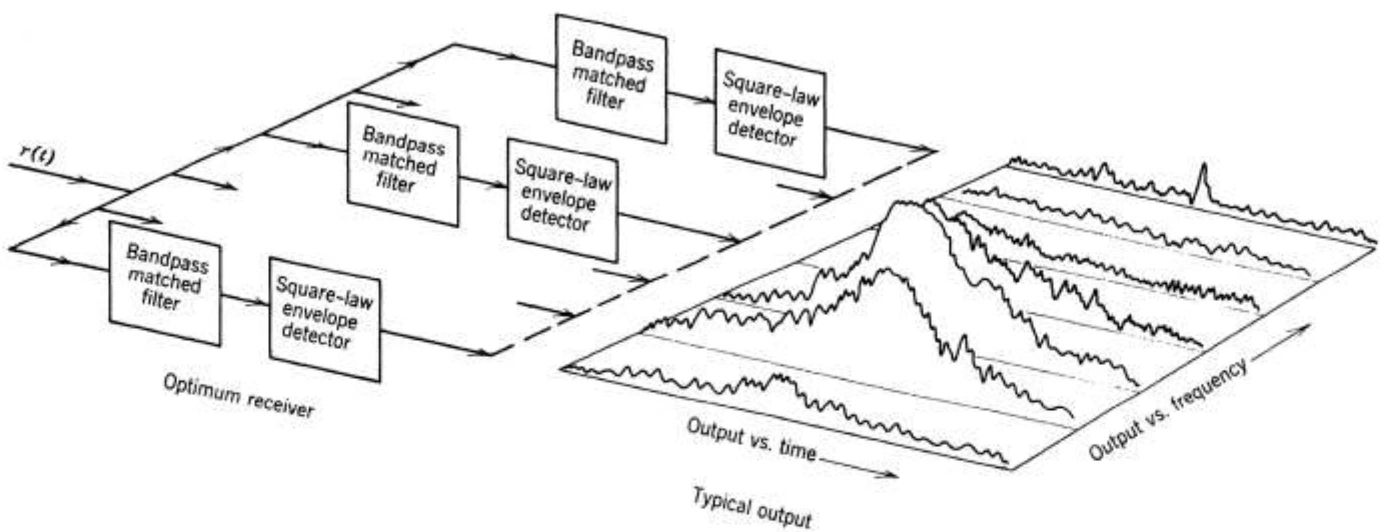
- Target velocity has two effects:
  - Compression or stretching of the complex envelope
  - Shift in carrier frequency
- In the famous Doppler shift equation, we ignore the first effect
  - Describes modes close to CW Doppler
  - May lead to errors when we deviate from ideal sinusoidal excitation
  - Particularly important source of error for PW Doppler

# Optimum Doppler Receiver

- Known Doppler Shift

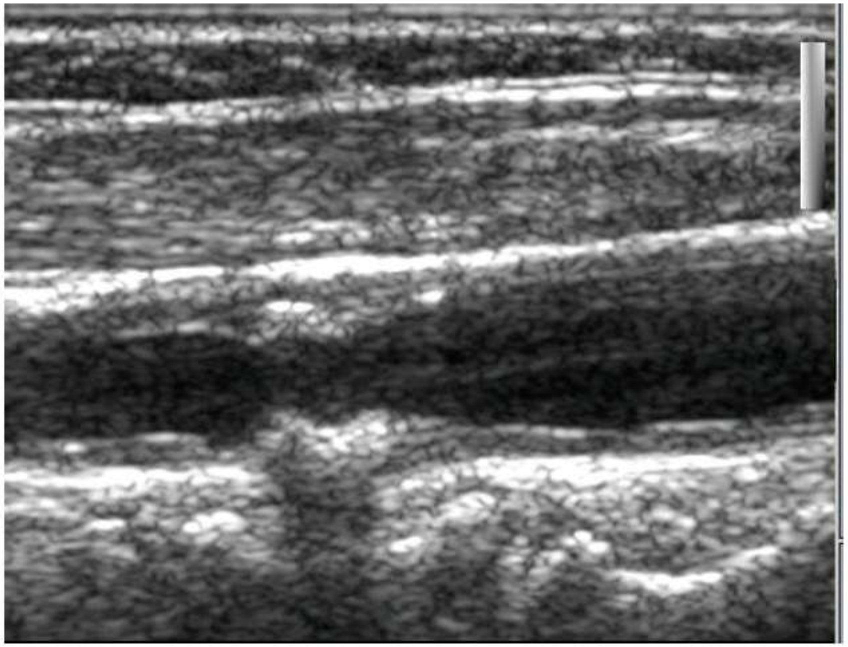



- Unknown Doppler Shift

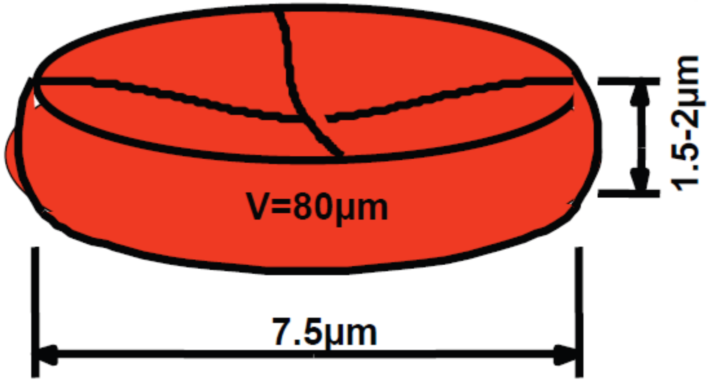


# RBCs in Ultrasound Imaging

- Hardly visible in ultrasound images
  - Scattering because of its very small size

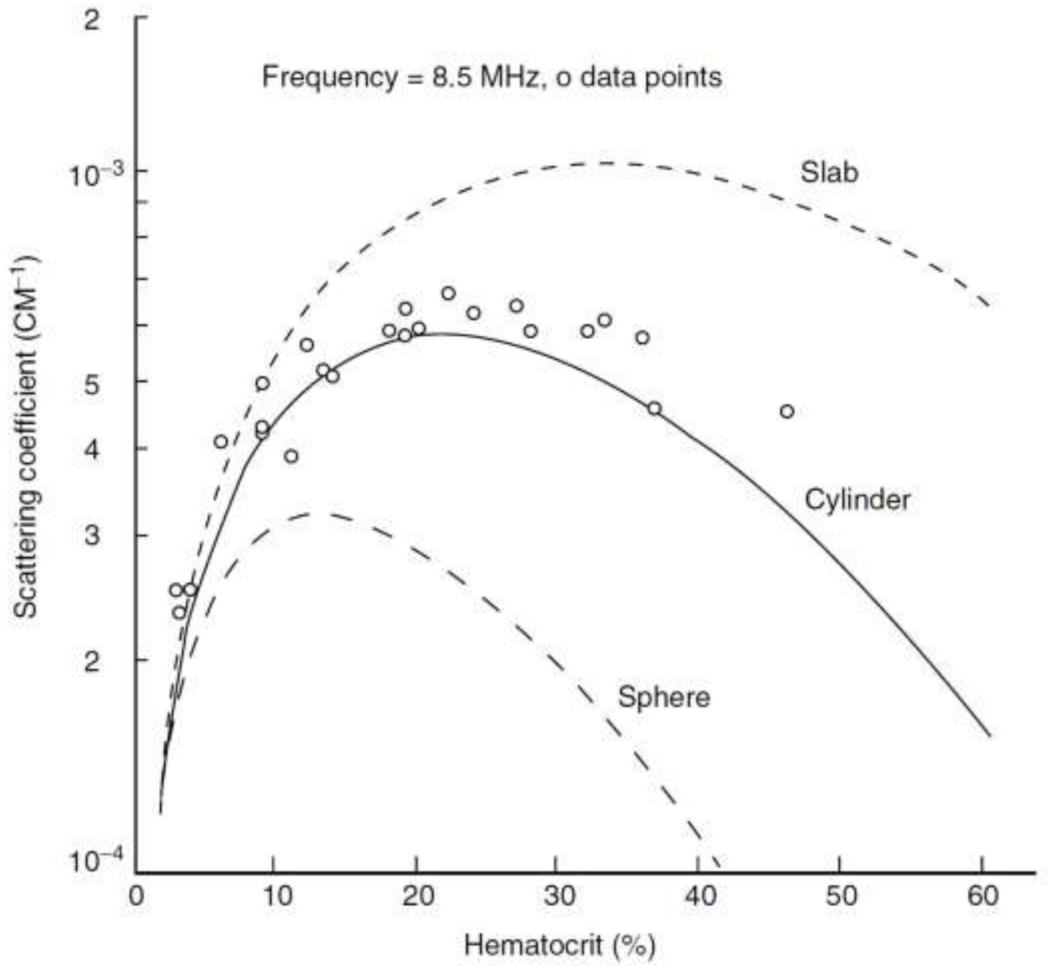


Carotid artery with calcified plaque

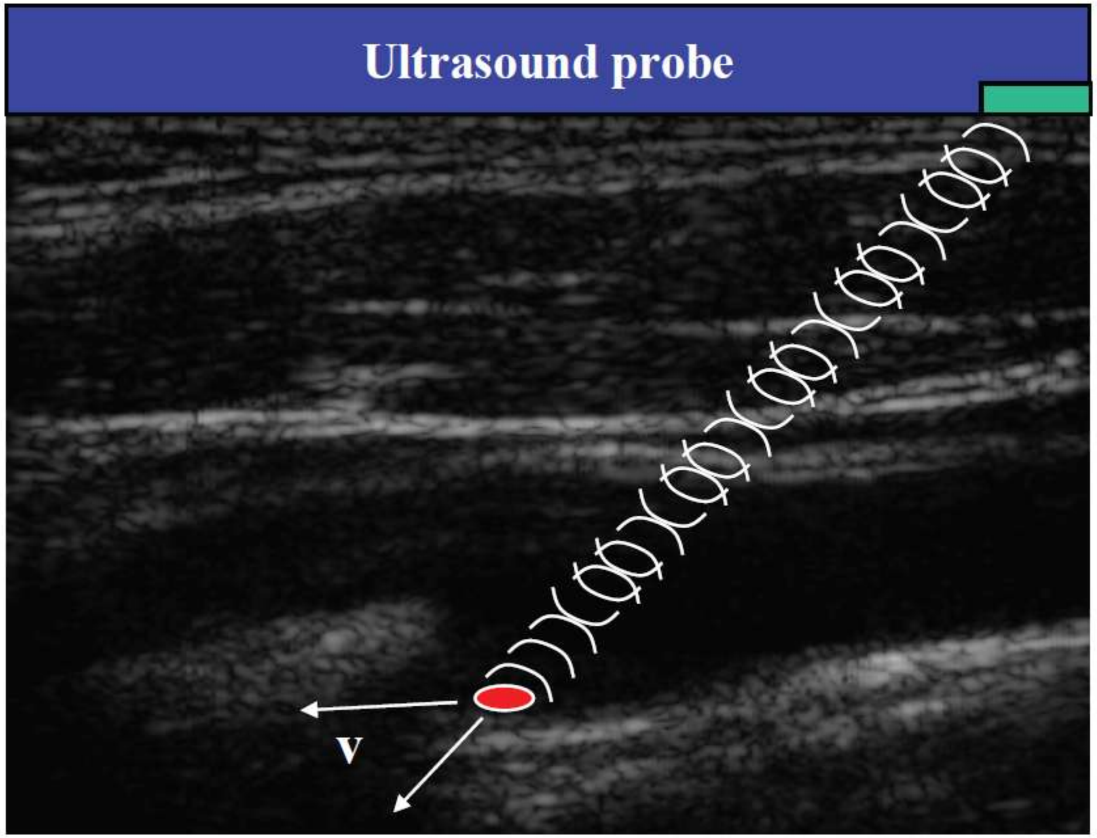



Red blood cell

# Scattering from Blood



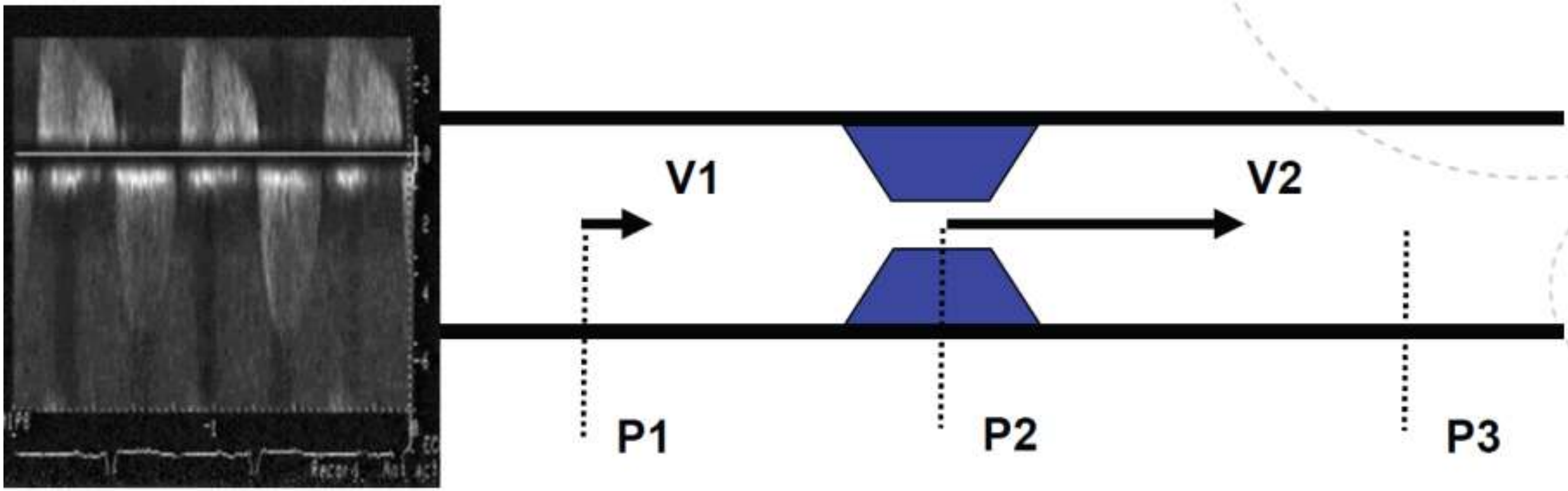
# Doppler Shift from Moving Blood




$$\Delta \nu = 2\nu_0 \left(\frac{v}{c}\right) \cos\theta$$

# Estimation of Pressure in Vessels

Bernouli's equation describes pressure drop due to increase in kinetic energy



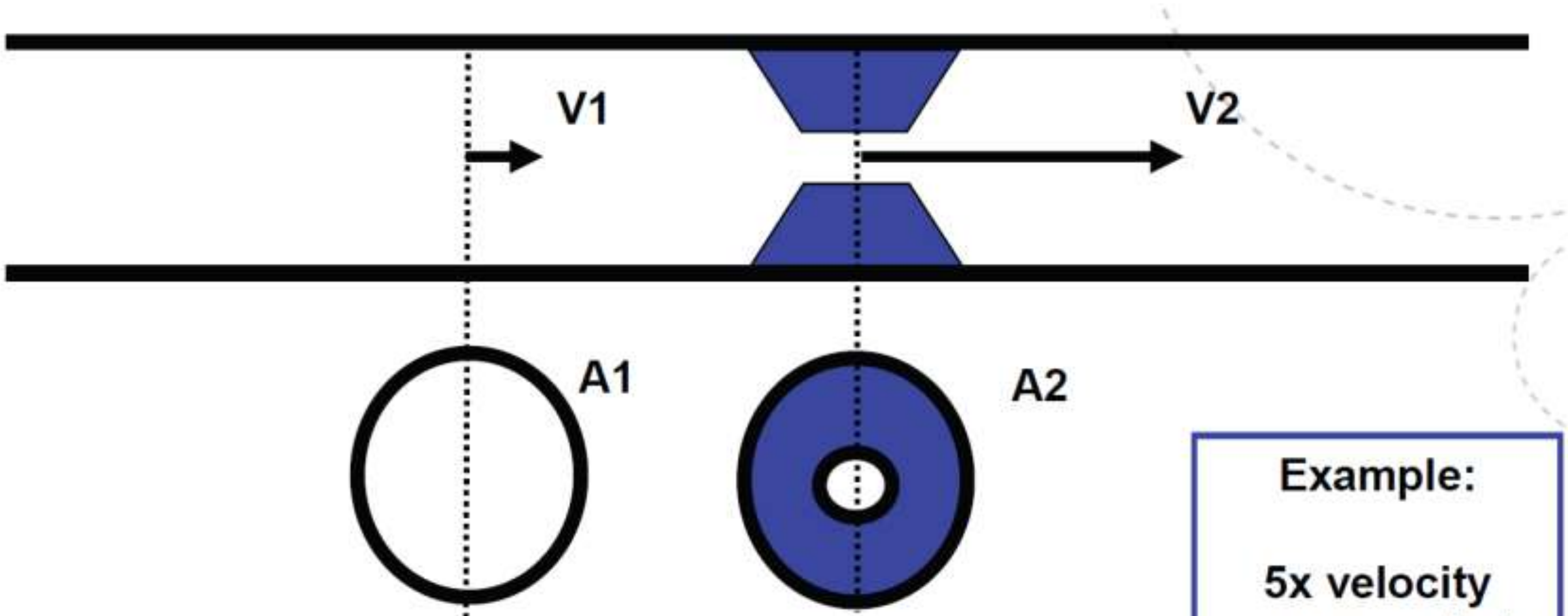

**Pressure drop (gradient) : P1 - P3 = 4 V2 ²**

**Example**: 80% aortic-stenosis

V1= 1 m/s V2 = 5 m/s pressure-gradient 4*5*5 = 100 mmHg

Normal aortic pressure 120 mmHg corresponds to 220 mmHg ventricular pressure!

# Continuity Equation to Assess Flow




$V1 * A1 = V2 * A2$

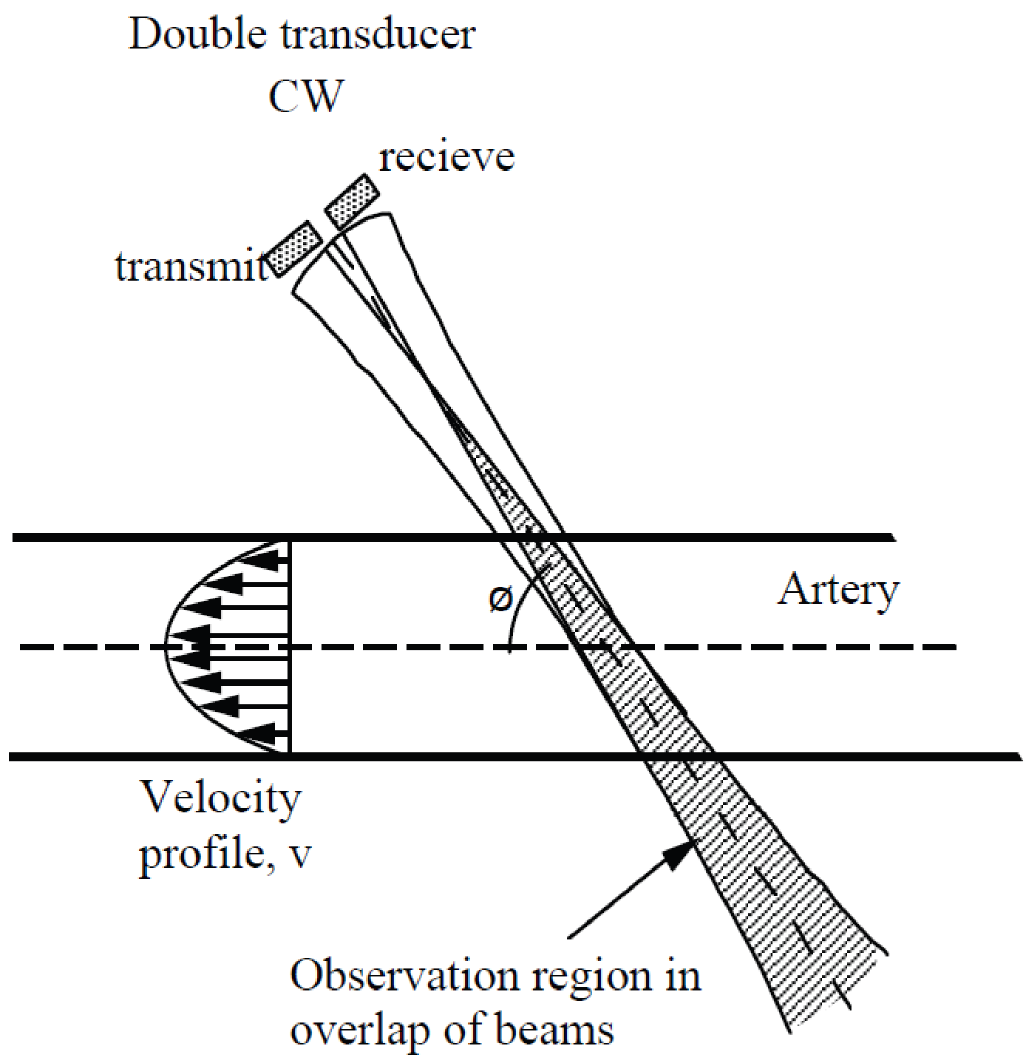
% reduction $\dfrac{A1 - A2}{A1} = \dfrac{V2 - V1}{V2}$

**Example:**

**5x velocity corresponds to 80% stenose**

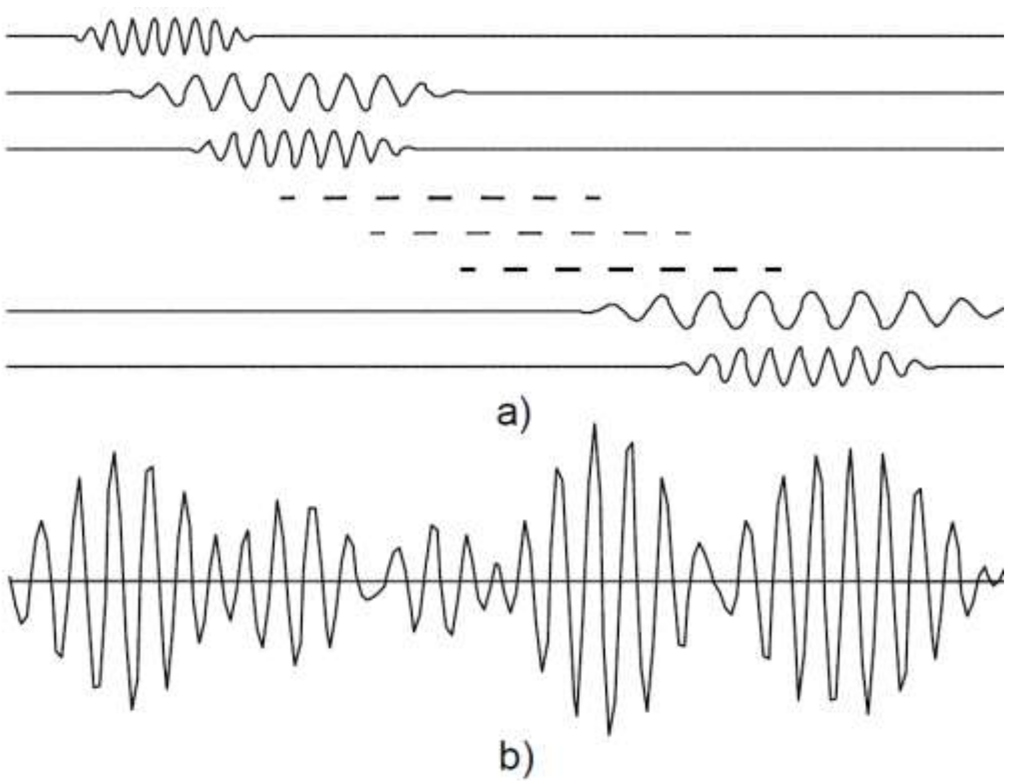
*Area reduction depends only on the velocity V2/V1; independent of diameter and angle*

# Continuous Wave Doppler

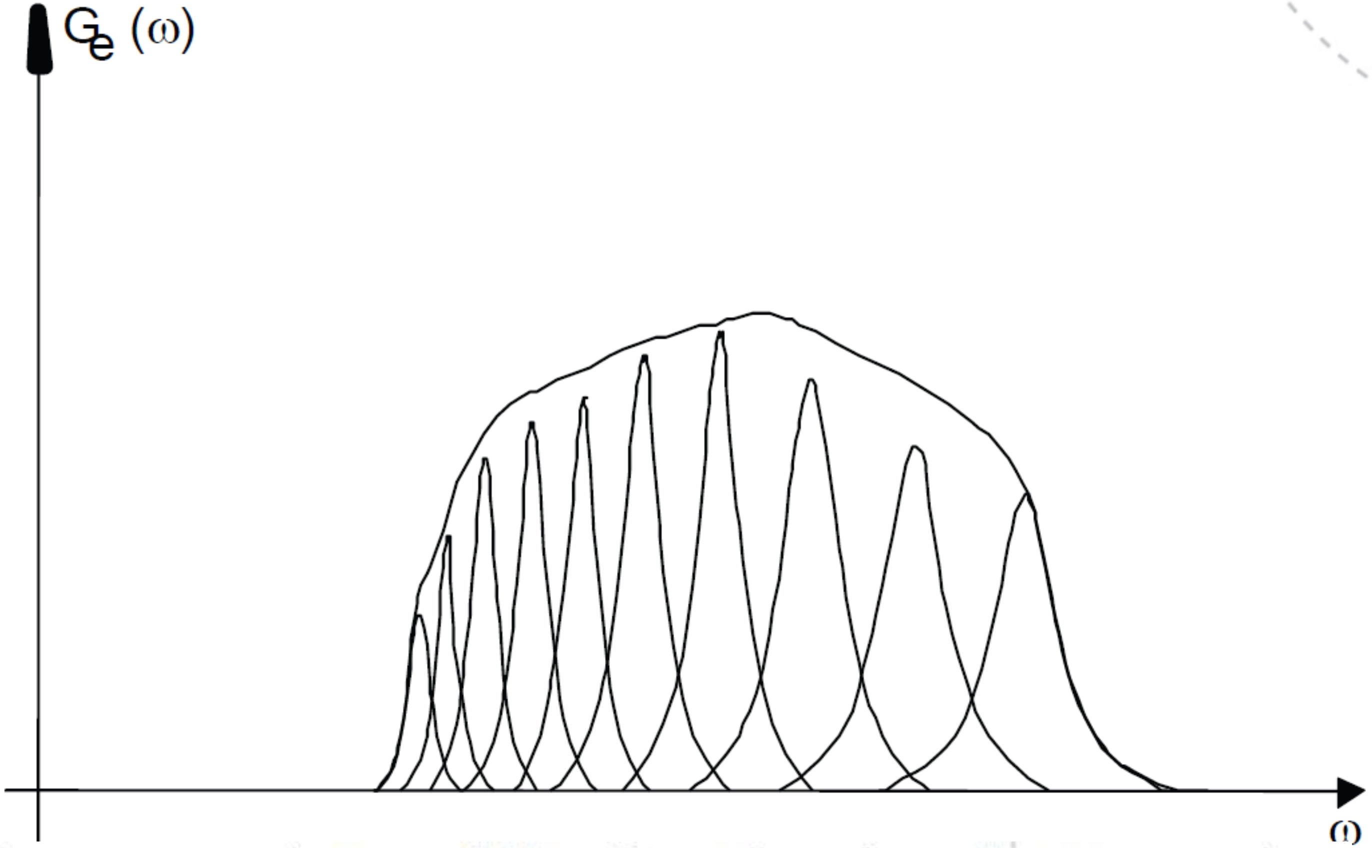


# Doppler Power Spectrum Model

- Central Limit theorem
  - Signals from RBCs add up to Gaussian distribution

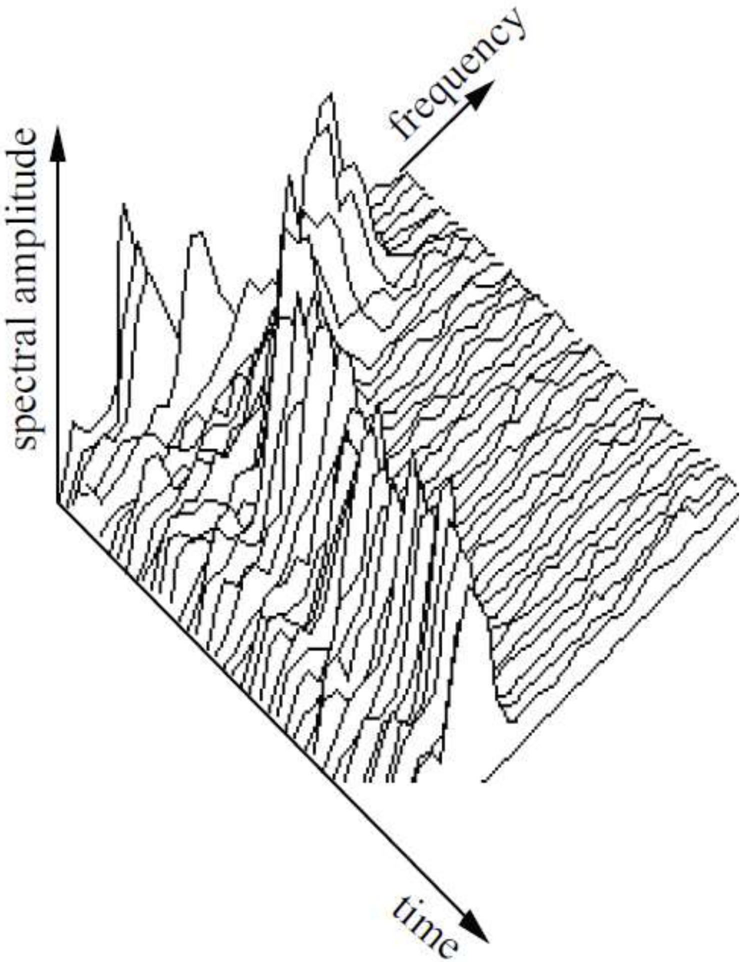


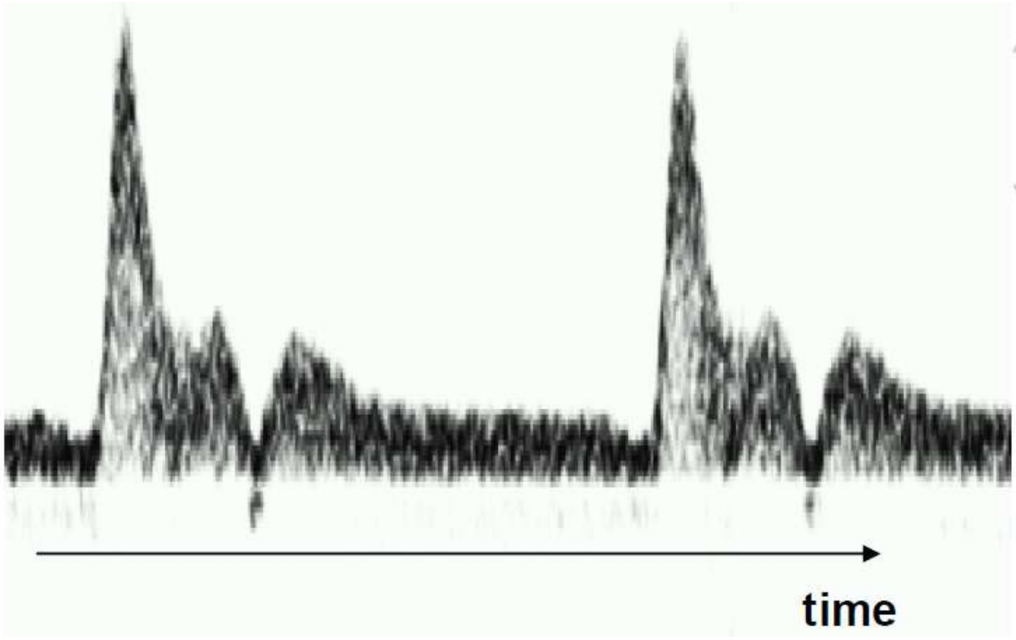
# Doppler Power Spectrum Model



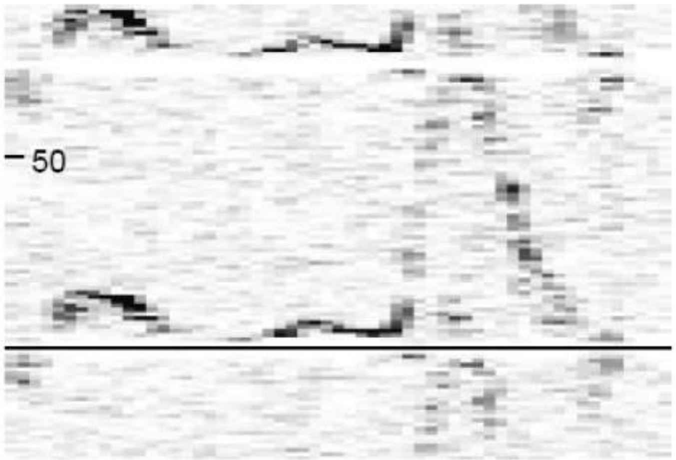

Power spectrum of the Doppler signal represents the distribution of velocities within the blood vessel

# Doppler Spectrogram

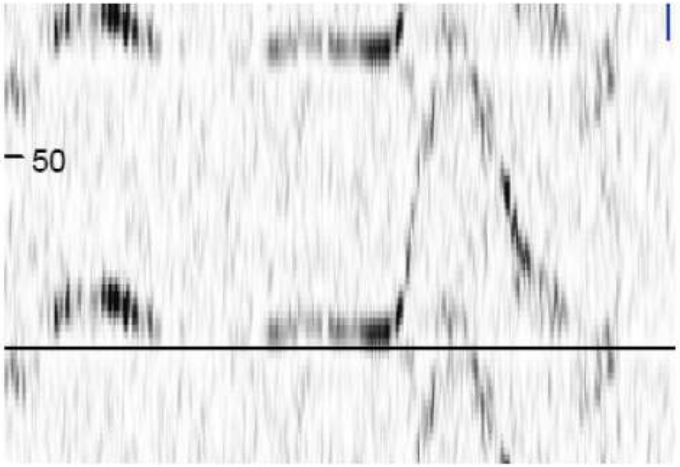


# Frequency vs. Time Resolution Trade-Off



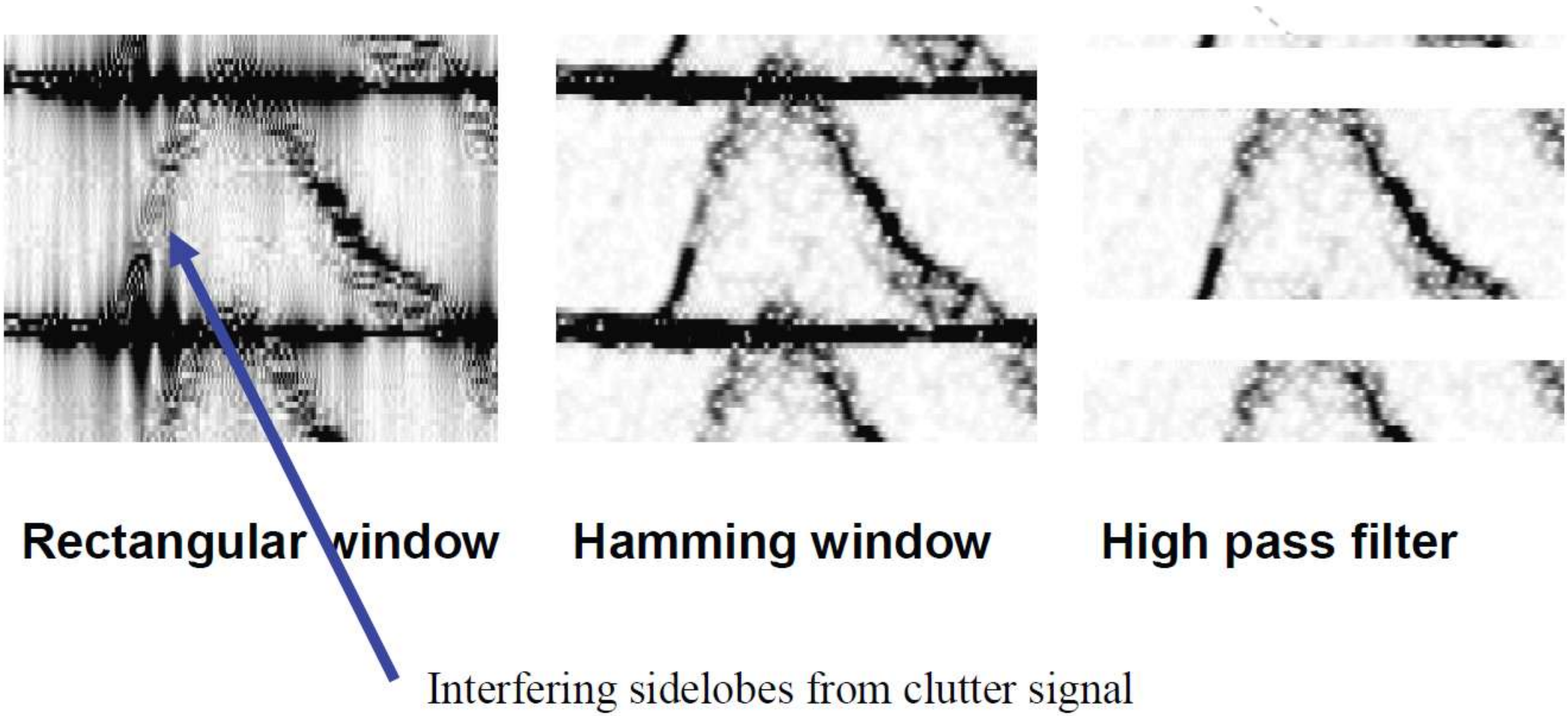

Window length 64



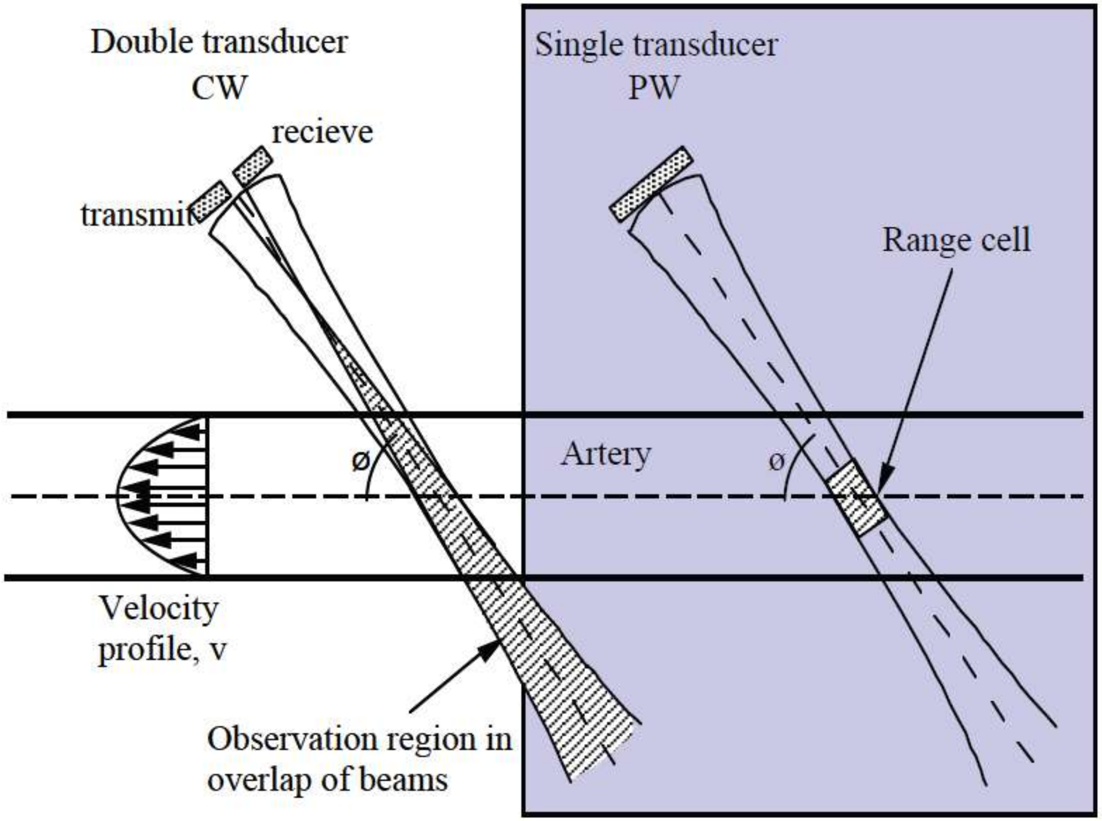

Window length 16

Time-bandwidth product: $\Delta f_d * \Delta T = 1$

# Clutter Noise



# Pulsed Wave (PW) Doppler



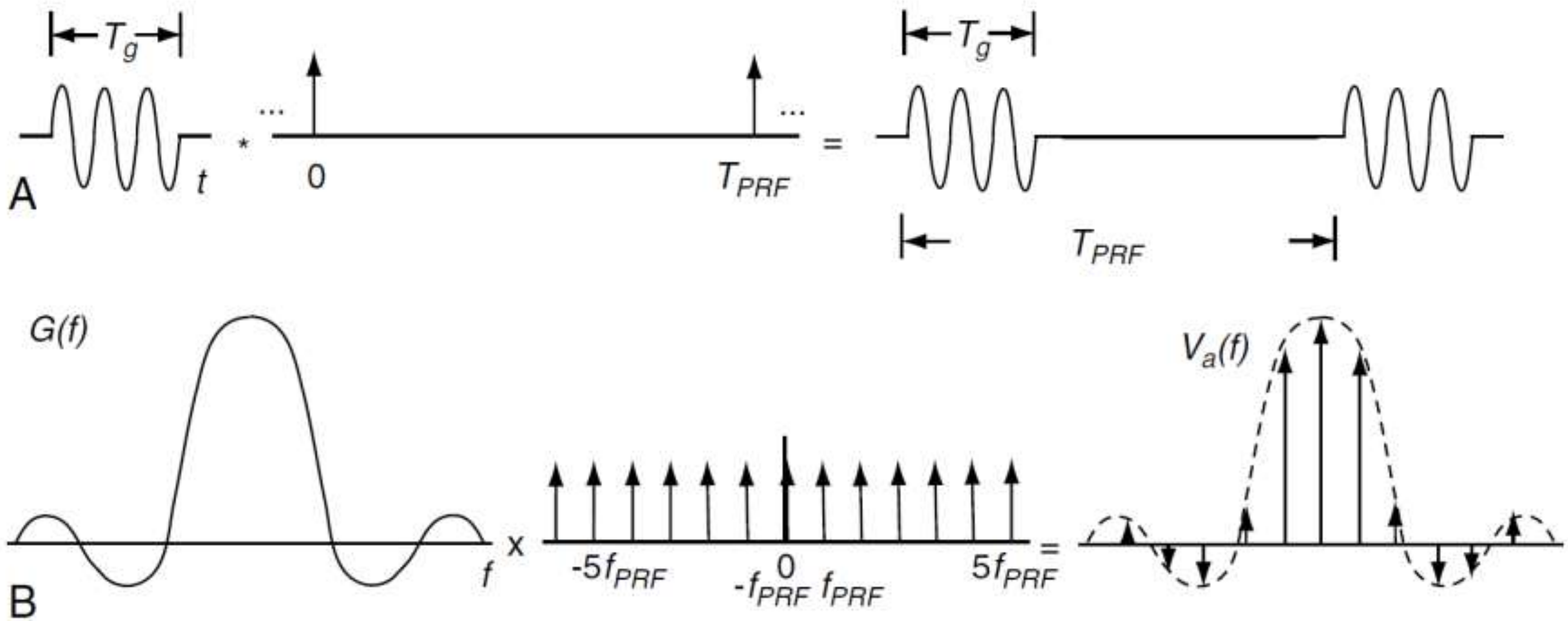

Sample volume length = cT/2

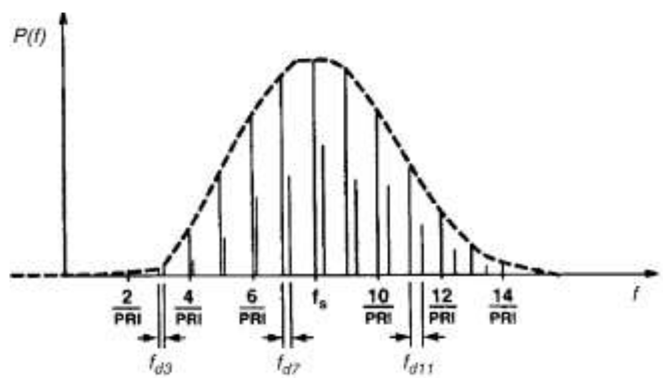
T = pulse length

Signal from all scatterers within the ultrasound beam

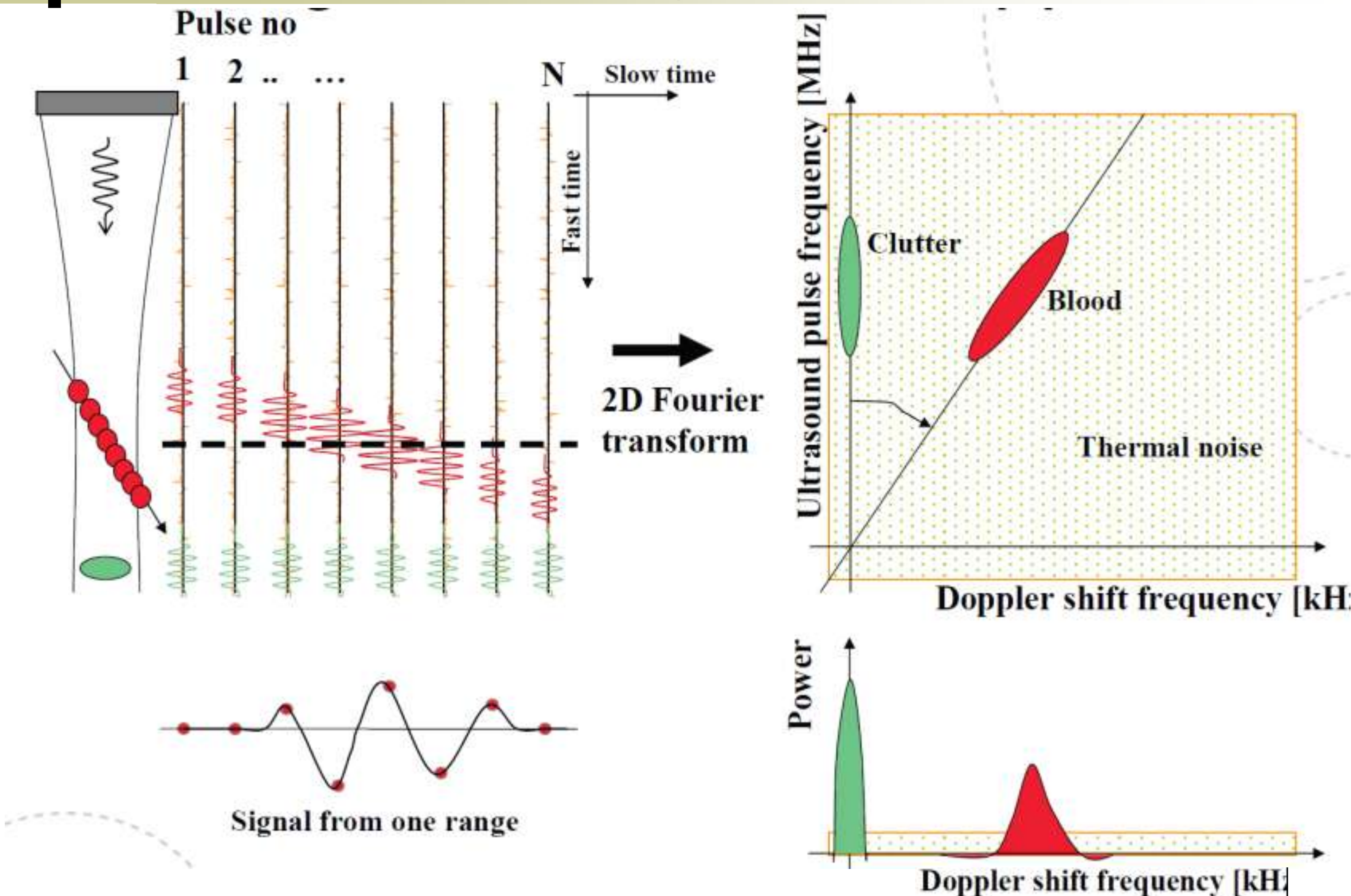
Signal from a limited sample volume

# PW Doppler Signal Model

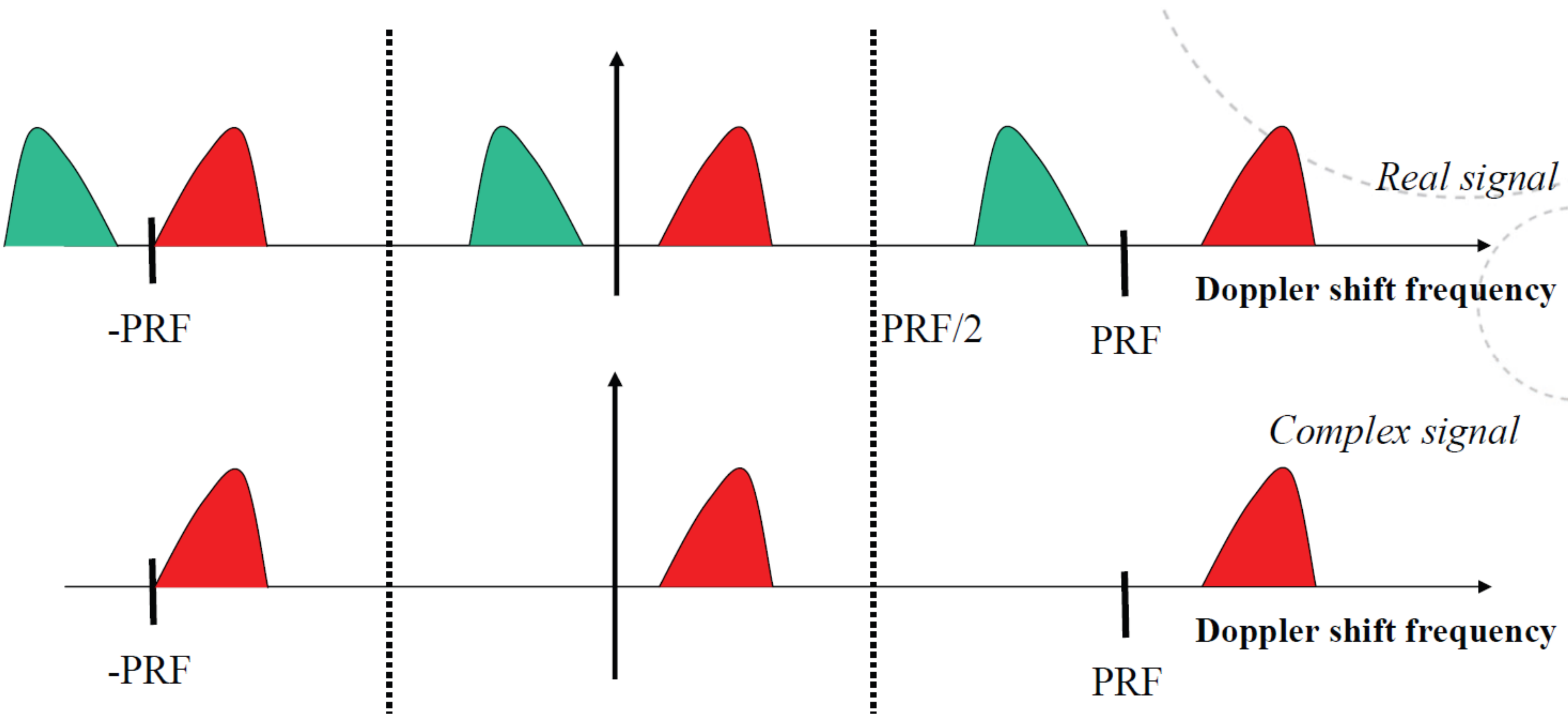


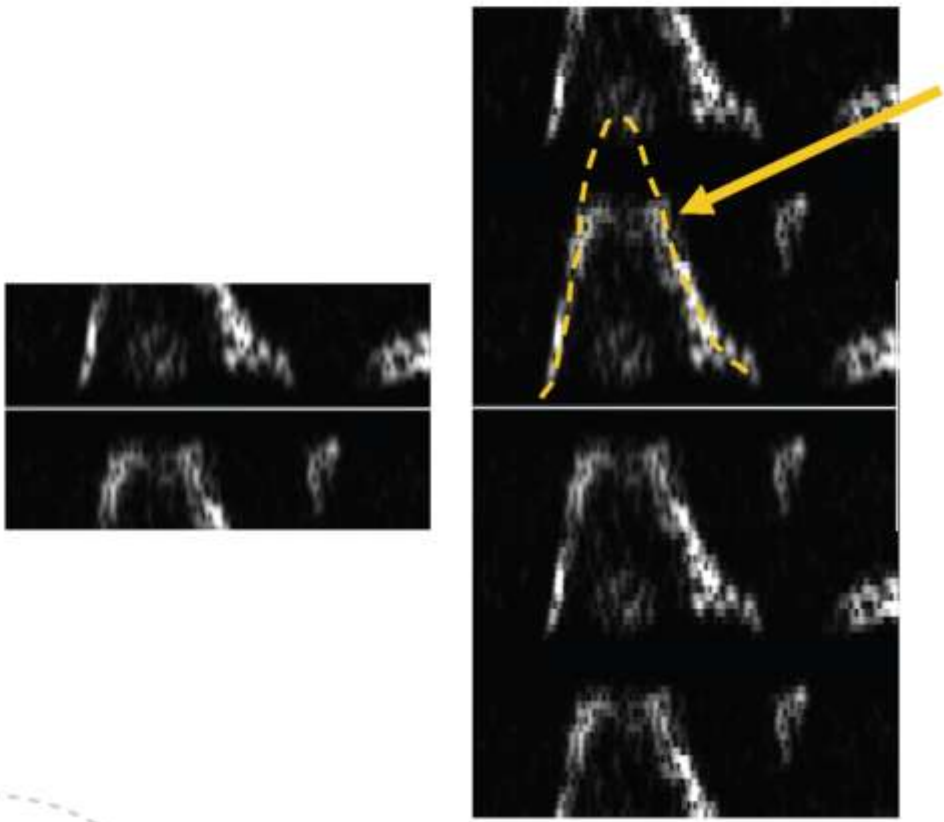
# PW Doppler 2D Signal Model



# Aliasing in PW Doppler



# Aliasing Example: Subclavian Artery



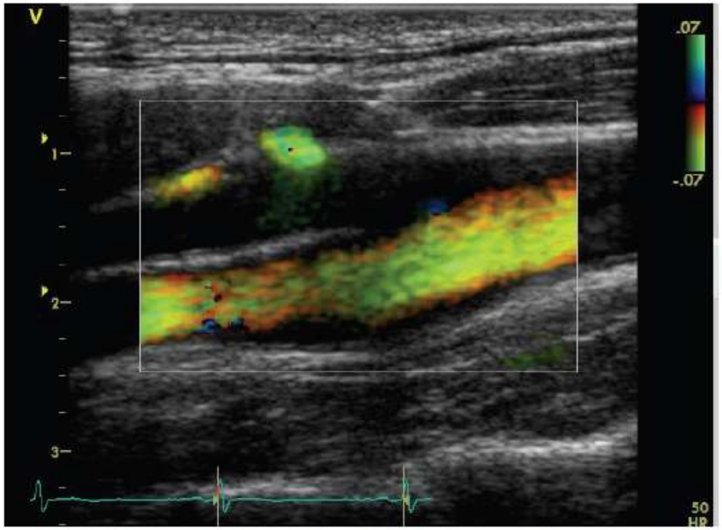
Velocity waveform restored by stacking

# Comparison of Doppler Technologies

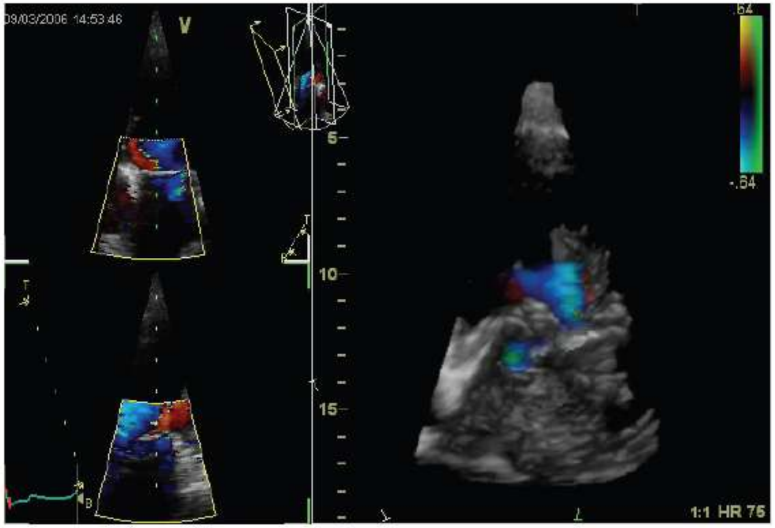
| Topic | Fixed CW Doppler | Steerable CW Doppler | PW Doppler |
|---|---|---|---|
| Resolution | At intersection of transmit and receive | At intersection of transmit and receive | Range-gated |
| Focusing | Fixed focus and amplitude | Electronic focusing with gain | Electronic focusing with gain |
| Steering | Mechanical | Electronic | Electronic |
| Visual aid for placement | None | Line, duplex, triplex imaging | Gate, duplex, triplex imaging |
| Aliasing | No | No | Yes |
| Absorption and diffraction | At intersection of transmit and receive | At intersection of transmit and receive | At gate position |

# Color Flow Imaging (CFI)

- Displays a *color coded map* of the *axial* blood velocity in a 2-D or 3-D region of interest
- Used in a wide range of diagnostic contexts in today's hospitals



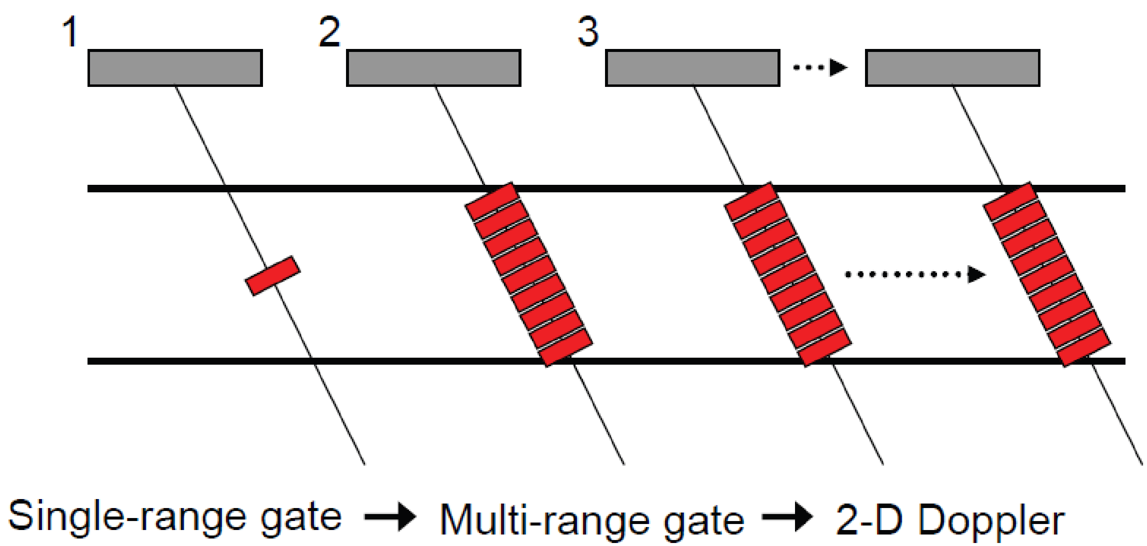
2-D CFI of a carotid bifurcation



3-D CFI of a mitral regurgitation jet

# CFI Technology History

- Technology and research progressed from single-range gate to multi-range gated Doppler, and further to 2-D Doppler imaging



# Color Flow Imaging (CFI) Chain

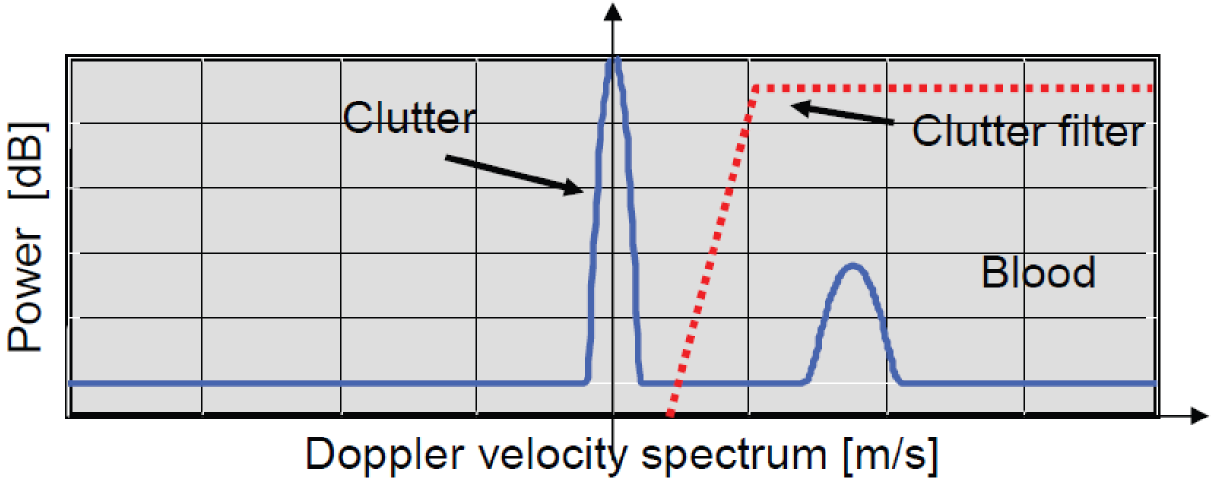
**Processing chain of conventional CFI**

Data acquisition → Clutter filtering → Parameter estimation → Display

- CFI data acquisition
  - Scanning operation and pulse sequence
- Clutter filtering (wall filtering)
  - Attenuating interfering signal from stationary tissue
- Doppler parameter estimation
  - Estimation of Doppler power, mean-frequency, and bandwidth
- Display
  - Color encoding of Doppler parameters

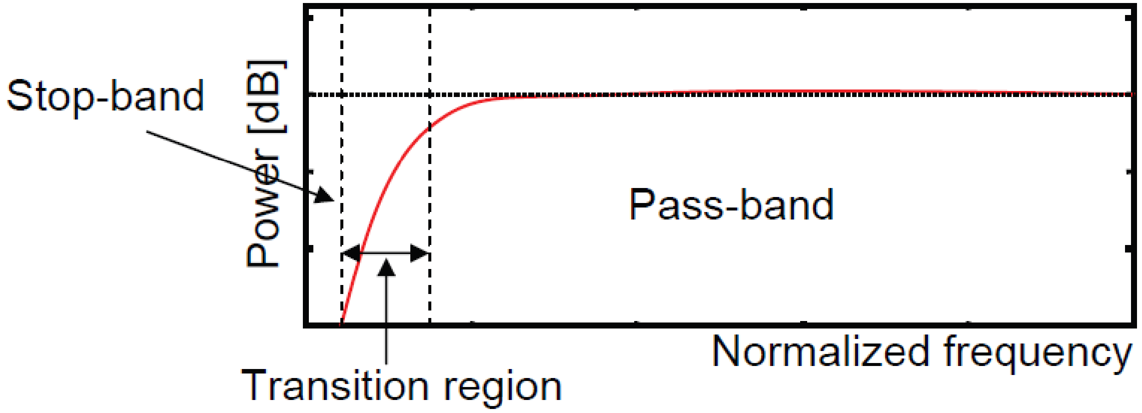
# Clutter Filtering

- Clutter is signal from surrounding tissue due to beam side lobes and reverberations
  - Can have 60-80 dB higher signal power than blood
- Blood typically has a higher velocity than tissue, i.e. higher Doppler shifts
  - The two components can thus be separated by high-pass filtering the Doppler signal

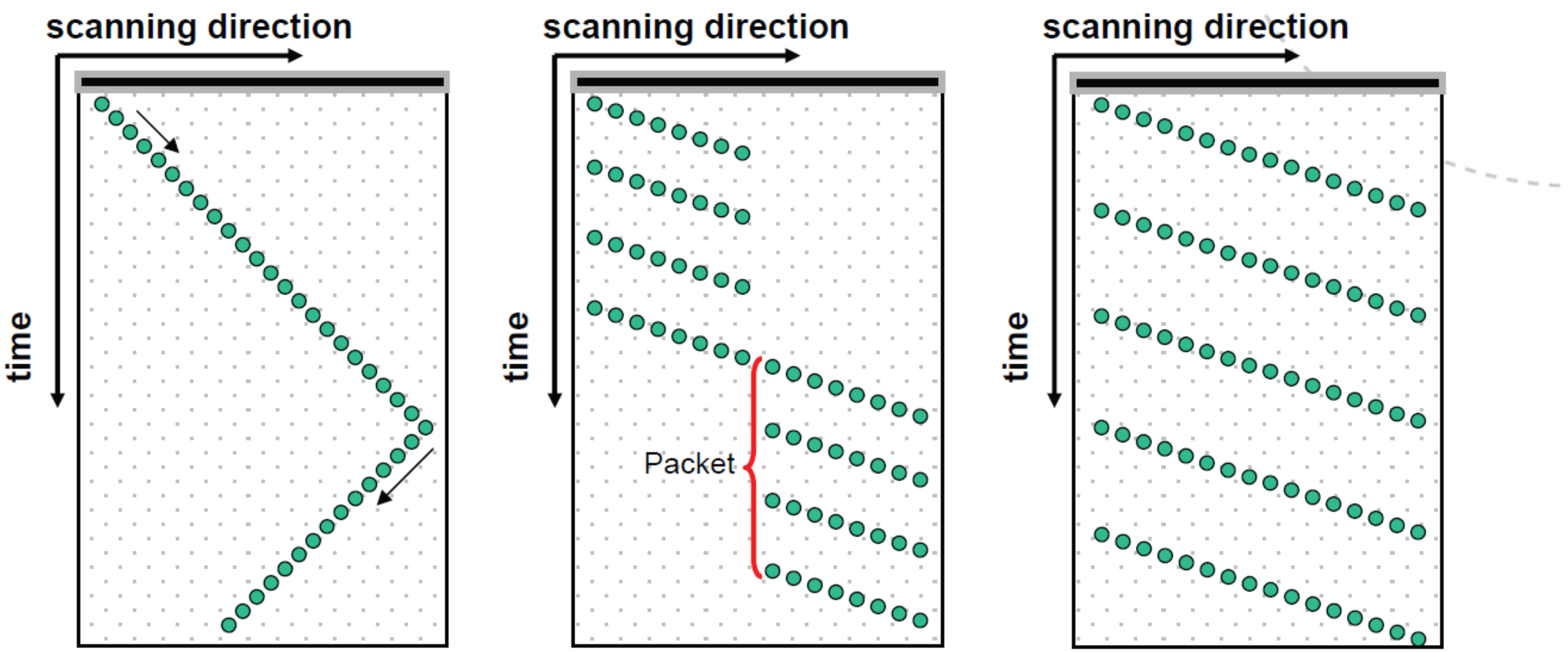


# Clutter Filter Design

- Clutter filters should have a *high stop-band attenuation* (60-80 dB) to sufficiently attenuate clutter
- Clutter filters should have a *short transition region* to avoid removing signal from blood



# Data Acquisition in CFI

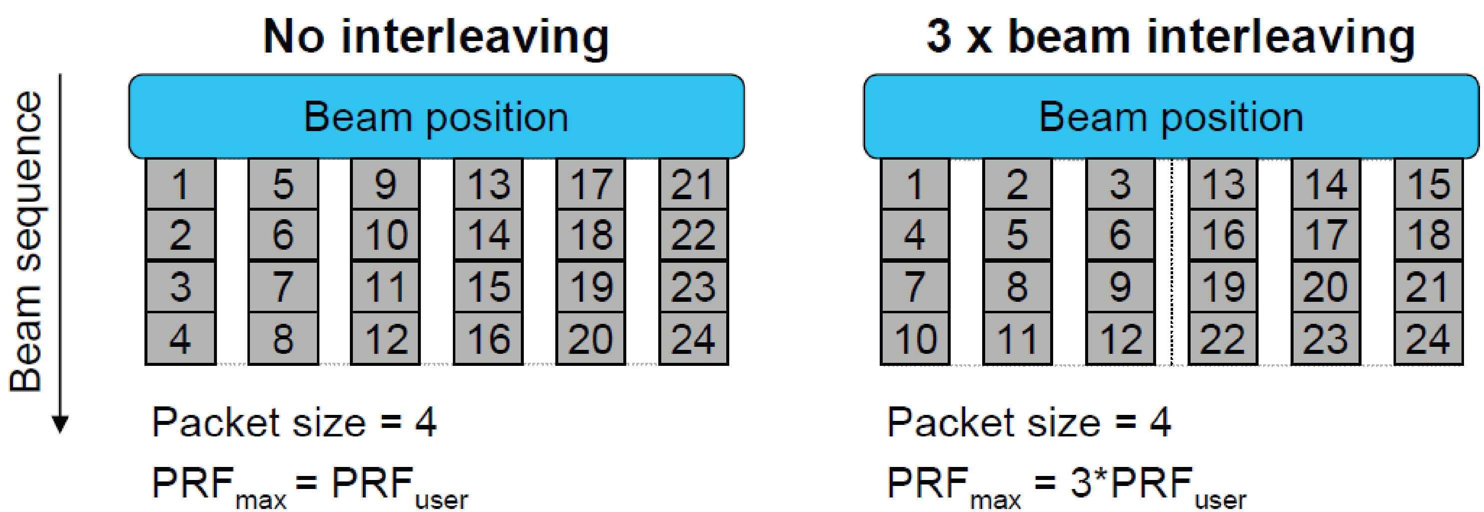



**Mechanical scanning**
+ No settling time for clutter filter
- Low frame rate

**Electronic packet scanning**
- Settling time for clutter filter
+ Flexible PRF without loss in frame rate

**Electronic continuous scanning**
+ No settling time for clutter filter
- High frame rate, but low PRF

# CFI Interleaved Packet Acquisition




- Overall frame rate increased by the interleave factor
- Maximum PRF limited by image depth, multiple reflections, hardware
- Especially important when imaging low-velocity flow, i.e. with a low user chosen PRF such as peripheral vascular imaging

# Packet / Ensemble Size in CFI

- Increasing the packet size in CFI will:
  - \+ Lead to more efficient clutter filtering
  - \+ Lower the variance of the Doppler parameters
  - – Reduce the overall frame rate dramatically
  - – May lead to visible artificial lags of the flow field in the image from one side of the image to the other
- How many samples are necessary?
  - Application dependent, typically 8-16 samples
  - Cardiac imaging (deep, high dynamics): packet size = 8-10
  - Vascular imaging (shallow, lower dynamics): packet size = 10-16
  - Abdominal imaging (deep, lower dynamics): packet size = 10-12

# SNR Considerations

*Signal-to-noise is one of the most crucial design criteria in Doppler acquisition*

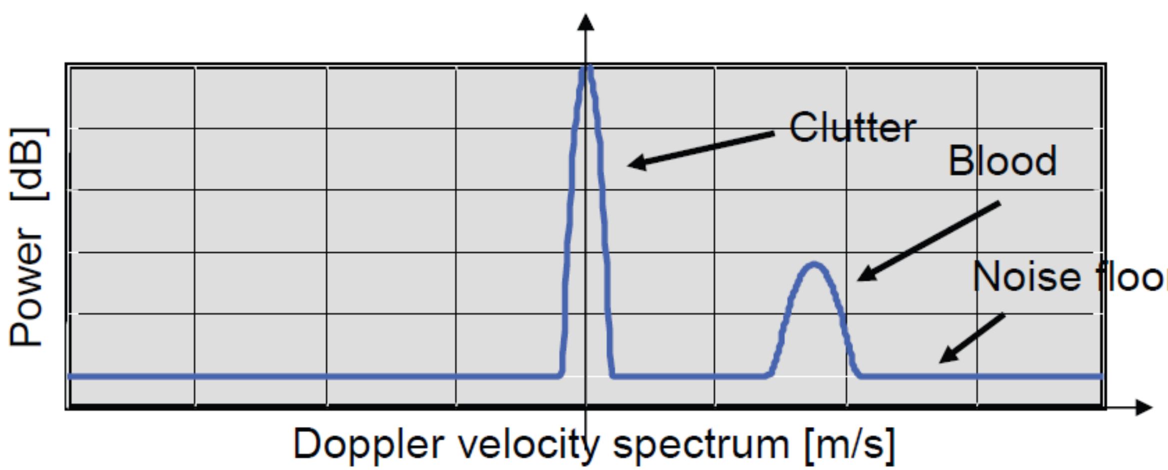
- Similar as for PW-Doppler: total SNR ~ 1/B, spectral SNR ~ $1/B^2$
- The received signal from blood is proportional to the length of the transmitted pulse (incoherent sum of burstlets)
  - Increase the number of pulse cycles
  - Decrease the pulse center frequency
- The optimal receive filter is approximatively given by a rectangular filter with length equal to the emitted pulse
  - Boxcar integrator over the pulse length
- Optimize TGC to achieve a constant noise floor throughout depth

# Doppler Signal Model

- The Doppler spectrum may consists of three components, clutter *c*, blood *b*, and thermal noise *n*

$$\mathbf{x} = \mathbf{c} + \mathbf{n} + \mathbf{b} \qquad \mathbf{x} = [x(1),\ldots,x(N)]^T$$

- Typical clutter/signal level: 20 – 80 dB
- Signal from blood is characterized by a complex Gaussian process



# Doppler Parameter Estimation

Doppler parameter estimation in CFI has focused on the first three moments of the Doppler spectrum, which equals the mean power, mean frequency, and bandwidth (rms):

$$P = \int_{-\infty}^{\infty} G(\omega) d\omega \qquad \overline{\omega} = \frac{\int_{-\infty}^{\infty} \omega \cdot G(\omega) d\omega}{\int_{-\infty}^{\infty} G(\omega) d\omega} \qquad B_{rms}^2 = \frac{\int_{-\infty}^{\infty} (\omega - \overline{\omega})^2 \cdot G(\omega) d\omega}{\int_{-\infty}^{\infty} G(\omega) d\omega}$$

**However**: estimating the Doppler power spectrum and integrating is not a practical solution.

Time (phase) domain approaches has several qualities

- Less computationally expensive
- Robust in low signal-to-noise ratios
- Velocity range covering the full Nyquist spectrum width

# Time Domain Formulation

The **Wiener-Khinchin** formula relates the autocorrelation function and the power spectral density function:

$$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{j\omega\tau}d\omega$$

Derivatives with respect to tau gives:

$$\dot{R}(\tau) = \frac{j}{2\pi}\int_{-\infty}^{\infty} \omega G(\omega)e^{j\omega\tau}d\omega \qquad \ddot{R}(\tau) = \frac{-1}{2\pi}\int_{-\infty}^{\infty} \omega^2 G(\omega)e^{j\omega\tau}d\omega$$

Yields time-domain expressions for power, mean frequency, and bandwidth (rms):

$$P = R(0) \qquad \overline{\omega} = -j\frac{\dot{R}(0)}{R(0)} \qquad B^2 = \left[\frac{\dot{R}(0)}{R(0)}\right]^2 - \frac{\ddot{R}(0)}{R(0)}$$

# Autocorrelation Method

**However**: Accurate estimates of the derivatives of the autocorrelation function can be difficult to achieve. Therefore an alternative formulation is used:

Correlation function in polar form: $R(\tau) = A(\tau)\exp\left[j\phi(\tau)\right]$

Yields the following mean frequency and bandwidth estimate:

$$\bar{\omega} = -j\frac{\dot{R}(0)}{R(0)} = \dot{\phi}(0) \qquad \bar{\omega} \cong \frac{\phi(T_{PRF}) - \phi(0)}{T_{PRF}} = \boxed{\frac{1}{T_{PRF}}\arg\left[R(T_{PRF})\right]}$$

$$B^2 = -\frac{\ddot{A}(0)}{A(0)} \approx \frac{2}{T_{PRF}{}^2}\left[1 - \frac{A(T_{PRF})}{A(0)}\right] = \boxed{\frac{2}{T_{PRF}{}^2}\left[1 - \frac{\left|R(T_{PRF})\right|}{R(0)}\right]}$$

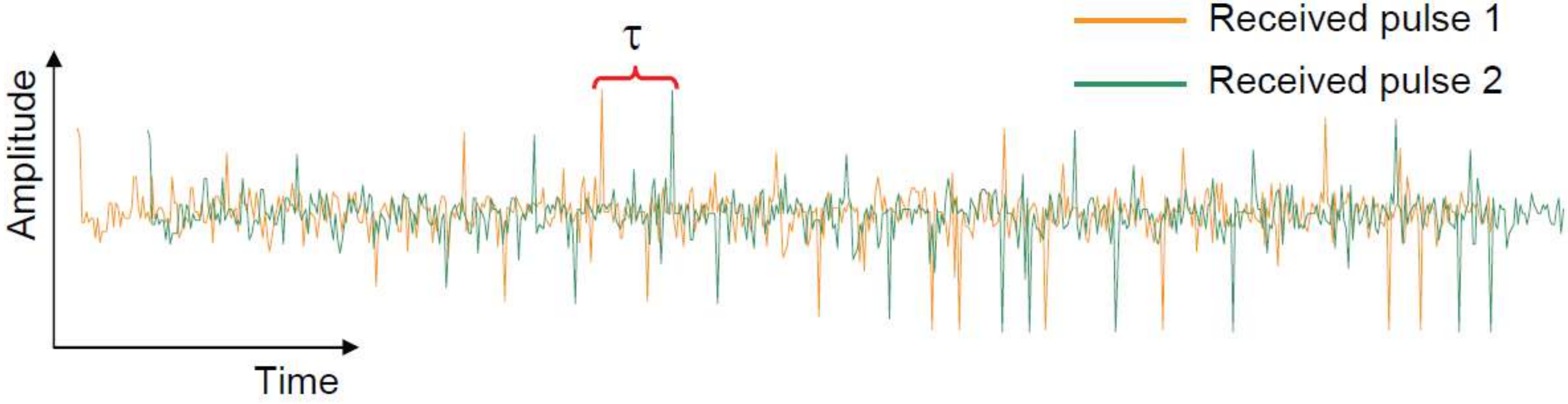
**In other words:** The power, mean frequency and bandwidth of the Doppler spectrum can be found using magnitude and phase estimates of the correlation function at lags 0 and 1 ($T_{PRF}$)

# Properties of Autocorrelation Method

- Robust in low signal-to-noise environments
  - Superior to FFT-based method below ~15 dB, similar above ~15dB
- Computationally inexpensive
  - Ideally, in a noise free environment only two complex samples are needed to estimate the mean frequency
  - In practice more samples are needed to 1) attenuate clutter, and 2) reduce the variance of the correlation estimates

# Cross-Correlation Method

- The velocity is proportional to the RF time shift between successive pulses



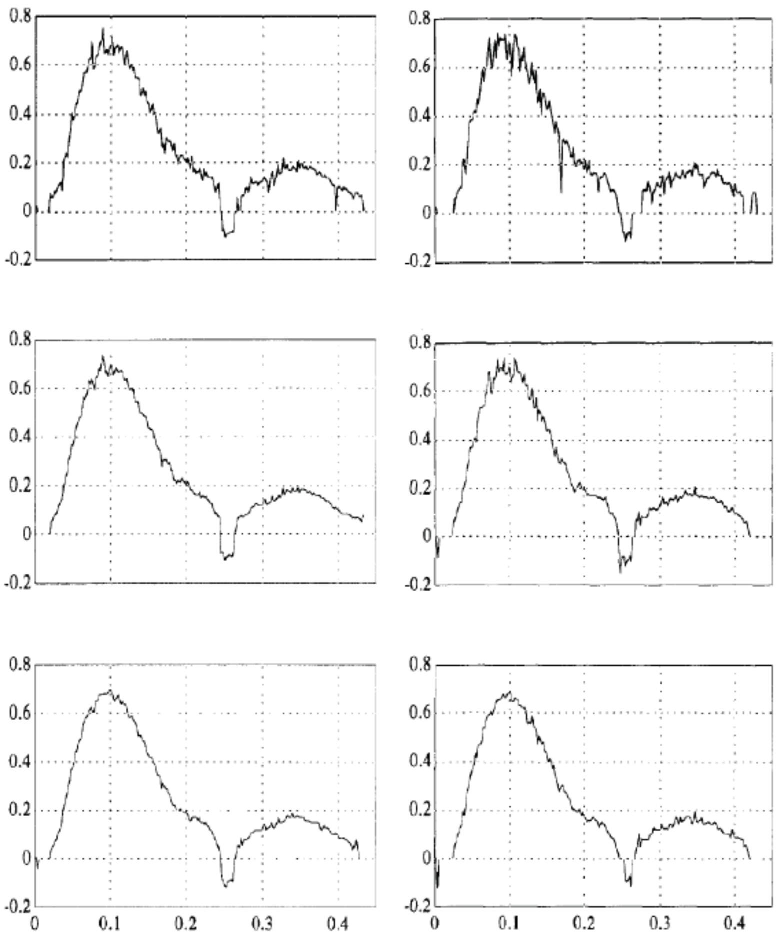

$$\tau = \frac{2\Delta z}{c} = \frac{2v\cos(\theta)T_{PRI}}{c} \qquad \hat{R}_{12}(m, m_0) = \frac{1}{N_S}\sum_{k=0}^{N_S-1} r_1(m_0+k)\,r_2(m_0+k+m)$$

$$\hat{\tau}_{max} = \arg\max \hat{R}_{12}(m)/F_s \qquad \hat{v}_z = \frac{c}{2}\frac{\hat{\tau}_{max}}{T_{PRI}}$$

# Properties of Cross-Correlation Method

- No aliasing under ideal circumstances
  - Signal decorrelation and lateral movement will limit this however
- Best performance for wide-band pulses
  - Higher resolution / lower penetration
- Axial sampling determines jitter error
  - Interpolation needed to find the true correlation maximum
- Time shift does not directly transfer to *mean-velocity*
- Computationally expensive compared to autocorrelation method

# Autocorrelation vs. Cross-Correlation




Narrow band

Wide band

Autocorr.
No averaging

Auto corr.
1.6us averaging

Cross corr.
1.6us averaging

**Example:**
*In vivo* Comparison of autocorrelation and cross-correlation for data from the human subclavian artery

*The two methods are approximatively equal for narrow-band pulses and with radial averaging*

# Doppler Velocity Estimation Limitations

- Only the axial velocity component is measured
  - Measured velocities depend on the angle between the ultrasound beam and vessel of interest →*angle-dependency artifacts*
  - For transverse flow, the mean Doppler shift is centered around zero, and a substantial part is removed by the clutter filter
- Aliasing artifacts
  - A limited velocity range can be measured before *aliasing* occurs, determined by the PRF
  - Using a higher PRF increases the maximum limit, but sensitivity to low velocities is then reduced (in practice due to clutter filter)
- Bias / variance
  - The bias and variance can in practice be quite high, and varies in space and time → the use of CFI has mainly been of qualitative nature

# Patient Safety in Doppler Ultrasound

- Potential hazardous heating and mechanical effects restrict the allowed acoustic output of ultrasonic imaging equipment
- Acoustic output is restricted by one of the following:
  - Mechanical index (MI), a measure of mechanical effects
  - Spatial peak temporal average intensity (Ispta), total output power, or thermal index (TI)
  - Transducer surface temperature
- Heating effects are averaged for combinations of modes (duplex / triplex modes). Mechanical effects are determined by the modality with the highest value.

**The outcome:** The sensitivity / penetration in Doppler modes may be severely punished from these restrictions

# Next Lecture

- New Ultrasound Imaging System Design Trends